Coherent backscattering in the Fock space of disordered Bose- and Fermi-Hubbard systems

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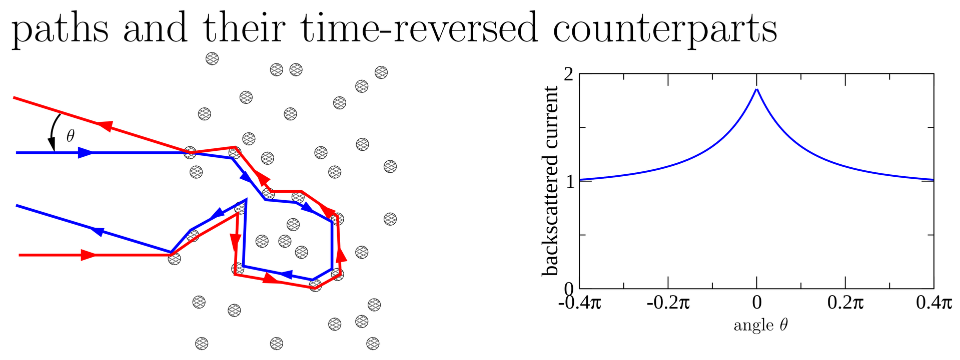
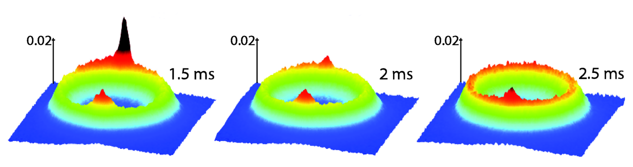
We present numerical evidence for the manifestation of coherent backscattering within the Fock space of disordered Bose- and Fermi-Hubbard systems. Preparing a Bose-Hubbard system in a Fock state and letting it evolve for a sufficiently long time will give rise to an enhancement of the detection probability of this state as compared to other Fock states with similar total energy. This constitutes a significant departure from the principle of ergodicity in the microcanonical context. In spin 1/2 Fermi-Hubbard systems with Rashba hopping terms, coherent backscattering gives rise to spin echoes on the initial Fock state and its spin-flipped counterpart.

Coherent backscattering

→ constructive wave interference between reflected classical paths and their time-reversed counterparts



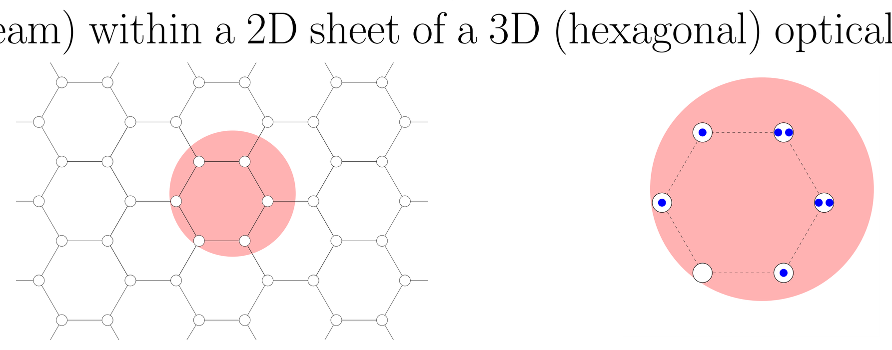
- first observation with laser light
  M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985);
  P.-E. Wolf and G. Maret, PRL 55, 2696 (1985)
- recent verification with Bose-Einstein condensates
  F. Jendrzejewski *et al.*, PRL 109, 195302 (2012)



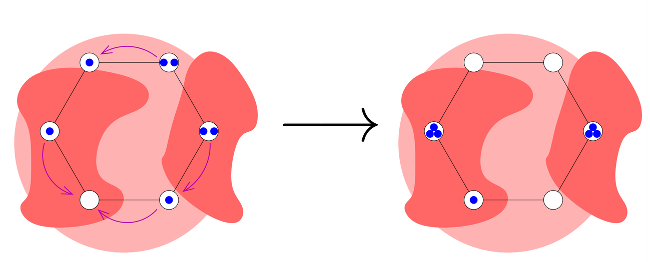
- inversion in the presence of nonlinearity
  M. Hartung *et al.*, PRL 101, 020603 (2008)

Proposed experimental procedure

1. Isolate a single plaquette (by a focused red-detuned laser beam) within a 2D sheet of a 3D (hexagonal) optical lattice



2. Load the lattice with a well-defined number of atoms in the deep Mott-insulator regime
3. Add disorder (by means of an optical speckle field) and randomly displace the focus of the red-detuned laser beam
4. Switch on the inter-site hopping and let the atoms move



5. quench back to the Mott regime after a given evolution time and detect the atomic population on each site
   W. Bakr *et al.*, Nature 462, 74 (2009)
   J. Sherson *et al.*, Nature 467, 68 (2010)
   S. Fölling *et al.*, Nature 448, 1029 (2007)
6. Repeat the experiment with the same initial state but for a different disorder configuration

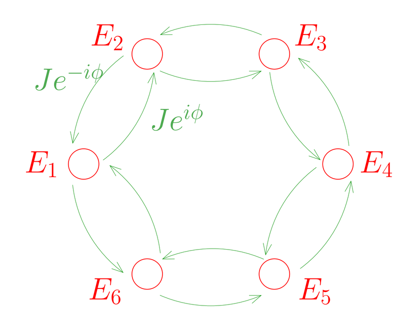
→ the initial state is twice as often detected as other Fock states with comparable total energy

Spinless bosons

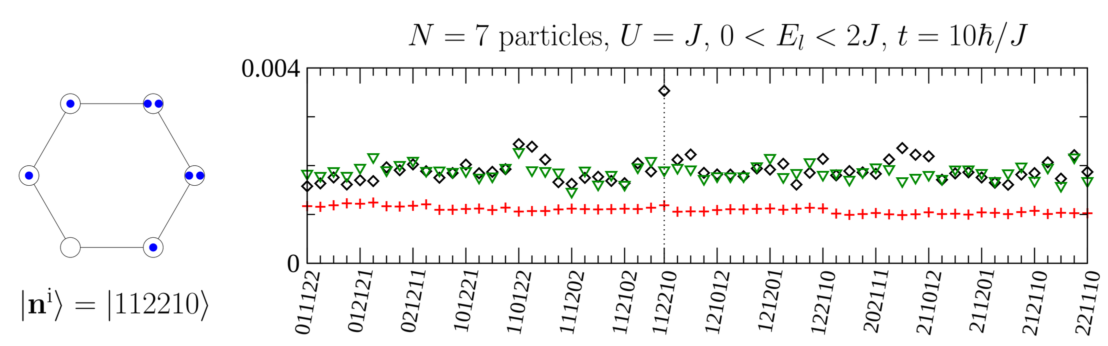
Bose-Hubbard Hamiltonian for a ring lattice of L sites:

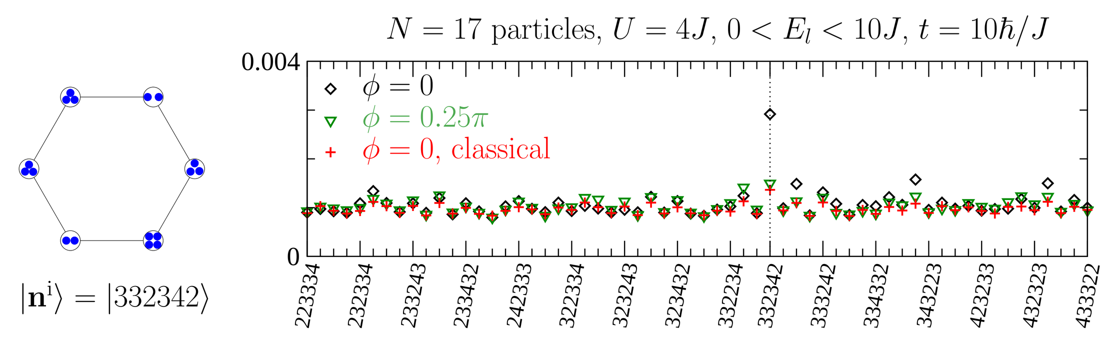
$$\hat{H} = \sum_{l=1}^{L}\left[E_l \hat{b}_l^\dagger \hat{b}_l - J\left(\hat{b}_l^\dagger \hat{b}_{l-1} e^{i\phi} + \hat{b}_{l-1}^\dagger \hat{b}_l e^{-i\phi}\right) + \frac{U}{2}\hat{b}_l^\dagger \hat{b}_l^\dagger \hat{b}_l \hat{b}_l\right]$$

E_l = (random) energy on site l
U = on-site interaction strength
J = inter-site hopping amplitude
ϕ = inter-site hopping phase induced by an artificial gauge field
J. Struck *et al.*, Science 333, 996 (2011)

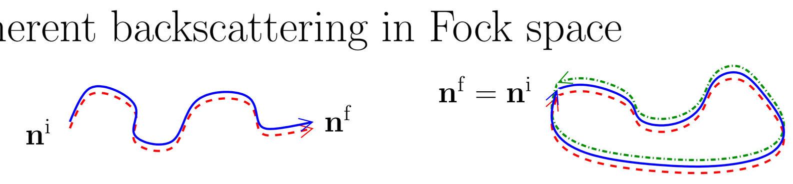


Disorder-averaged detection probability $\bar{P}(\mathbf{n}^f, \mathbf{n}^i, t)$ of the Fock state $|\mathbf{n}^f\rangle = |n_1, \cdots, n_L\rangle$ after the evolution time t:





⟶ coherent backscattering in Fock space



Semiclassical (van-Vleck Gutzwiller) prediction:

$$\bar{P}(\mathbf{n}^f, \mathbf{n}^i, t) = \begin{cases} 2\bar{P}^{cl}(\mathbf{n}^f, \mathbf{n}^i, t) : \mathbf{n}^f = \mathbf{n}^i, \phi = 0, t \gg \hbar/J \\ \bar{P}^{cl}(\mathbf{n}^f, \mathbf{n}^i, t) : \text{otherwise} \end{cases}$$

with the classical detection probabilities

$$\bar{P}^{cl}(\mathbf{n}^f, \mathbf{n}^i, t) = \prod_{l=2}^{L}\int_0^{2\pi}\frac{d\theta_l^i}{2\pi}\prod_{l=2}^{L}\delta\left(n_l^f + 0.5 - |\psi_l(t; \mathbf{n}^i, \boldsymbol{\theta}^i)|^2\right)$$

(fixing $\theta_1^i = 0$) where ψ_l evolves according to

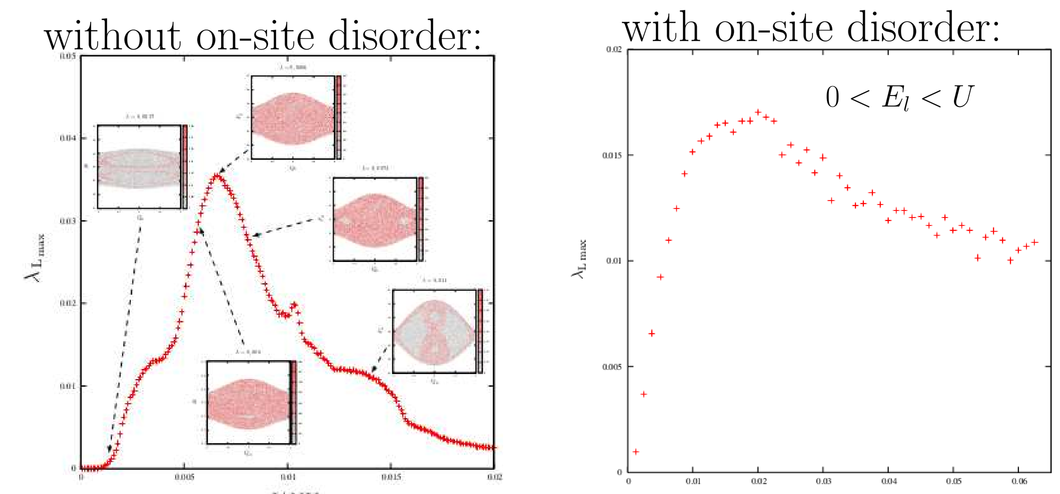
$$i\hbar\frac{\partial}{\partial t}\psi_l(t) = E_l\psi_l(t) - J\left[e^{i\phi}\psi_{l-1}(t) + e^{-i\phi}\psi_{l+1}(t)\right] + U\left(|\psi_l(t)|^2 - 1\right)\psi_l(t)$$

with the initial condition $\psi_l(0; \mathbf{n}^i, \boldsymbol{\theta}^i) = \sqrt{n_l^i + 0.5}\, e^{i\theta_l}$.

Is the classical phase space ergodic?

⟶ investigation of a 3-sites Bose-Hubbard ring: calculation of the mean Lyapunov exponent

without on-site disorder: with on-site disorder:



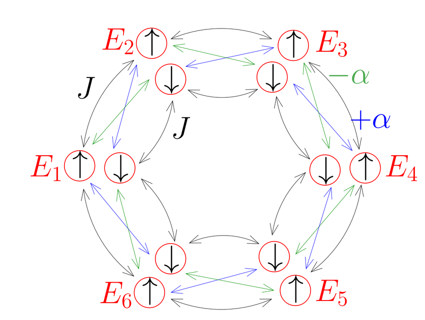
⟶ a mixed regular-chaotic phase-space structure should generally be expected for disordered Bose-Hubbard systems

Spin 1/2 fermions

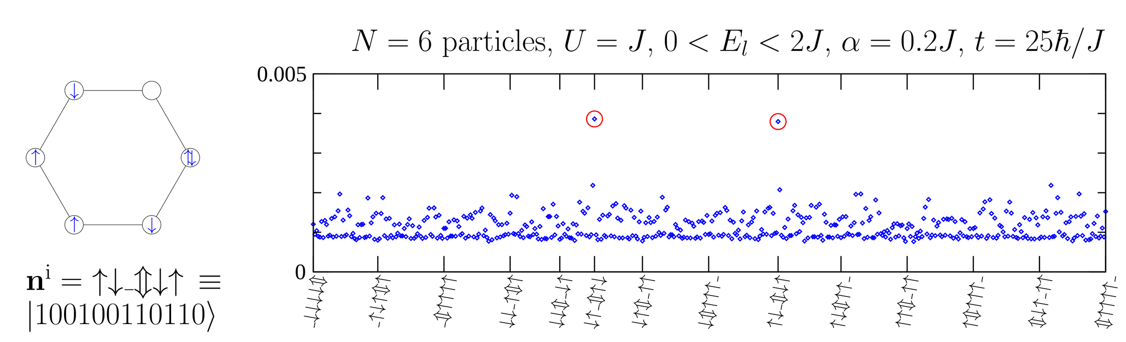
Spin 1/2 Fermi-Hubbard Hamiltonian with Rashba hopping:

$$\hat{H} = \sum_{l=1}^{L}\Bigg\{\sum_{\sigma=\uparrow,\downarrow}\left[E_l \hat{c}_{l,\sigma}^\dagger \hat{c}_{l,\sigma} - J\left(\hat{c}_{l,\sigma}^\dagger \hat{c}_{l-1,\sigma} + \hat{c}_{l-1,\sigma}^\dagger \hat{c}_{l,\sigma}\right)\right] + \alpha\left(\hat{c}_{l,\uparrow}^\dagger \hat{c}_{l-1,\downarrow} + \hat{c}_{l-1,\downarrow}^\dagger \hat{c}_{l,\uparrow} - \hat{c}_{l,\downarrow}^\dagger \hat{c}_{l-1,\uparrow} - \hat{c}_{l-1,\uparrow}^\dagger \hat{c}_{l,\downarrow}\right) + \frac{U}{2}\hat{c}_{l,\uparrow}^\dagger \hat{c}_{l,\uparrow}\hat{c}_{l,\downarrow}^\dagger \hat{c}_{l,\downarrow}\Bigg\}$$

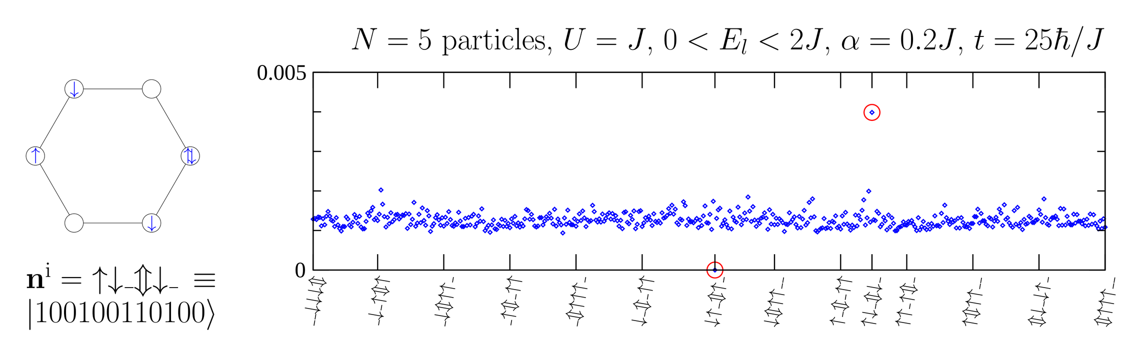
E_l = (random) energy on site l
U = on-site spin-spin interaction
J = inter-site hopping amplitude
α = inter-site Rashba hopping involving a spin flip



Disorder-averaged detection probability of the Fock state $|\mathbf{n}^f\rangle = |n_{1,\uparrow}, n_{1,\downarrow}, \cdots, n_{L,\uparrow}, n_{L,\downarrow}\rangle$ after the evolution time t:



→ enhanced detection probability for the initial state and its spin-flipped counterpart

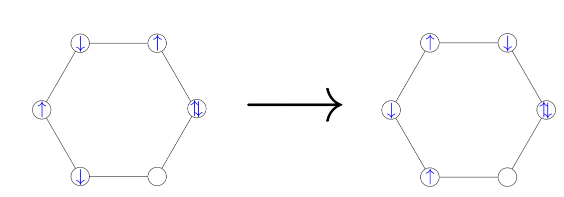


→ enhanced detection probability for the initial state

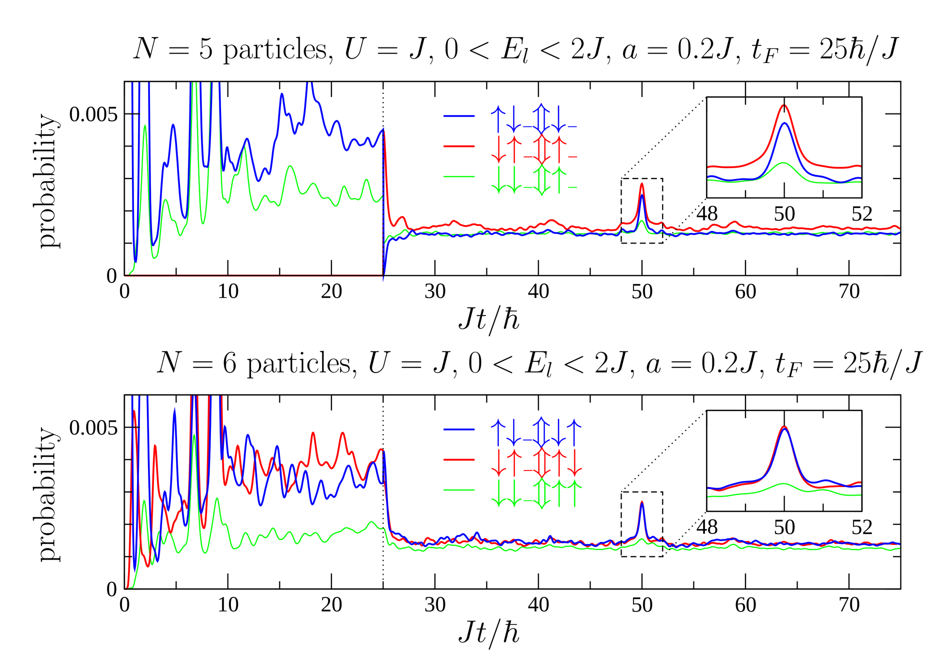
→ vanishing probability for its spin-flipped counterpart, due to a symplectic time-reversal symmetry of the Hamiltonian

Spin echo

→ flip the spins on all sites at $t = t_F$:



⟶ revival of coherent backscattering at $t = 2t_F$, both for the initial state and its spin-flipped counterpart:



T. Engl, J. Dujardin, A. Argüelles, P. Schlagheck, K. Richter, and J. D. Urbina, Phys. Rev. Lett. 112, 140403 (2014).
T. Engl, P. Plößl, J. D. Urbina, and K. Richter, Theoretical Chemistry Accounts 133, 1563 (2014).
T. Engl *et al.*, in preparation (arXiv:1409.5684).