Aharonov-Bohm oscillations of bosonic matter-wave beams in the presence of disorder and interaction

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Abstract
We study the one-dimensional (1D) transport properties of an ultracold gas of Bose-Einstein condensed atoms through Aharonov-Bohm (AB) rings. Our system consists of a Bose-Einstein condensate (BEC) that is entangled from a magnetic trap into a 1D waveguide which is made of two semi-infinite leads that join a ring geometry exposed to a synthetic magnetic flux. We specifically investigate the effects of a disorder potential and of a small atom-atom contact interaction strength on the AB oscillations. The main numerical tools that we use for this purpose are a mean-field Gross-Pitaevskii (GP) description and the truncated Wigner (tW) method. We find that a correlated disorder suppress the AB oscillations leaving thereby place to weaker amplitude, half period oscillations on transmission, namely the Arozhov-Al'tshuler-Spivak (AAS) oscillations. The competition between disorder and interaction leads to a dip of the transmission at the AB flux \( \phi = \pi \). This dip could be a possible signature of an inversion of the coherent backscattering (CBS) peak. Our study paves the way to an analytical description of the inversion of that peak.

Aharonov-Bohm rings

- Toroidal optical dipole trap
  [A. Romannsh-enh et al. PRL 106, 130401 (2011)]
  [L. Amico et al. PRL 95, 083201 (2005)]
- Interaction of two red-detuned beams
- Connection to two waveguides

Theoretical description

- Ring geometry connected to two semi-infinite homogeneous leads
- Perfect condensation of the reservoir (\( T = 0 \) K) with chemical potential \( \mu \)
- Discretisation of a 1D Bose-Hubbard system
- Hamiltonian
  \[ H = H_L + H_R + H_{\text{inter}} \]
  where
  \[ H_L = \sum_{\alpha \in \mathbb{L}} \left( E_{\alpha} \hat{a}_{\alpha}^\dagger \hat{a}_{\alpha} - \frac{\mu}{2} \left( \hat{a}_{\alpha+1}^\dagger \hat{a}_{\alpha} + \hat{a}_{\alpha}^\dagger \hat{a}_{\alpha+1} \right) \right) \]
  \[ H_R = \frac{\mu}{2} \left( \hat{a}_{0}^\dagger \hat{a}_{0} + \hat{a}_{0+1}^\dagger \hat{a}_{0+1} + \hat{a}_{0}^\dagger \hat{a}_{0+1} + \hat{a}_{0+1}^\dagger \hat{a}_{0} \right) \]
  \[ H_{\text{inter}} = \sum_{\alpha \in \mathbb{R}} (E_{\alpha} + V_{\alpha}) \hat{a}_{\alpha}^\dagger \hat{a}_{\alpha} - \frac{\mu}{2} \left( \hat{a}_{\alpha+1}^\dagger \hat{a}_{\alpha} + \hat{a}_{\alpha}^\dagger \hat{a}_{\alpha+1} \right) \]
  \[ + g \hat{a}_{\alpha}^\dagger \hat{a}_{\alpha} \hat{b}_{\alpha} \]
  \[ + \kappa (\hat{b}_{\alpha}^\dagger \hat{b}_{\alpha} + \kappa (\hat{b}_{\alpha}^\dagger \hat{b}_{\alpha} + \mu d_{\alpha} d_{\alpha}) \]
  with:
  - \( \hat{a}_{\alpha} \) and \( \hat{a}_{\alpha}^\dagger \) the annihilation and creation operators at site \( \alpha \) (of the source),
  - \( E_{\alpha} \propto 1/\delta \) the on-site energy,
  - \( V_{\alpha} \) the disorder potential at site \( \alpha \),
  - \( g \) the interaction strength,
  - \( N \rightarrow \infty \) the number of Bose-Einstein condensed atoms within the source,
  - \( \kappa(t) \rightarrow 0 \) the coupling strength, which tends to zero such that \( N(\kappa(t))^{2} \) remains finite.

Aharonov-Bohm effect

- Periodic transmission oscillation with \( \phi \)
  \[ \mu/E_0 = 1, g = 0 \]

- Clear signature of the AB effect
- Incoherent transmission when \( \phi = 0 \)

- Oscillations in transport properties due to interferences of partial waves crossing each arm of the ring
- Transmission periodic w.r.t. the AB flux \( \phi \)
  \[ T = |t_1|^2 + |t_2|^2 + 2|t_1||t_2| \cos \Delta \phi \]
  with \( \phi = h/2e \) the magnetic flux quantum
- Same effect within a two-arm ring

Aharonov-Bohm oscillations

- Periodic transmission oscillation with \( \phi \)
  \[ \mu/E_0 = 1, g = 0 \]

- With interaction
  - Total
  - Incoh

- Resonant transmission peaks move with \( g \) and disappear if \( g \) is strong enough

- More incoherent particles created as \( g \)

Higher order interferences

- Presence of higher harmonics of weak intensity
- Diagrammatic approach of the problem
  \[ R = |r_1 + r_2 e^{i\phi} + r_3 e^{-i\phi} + \cdots |^2 \]
  \[ = |r_1|^2 + |r_2|^2 + 2|r_1||r_2| \cos A \Phi + \cdots \]

- Interference pattern shifted due to the presence of vector potential \( A \) with depasing
  \[ \Delta \phi = k \Delta l + \frac{\phi}{\hbar} \int A \cdot d\ell = k \Delta l + 2 \pi \phi \]

- Transmission
  \[ T = |t_1|^2 + |t_2|^2 + 2|t_1||t_2| \cos \Delta \phi \]
  with \( \phi = h/2e \) the magnetic flux quantum
- Same effect within a two-arm ring

Towards coherent backscattering

- Same origin for CBS and AAS
- Constructive wave interference between reflected classical paths and their time-reversed counterparts
- Recent verification with BEC
  [J. Jendowycz et al., PRL 109, 195302 (2012)]

AAS oscillations

- Averages over the disorder suppress Aharonov-Bohm oscillations
  \[ \mu/E_0 = 0.25, V/E_0 = 0 \]

- Appearance of \( \phi/2 \) periodic oscillations : Al'tshuler-Aronov-Spivak oscillations
- What happens if we set a weak interaction ?
  \[ \mu/E_0 = 0.75, V/E_0 = 2 \]

- The oscillations amplitude is reduced
- The minimum at \( \phi = \pi \) becomes a maximum

AAS oscillations

- Total density
- No disorder
- Disorder

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