ICOMP International Conference on COmputational methods in Manufacturing Processes

Comparison of interval and stochastic methods for uncertainty quantification in metal forming

M. Arnst, K. Liegeois, R. Boman, and J.-P. Ponthot

May 18, 2016

Motivation

This presentation compares interval and stochastic methods
for studying the impact of manufacturing variability in series production in sheet metal forming.

Outline

- Motivation.
- Outline.
- Interval and stochastic methods.
- Application to sheet metal forming.
- Conclusion.

Interval and stochastic methods

Raw materials variability:
- Material properties.

. . .

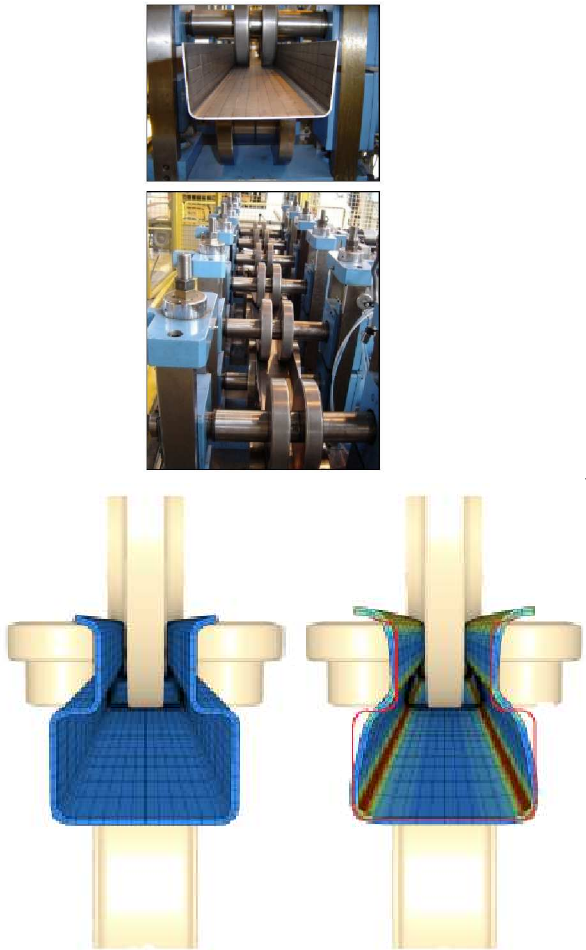
Process variability:
- Blank holder force.
- Initial dimensions.
- Friction.

. . .

Modeling limitations:
- Constitutive model.
- FE discretization.

. . .

→



→

Product variability:
- Final dimensions.
- Springback.

. . .

Prediction limitations:
- Numerical noise.

. . .

Input variables

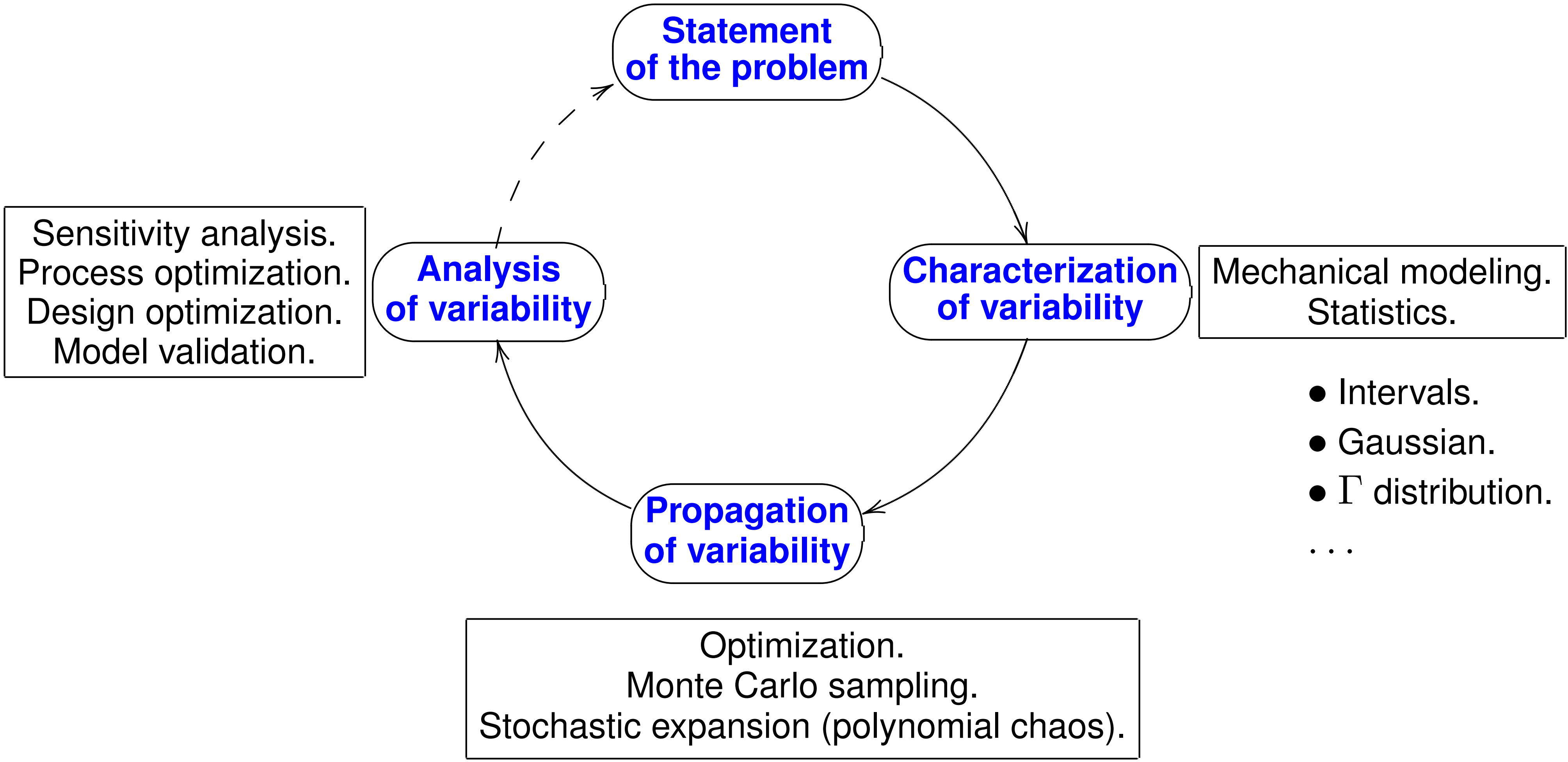
$(x_1, x_2, \ldots, x_m)$

Mapping

$y = g(x_1, x_2, \ldots, x_m)$

Output variable

y



The computational cost of both interval and stochastic methods can be lowered via the use of a surrogate model as a substitute for a FE model or the real process.

- **Characterization of variability**:

Available information		Intervals
Catalogue bounds Production data ...	$\longrightarrow$	$[\underline{x}_1, \overline{x}_1], \ldots, [\underline{x}_m, \overline{x}_m]$

- **Propagation of variability**:

Intervals		Interval
$[\underline{x}_1, \overline{x}_1], \ldots, [\underline{x}_m, \overline{x}_m]$	$\longrightarrow$ optimization	$[\underline{y}, \overline{y}]$

$$\underline{y} = \min_{\substack{\underline{x}_1 \leq x_1 \leq \overline{x}_1 \\ \vdots \\ \underline{x}_m \leq x_m \leq \overline{x}_m}} g(x_1, \ldots, x_m) \quad \text{and} \quad \overline{y} = \max_{\substack{\underline{x}_1 \leq x_1 \leq \overline{x}_1 \\ \vdots \\ \underline{x}_m \leq x_m \leq \overline{x}_m}} g(x_1, \ldots, x_m).$$

- **Sensitivity analysis**:
  - Local sensitivity descriptors, such as $\frac{\partial \underline{y}}{\partial \underline{x}_1}, \frac{\partial \underline{y}}{\partial \overline{x}_1}, \ldots, \frac{\partial \overline{y}}{\partial \underline{x}_m}, \frac{\partial \overline{y}}{\partial \overline{x}_m}$, and their variants.

Stochastic methods

- **Characterization of variability**:

Available information	$\xrightarrow[\text{mechanical modeling, statistics}]{}$	Probability density function
Catalogue bounds Production data ...	mechanical modeling statistics	$\rho_{(x_1,\ldots,x_m)}$

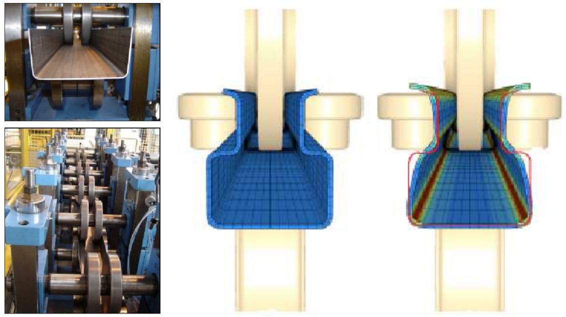
- **Propagation of variability**:

Probability density function	$\longrightarrow$	Probability density function
$\rho_{(x_1,\ldots,x_m)}$	Monte Carlo	ρ_y

Generate an ensemble of samples of $x_1,\ldots,x_m$ following probability density function $\rho_{(x_1,\ldots,x_m)}$.
Map each sample of $x_1,\ldots,x_m$ into the corresponding sample of y.
Deduce probability density function ρ_y and other statistical descriptors of y.

- **Sensitivity analysis**:
  - Local sensitivity descriptors, such as $\frac{\partial y}{\partial x_1}, \ldots, \frac{\partial y}{\partial x_m}$, and their variants.
  - Global sensitivity descriptors that indicate the significance of the variability in each input.

Surrogate model



Real process or FE model.

$$\approx \quad y = \sum_{(\alpha_1,\ldots,\alpha_m)} c_{(\alpha_1,\ldots,\alpha_m)} x_1^{\alpha_1} \times \ldots \times x_m^{\alpha_m}$$

Surrogate model.

Training set

$$y^{(1)} = g(x_1^{(1)}, \ldots, x_m^{(1)})$$

$$\vdots$$

$$y^{(n)} = g(x_1^{(n)}, \ldots, x_m^{(n)})$$

$\xrightarrow{\text{regression}}$

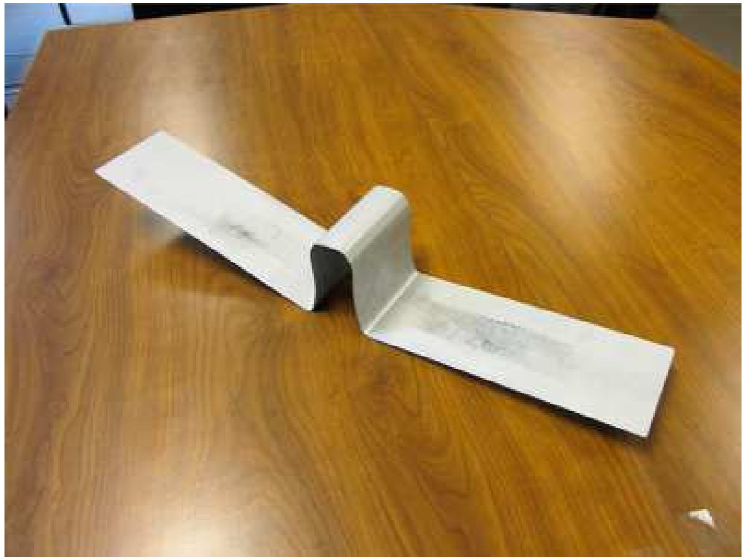
Surrogate model

$$y = \sum c_{(\alpha_1,\ldots,\alpha_m)} x_1^{\alpha_1} \times \ldots \times x_m^{\alpha_m}$$

The computational cost of interval and stochastic methods can be lowered via the use of a surrogate model as a substitute for the real process or a FE model.

Application to sheet metal forming

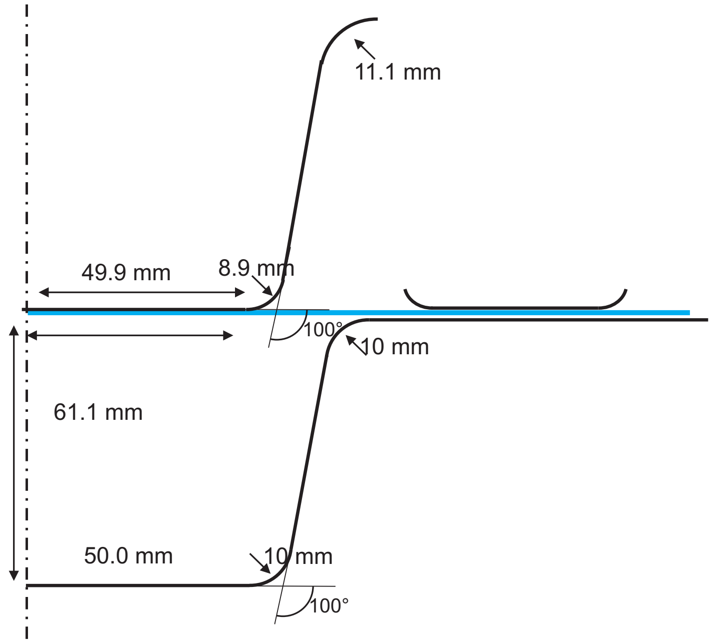
- We consider an "omega" sheet metal forming process:



- We assume that manufacturing variability manifests itself as variability in the ultimate tensile stress R_m and the strain hardening coefficient n, and we are interested in the impact of this manufacturing variability on a geometrical characteristic of the springback, which we denote by y:

R_m: ultimate tensile stress
n: strain hardening coefficient → Sheet metal forming process → y: geometrical characteristic of springback

- We assume that the following information is available:



FE model.

$$600\,\text{MPa} \leq R_\text{m} \leq 700\,\text{MPa}$$
$$0.14 \leq n \leq 0.22$$

Catalogue bounds (DP600).

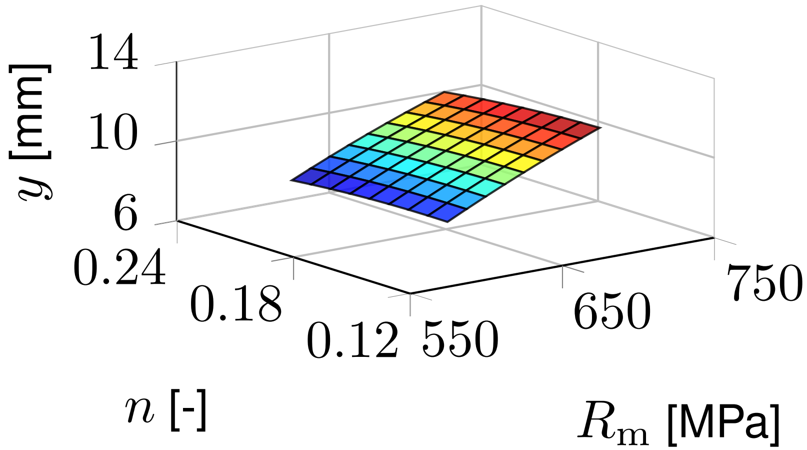
j [-]	R_m [MPa]	$n\,[-]$
1	659	0.164
⋮	⋮	⋮
25	646	0.162

Production data (fictitious).

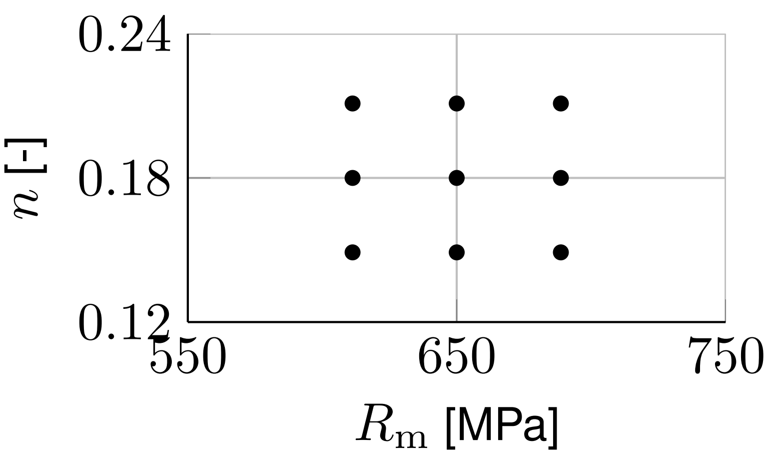
- In order to lower the computational cost, we construct a surrogate model to serve as a substitute for the FE model in the interval and stochastic analyses:

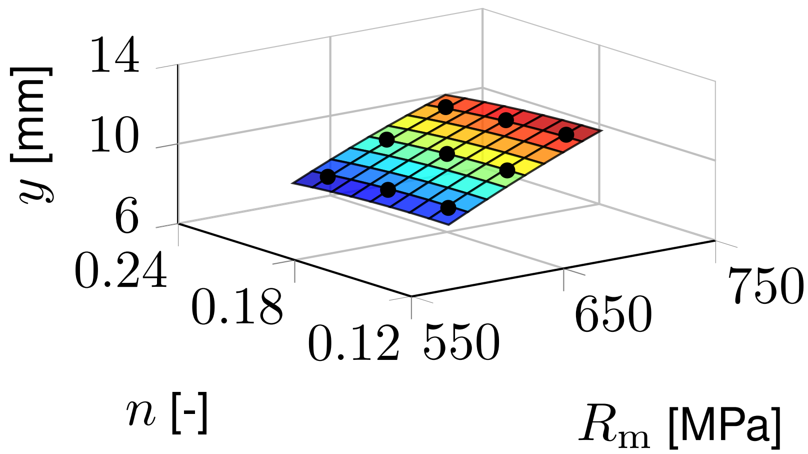
FE model.

$\approx$



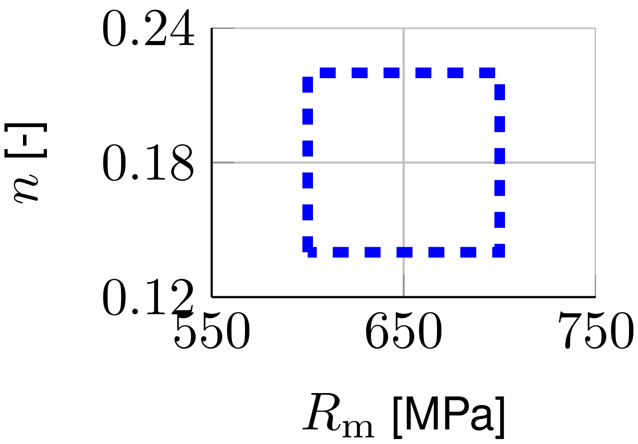
We construct the surrogate model by using the FE model to evaluate for a small number of well chosen values of R_m and n corresponding values of y and then applying a regression method:





Characterization of variability

Available information



Catalogue bounds

Intervals

R_m in $[600, 700]$ MPa
n in $[0.14, 0.22]$

Propagation of variability

Intervals
R_m in $[600, 700]$ MPa
n in $[0.14, 0.22]$

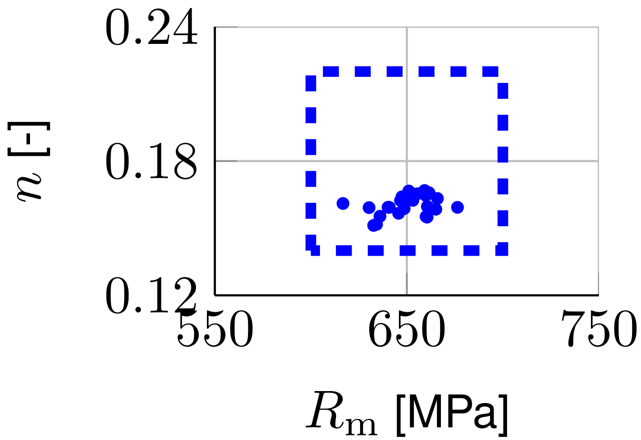
$\xrightarrow[\text{optimization}]{}$

Interval
y in $[7.93, 11.59]$ mm

$$7.93\,\text{mm} = \min_{\substack{600 \le R_\mathsf{m} \le 700 \\ 0.14 \le n \le 0.22}} g(R_\mathsf{m}, n) \quad \text{and} \quad 11.59\,\text{mm} = \max_{\substack{600 \le R_\mathsf{m} \le 700 \\ 0.14 \le n \le 0.22}} g(R_\mathsf{m}, n).$$

Characterization of variability

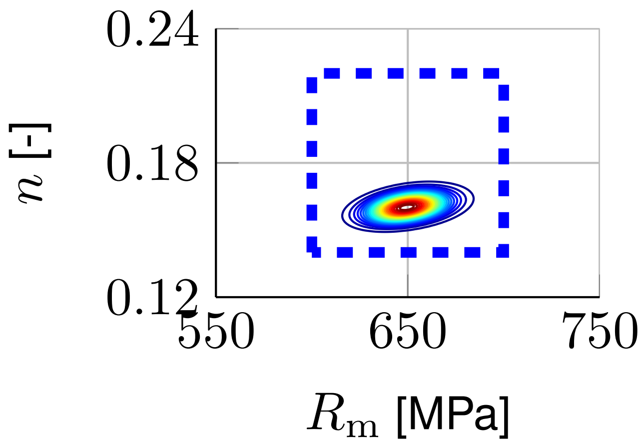
Available information



Catalogue bounds
Production data

mechanical modeling
statistics

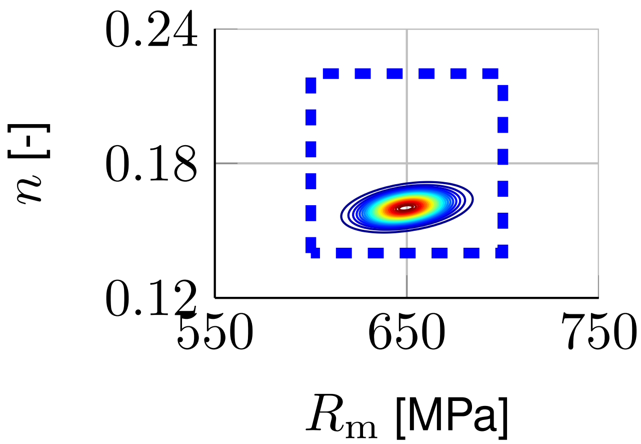
Probability density function



$$P(600\,\text{MPa} \leq R_\text{m} \leq 700\,\text{MPa}) = 1$$
$$P(0.14 \leq n \leq 0.22 = 1$$

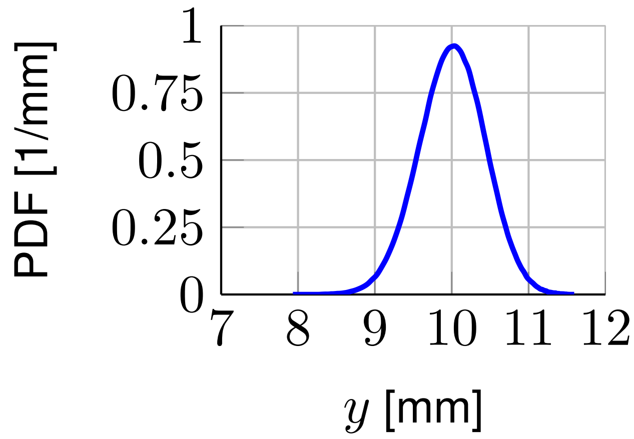
Propagation of variability

Probability density function



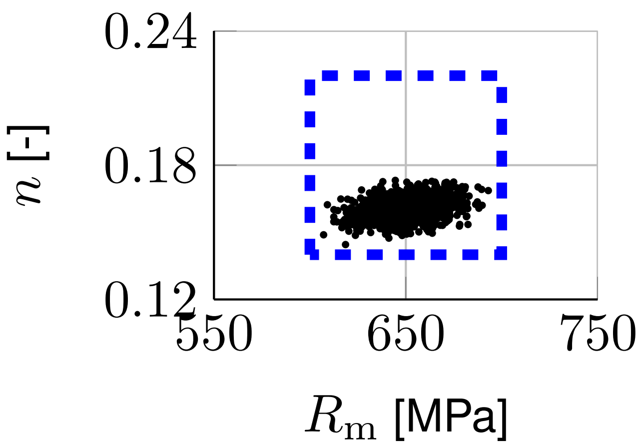
Monte Carlo

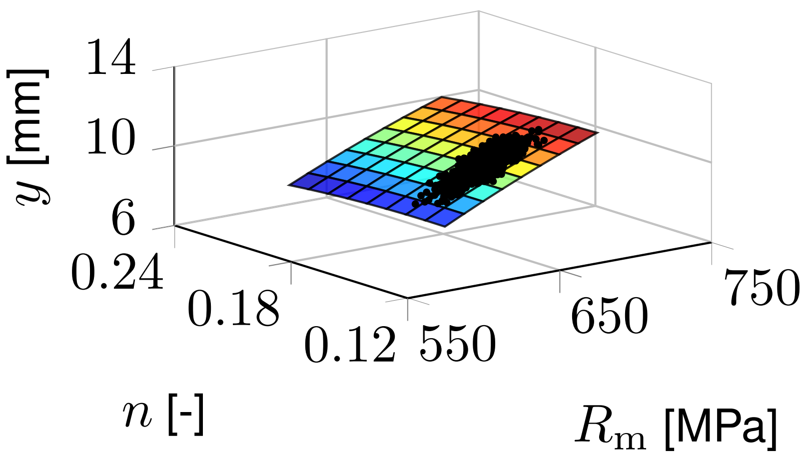
Probability density function



$$P(7.93\,\text{mm} \leq y \leq 11.59\,\text{mm}) = 1$$
$$P(9.16\,\text{mm} \leq y \leq 10.84\,\text{mm}) = 95\%$$

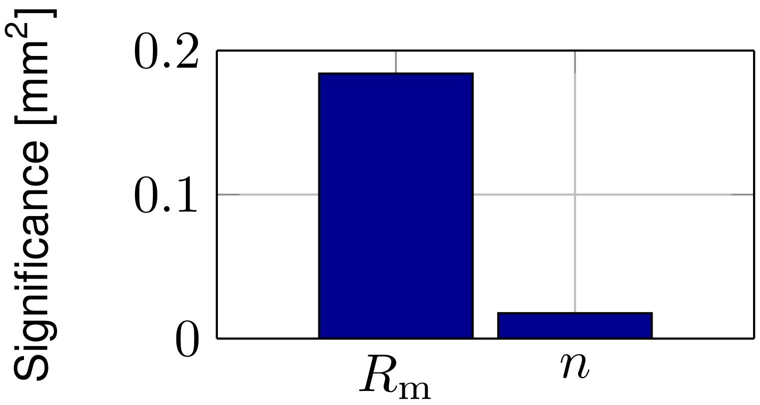
We generated an ensemble of samples of R_m and n following their probability density function, mapped each sample of R_m and n into a corresponding value of y, and applied mathematical statistics method to deduce the probability density function and other statistical descriptors of y:





Sensitivity analysis

In addition to a local sensitivity analysis, for which we do not show results here, stochastic methods allow a global sensitivity analysis, which can indicate the significance of the variability in each input:



A global sensitivity analysis based on the analysis-of-variance method, about which we do not provide details here, indicates that the variability in R_{m} is most significant in inducing variability in y.

Stochastic methods

- Interval and stochastic methods involve **different representations of variability**:
  - Intervals describe ranges of values.
  - Probability density functions functions describe frequencies of occurrence.
- Interval and stochastic methods **require different types of information to be available**:
  - Interval methods require that the available information allows the ranges of values of the variable properties of the manufacturing process to be bounded.
  - Stochastic methods require that the available information allows the frequencies of occurrence of the variable properties of the manufacturing process to be modeled.
- Interval and stochastic methods **provide different types of insight**:
  - Interval methods describe the impact of the manufacturing variability in terms of ranges of values for properties of the formed object.
  - Stochastic methods describe the impact of the manufacturing variability in terms of frequencies of occurrence of properties of the formed object..
- The computational cost of both interval and stochastic methods can be lowered via the use of a surrogate model as a substitute for a FE model or the real process.

References and acknowledgements

- L. Papeleux and J.-P. Ponthot. Finite element simulation of springback in sheet metal forming. Journal of Materials Processing Technology, 125–126:785–791, 2002.

- M. Arnst and J.-P. Ponthot. An overview of nonintrusive characterization, propagation, and sensitivity analysis of uncertainties in computational mechanics. International Journal for Uncertainty Quantification, 4:387–421, 2014.

- M. Arnst, B. Abello Alvarez, J.-P. Ponthot, and R. Boman. Itô-SDE-based MCMC method for Bayesian characterization and propagation of errors associated with data limitations. SIAM/ASA Journal on Uncertainty Quantification, Submitted, 2016.

- Support of ArcelorMittal and support of the University of Liège through a starting grant are gratefully acknowledged.