Coherent backscattering in the Fock space of disordered Bose- and Fermi-Hubbard systems

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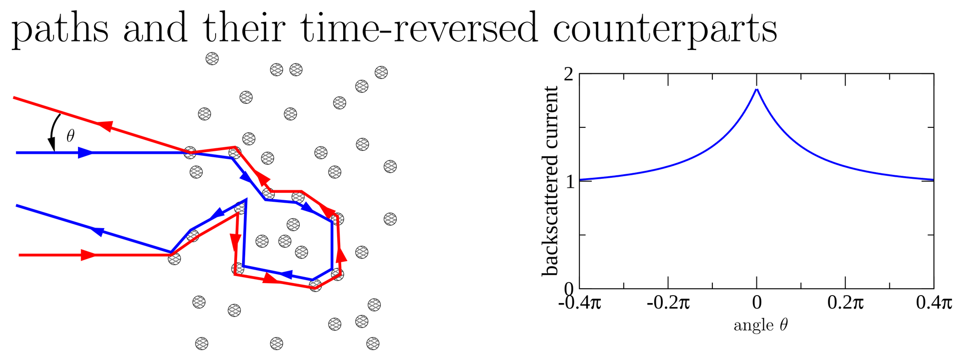
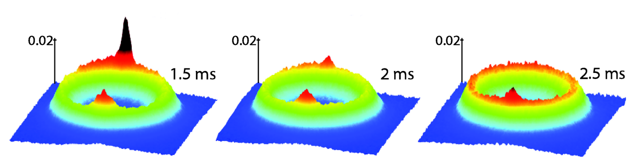
We present numerical evidence for the manifestation of coherent backscattering within the Fock space of disordered Bose- and Fermi-Hubbard systems. Preparing a Bose-Hubbard system in a Fock state and letting it evolve for a sufficiently long time will give rise to an enhancement of the detection probability of this state as compared to other Fock states with similar total energy. This constitutes a significant departure from the principle of ergodicity in the microcanonical context. In spin 1/2 Fermi-Hubbard systems with Rashba hopping terms, coherent backscattering gives rise to spin echoes on the initial Fock state and its spin-flipped counterpart.

Coherent backscattering

→ constructive wave interference between reflected classical paths and their time-reversed counterparts



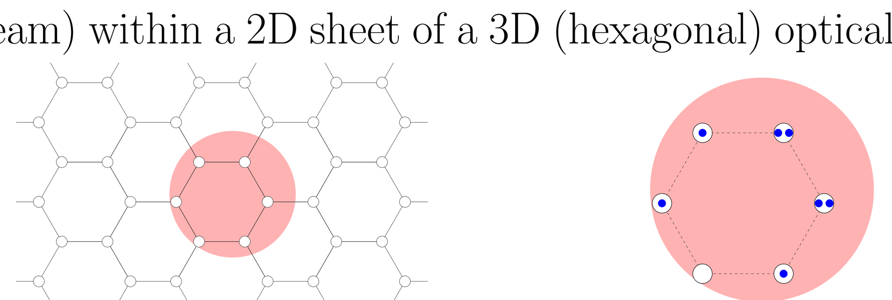
- first observation with laser light
  M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985);
  P.-E. Wolf and G. Maret, PRL 55, 2696 (1985)
- recent verification with Bose-Einstein condensates
  F. Jendrzejewski *et al.*, PRL 109, 195302 (2012)



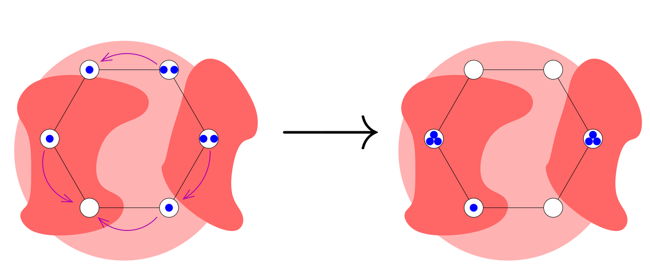
- inversion in the presence of nonlinearity
  M. Hartung *et al.*, PRL 101, 020603 (2008)

Proposed experimental procedure

1. Isolate a single plaquette (by a focused red-detuned laser beam) within a 2D sheet of a 3D (hexagonal) optical lattice



2. Load the lattice with a well-defined number of atoms in the deep Mott-insulator regime
3. Add disorder (by means of an optical speckle field) and randomly displace the focus of the red-detuned laser beam
4. Switch on the inter-site hopping and let the atoms move



5. quench back to the Mott regime after a given evolution time and detect the atomic population on each site
   W. Bakr *et al.*, Nature 462, 74 (2009)
   J. Sherson *et al.*, Nature 467, 68 (2010)
   S. Fölling *et al.*, Nature 448, 1029 (2007)
6. Repeat the experiment with the same initial state but for a different disorder configuration

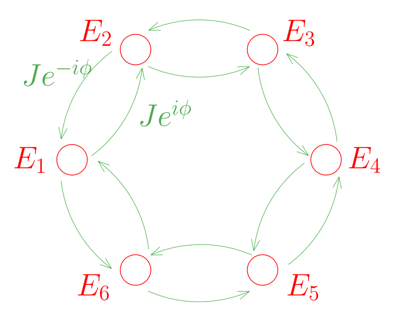
→ the initial state is twice as often detected as other Fock states with comparable total energy

Spinless bosons

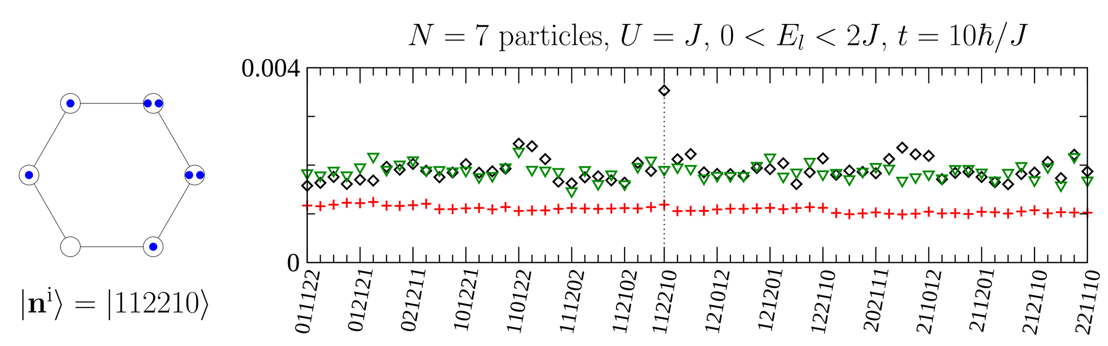
Bose-Hubbard Hamiltonian for a ring lattice of L sites:

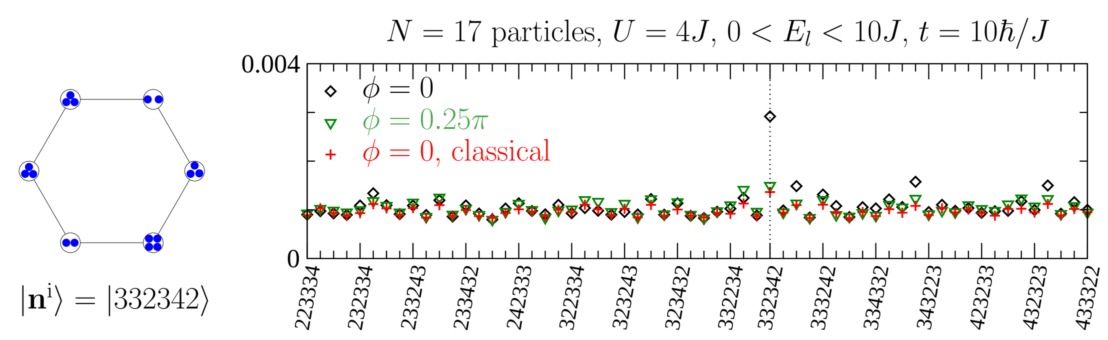
$$\hat{H} = \sum_{l=1}^{L}\left[E_l\hat{b}_l^\dagger\hat{b}_l - J\left(\hat{b}_l^\dagger\hat{b}_{l-1}e^{i\phi} + \hat{b}_{l-1}^\dagger\hat{b}_l e^{-i\phi}\right) + \frac{U}{2}\hat{b}_l^\dagger\hat{b}_l^\dagger\hat{b}_l\hat{b}_l\right]$$

E_l = (random) energy on site l
U = on-site interaction strength
J = inter-site hopping amplitude
ϕ = inter-site hopping phase induced by an artificial gauge field
J. Struck *et al.*, Science 333, 996 (2011)

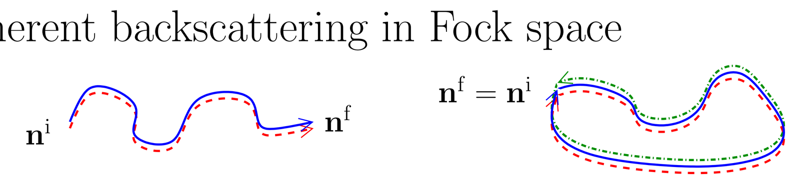


Disorder-averaged detection probability $\bar{P}(\mathbf{n}^f, \mathbf{n}^i, t)$ of the Fock state $|\mathbf{n}^f\rangle = |n_1, \cdots, n_L\rangle$ after the evolution time t:





⟶ coherent backscattering in Fock space



Semiclassical (van-Vleck Gutzwiller) prediction:

$$\bar{P}(\mathbf{n}^f, \mathbf{n}^i, t) = \begin{cases} 2\bar{P}^{cl}(\mathbf{n}^f, \mathbf{n}^i, t) : \mathbf{n}^f = \mathbf{n}^i, \phi = 0, t \gg \hbar/J \\ \bar{P}^{cl}(\mathbf{n}^f, \mathbf{n}^i, t) : \text{otherwise} \end{cases}$$

with the classical detection probabilities

$$\bar{P}^{cl}(\mathbf{n}^f, \mathbf{n}^i, t) = \prod_{l=2}^{L}\int_0^{2\pi}\frac{d\theta_l^i}{2\pi}\prod_{l=2}^{L}\delta\left(n_l^f + 0.5 - |\psi_l(t; \mathbf{n}^i, \boldsymbol{\theta}^i)|^2\right)$$

(fixing $\theta_1^i = 0$) where ψ_l evolves according to

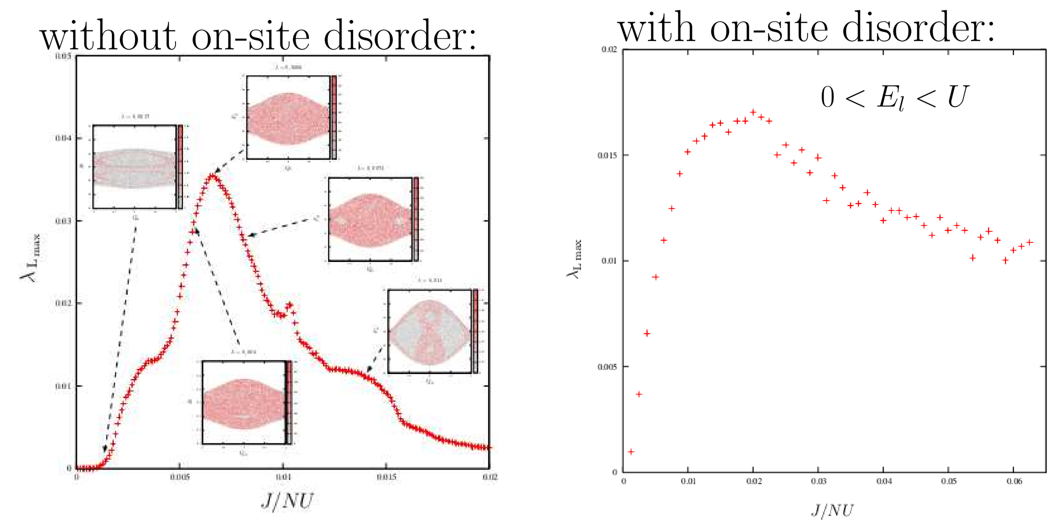
$$i\hbar\frac{\partial}{\partial t}\psi_l(t) = E_l\psi_l(t) - J\left[e^{i\phi}\psi_{l-1}(t) + e^{-i\phi}\psi_{l+1}(t)\right] + U\left(|\psi_l(t)|^2 - 1\right)\psi_l(t)$$

with the initial condition $\psi_l(0; \mathbf{n}^i, \boldsymbol{\theta}^i) = \sqrt{n_l^i + 0.5}\,e^{i\theta_l}$.

Is the classical phase space ergodic?

⟶ investigation of a 3-sites Bose-Hubbard ring: calculation of the mean Lyapunov exponent

without on-site disorder: with on-site disorder:



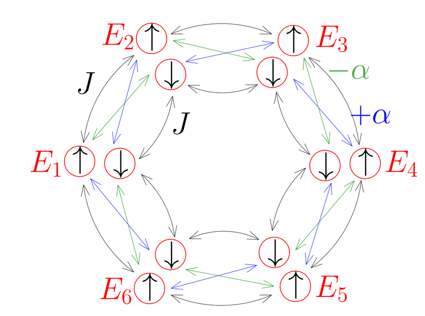
⟶ a mixed regular-chaotic phase-space structure should generally be expected for disordered Bose-Hubbard systems

Spin 1/2 fermions

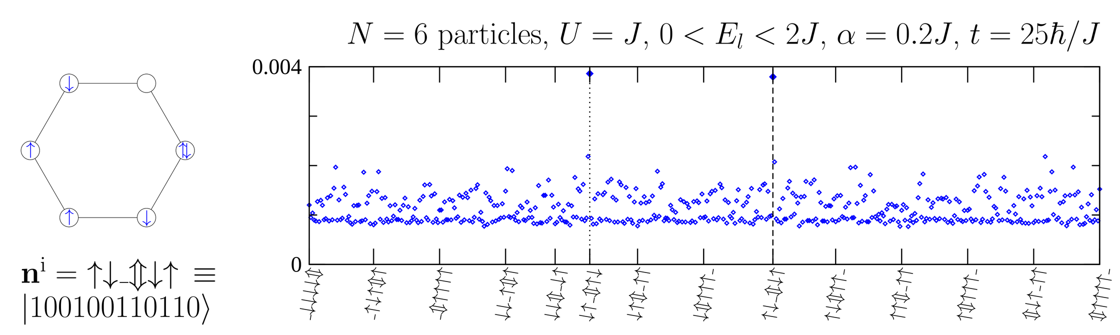
Spin 1/2 Fermi-Hubbard Hamiltonian with Rashba hopping:

$$\hat{H} = \sum_{l=1}^{L}\left\{\sum_{\sigma=\uparrow,\downarrow}\left[E_l\hat{c}_{l,\sigma}^\dagger\hat{c}_{l,\sigma} - J\left(\hat{c}_{l,\sigma}^\dagger\hat{c}_{l-1,\sigma} + \hat{c}_{l-1,\sigma}^\dagger\hat{c}_{l,\sigma}\right)\right] + \alpha\left(\hat{c}_{l,\uparrow}^\dagger\hat{c}_{l-1,\downarrow} + \hat{c}_{l-1,\downarrow}^\dagger\hat{c}_{l,\uparrow} - \hat{c}_{l,\downarrow}^\dagger\hat{c}_{l-1,\uparrow} - \hat{c}_{l-1,\uparrow}^\dagger\hat{c}_{l,\downarrow}\right) + \frac{U}{2}\hat{c}_{l,\uparrow}^\dagger\hat{c}_{l,\uparrow}\hat{c}_{l,\downarrow}^\dagger\hat{c}_{l,\downarrow}\right\}$$

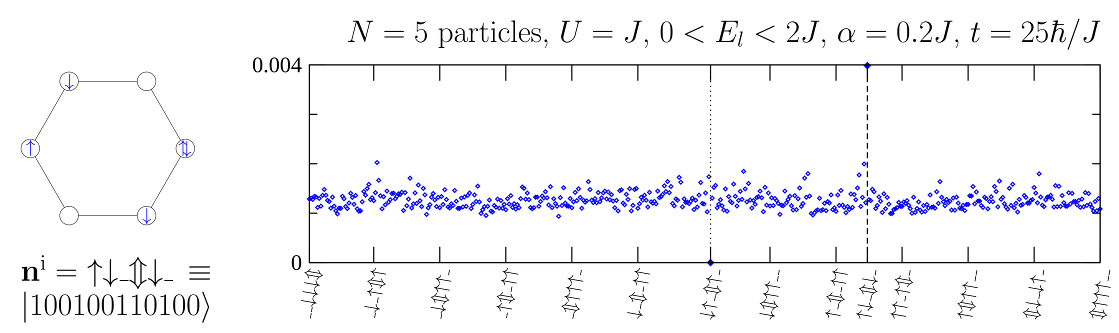
E_l = (random) energy on site l
U = on-site spin-spin interaction
J = inter-site hopping amplitude
α = inter-site Rashba hopping involving a spin flip



Disorder-averaged detection probability of the Fock state $|\mathbf{n}^f\rangle = |n_{1,\uparrow}, n_{1,\downarrow}, \cdots, n_{L,\uparrow}, n_{L,\downarrow}\rangle$ after the evolution time t:



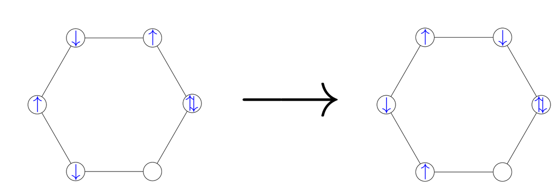
→ enhanced detection probability for the initial state and its spin-flipped counterpart



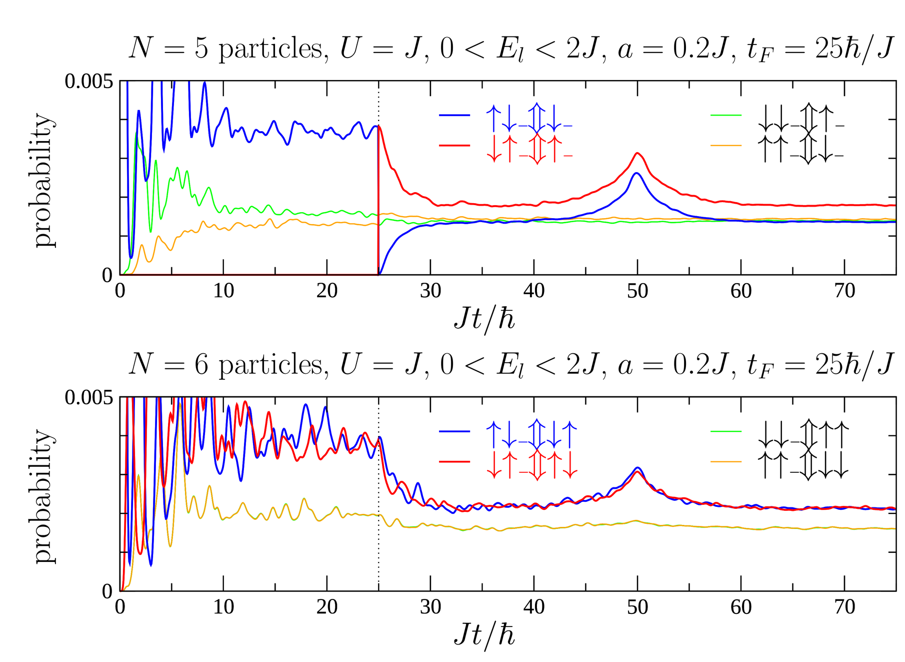
→ enhanced detection probability for the initial state
→ vanishing probability for its spin-flipped counterpart, due to a symplectic time-reversal symmetry of the configuration

Spin echo

→ flip the spins on all sites at $t = t_F$:



⟶ revival of coherent backscattering at $t = 2t_F$, both for the initial state and its spin-flipped counterpart:



T. Engl, J. Dujardin, A. Argüelles, P. Schlagheck, K. Richter, and J. D. Urbina, Phys. Rev. Lett. 112, 140403 (2014).
T. Engl, P. Plößl, J. D. Urbina, and K. Richter, Theoretical Chemistry Accounts 133, 1563 (2014).
T. Engl, J. D. Urbina, and K. Richter, arXiv:1409.5684.