Aharonov-Bohm oscillations of bosonic matter-wave beams in the presence of disorder and interaction

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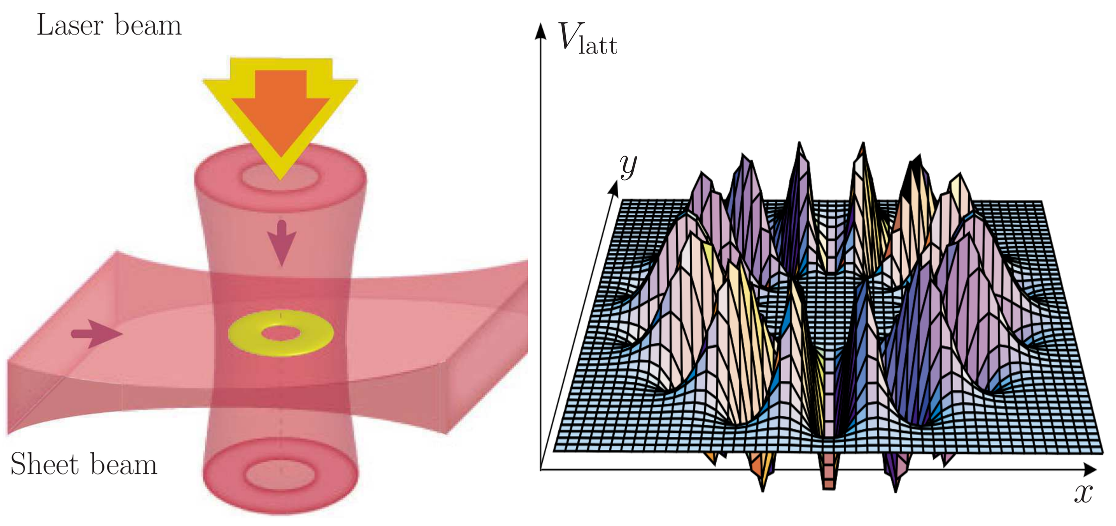
Abstract

We study the one-dimensional (1D) transport properties of an ultracold gas of Bose-Einstein condensed atoms through Aharonov-Bohm (AB) rings. Our system consists of a Bose-Einstein condensate (BEC) that is outcoupled from a magnetic trap into a 1D waveguide which is made of two semi-infinite leads that join a ring geometry exposed to a synthetic magnetic flux ϕ. We specifically investigate the effects both of a disorder potential and of a small atom-atom contact interaction strength on the AB oscillations. The main numerical tools that we use for this purpose are a mean-field Gross-Pitaevskii (GP) description and the truncated Wigner (tW) method. We find that a correlated disorder suppress the AB oscillations leaving thereby place to weaker amplitude, half period oscillations on transmission, namely the Aronov-Al'tshuler-Spivak (AAS) oscillations. The competition between disorder and interaction leads to a flip of the transmission at the AB flux $\phi = \pi$. This flip could be a possible preliminary signature of an inversion of the coherent backscattering (CBS) peak. Our study paves the way to an analytical description of the inversion of that peak.

Aharonov-Bohm rings

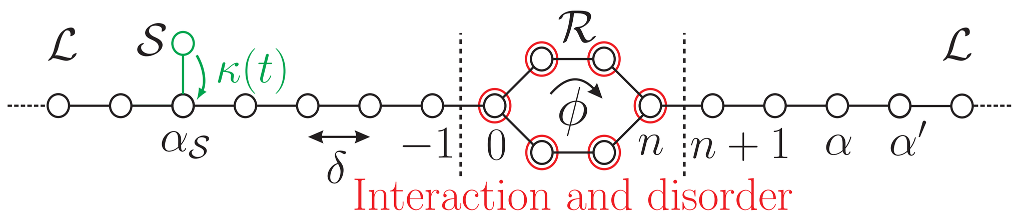
- Toroïdal optical dipole trap
  [A. Ramanathan *et al.* PRL **106**, 130401 (2011)]
  [L. Amico *et al.* PRL **95**, 063201 (2005)]



- Intersection of two red-detuned beams
- Connection to two waveguides

Theoretical description

- Ring geometry connected to two semi-infinite homogeneous leads
- Perfect condensation of the reservoir ($T = 0\,$K) with chemical potential μ
- Discretisation of a 1D Bose-Hubbard system
  [J. Dujardin *et al. Phys. Rev. A* **91**, 033614 (2015)]



- Hamiltonian

$$\hat{H} = \hat{H}_{\mathcal{L}} + \hat{H}_{\mathcal{LR}} + \hat{H}_{\mathcal{R}} + \hat{H}_{\mathcal{S}}$$

where

$$\hat{H}_{\mathcal{L}} = \sum_{\alpha\in\mathcal{L}} \left[E_\delta \hat{a}^\dagger_\alpha \hat{a}_\alpha - \frac{E_\delta}{2}\left(\hat{a}^\dagger_{\alpha-1}\hat{a}_\alpha + \hat{a}^\dagger_{\alpha+1}\hat{a}_\alpha\right)\right]$$

$$\hat{H}_{\mathcal{LR}} = -\frac{E_\delta}{2}\left(\hat{a}^\dagger_{-1}\hat{a}_0 + \hat{a}^\dagger_0\hat{a}_{-1} + \hat{a}^\dagger_n\hat{a}_{n+1} + \hat{a}^\dagger_{n+1}\hat{a}_n\right)$$

$$\hat{H}_{\mathcal{R}} = \Bigg[\sum_{\alpha\in\mathcal{R}} (E_\delta + V_\alpha)\,\hat{a}^\dagger_\alpha\hat{a}_\alpha - \frac{E_\delta}{2}\left(\hat{a}^\dagger_{\alpha-1}\hat{a}_\alpha + \hat{a}^\dagger_{\alpha+1}\hat{a}_\alpha\right) + g\hat{a}^\dagger_\alpha\hat{a}^\dagger_\alpha\hat{a}_\alpha\hat{a}_\alpha\Bigg]$$

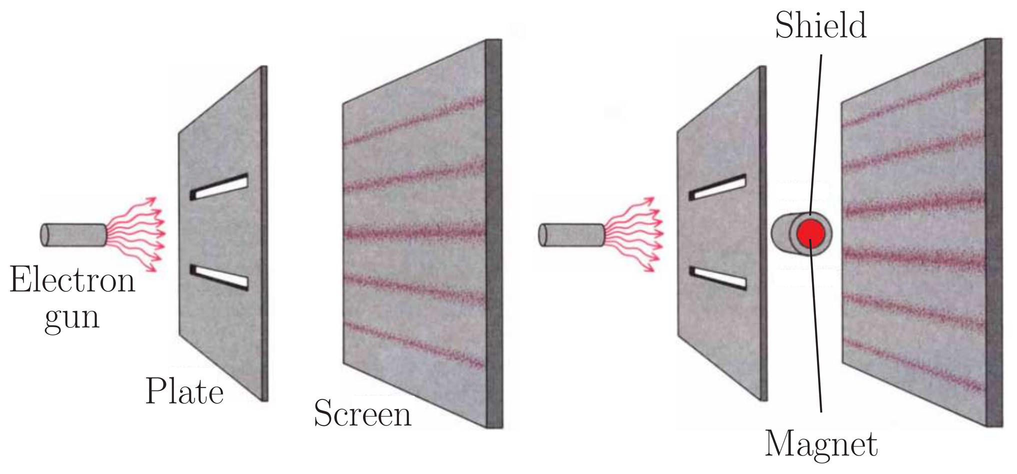
$$\hat{H}_{\mathcal{S}} = \kappa(t)\hat{a}^\dagger_{\alpha_S}\hat{b} + \kappa^*(t)\hat{b}^\dagger\hat{a}_{\alpha_S} + \mu\hat{b}^\dagger\hat{b}$$

with :

- $\hat{a}_\alpha$ ($\hat{b}$) and $\hat{a}^\dagger_\alpha$ ($\hat{b}^\dagger$) the annihilation and creation operators at site α (of the source),
- $E_\delta \propto 1/\delta^2$ the on-site energy,
- V_α the disorder potential at site α,
- g the interaction strength,
- $N \to \infty$ the number of Bose-Einstein condensed atoms within the source,
- $\kappa(t) \to 0$ the coupling strength, which tends to zero such that $N|\kappa(t)|^2$ remains finite.

Aharonov-Bohm effect

- Potentials act on charged particles even if all fields vanish
  [Y. Aharonov and D. Bohm, PR **115**, 485-491 (1959)]
  [Y. Imry and R.A. Webb, *Scient. Am.* **260**, 56 (1989)]

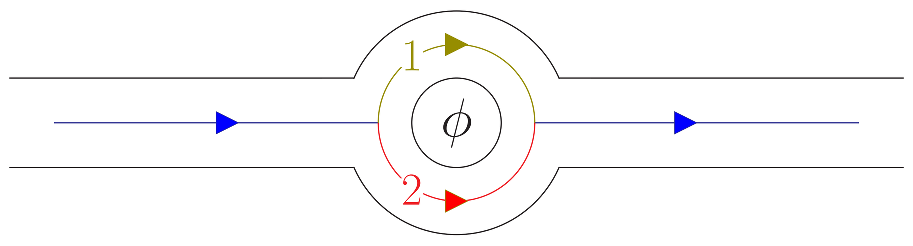


- Interference pattern shifted due to the presence of vector potential $\mathbf{A}$ with depashing

$$\Delta\varphi = k\Delta l + \underbrace{\frac{e}{\hbar}\oint \mathbf{A}\cdot d\mathbf{l}}_{\phi} = k\Delta l + 2\pi\underbrace{\frac{\phi}{\phi_0}}_{\Phi}$$

with $\phi_0 = h/2e$ the magnetic flux quantum

- Same effect within a two-arm ring



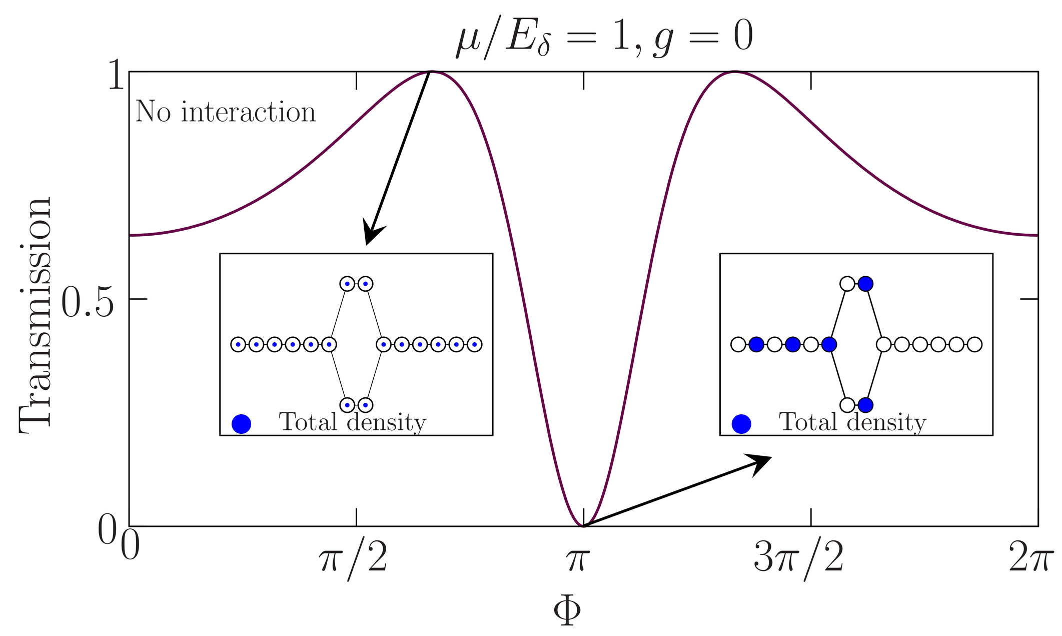
- Oscillations in transport properties due to interferences of partial waves crossing each arm of the ring
- Transmission periodic w.r.t. the AB flux ϕ

$$T = |t_1 + t_2|^2 = |t_1|^2 + |t_2|^2 + 2|t_1|\cdot|t_2|\cos\Delta\varphi$$

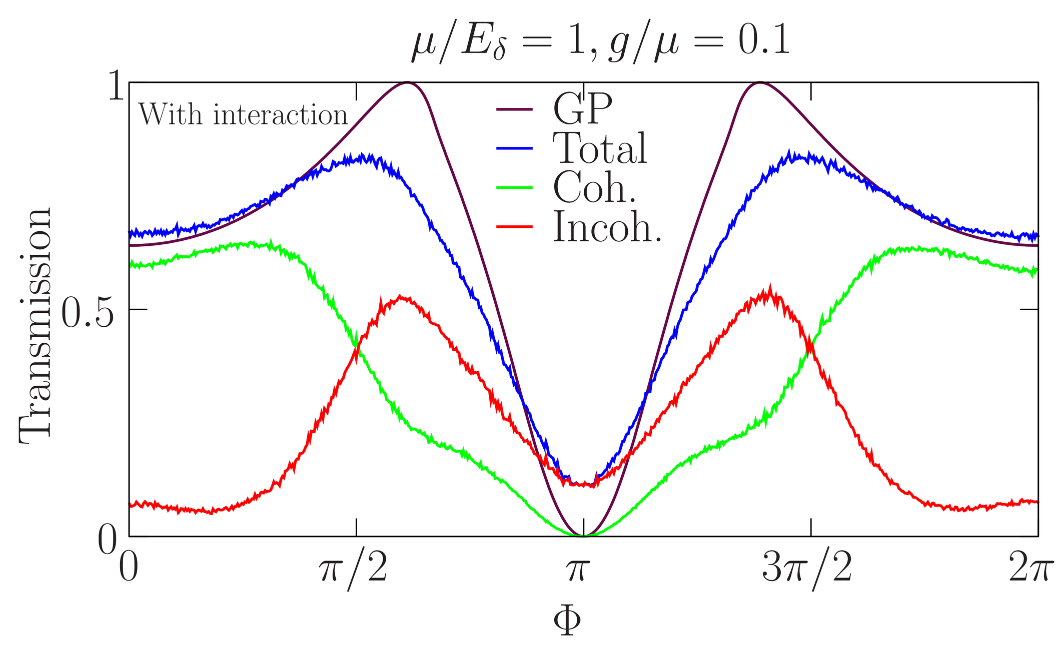
with period ϕ_0

Aharonov-Bohm oscillations

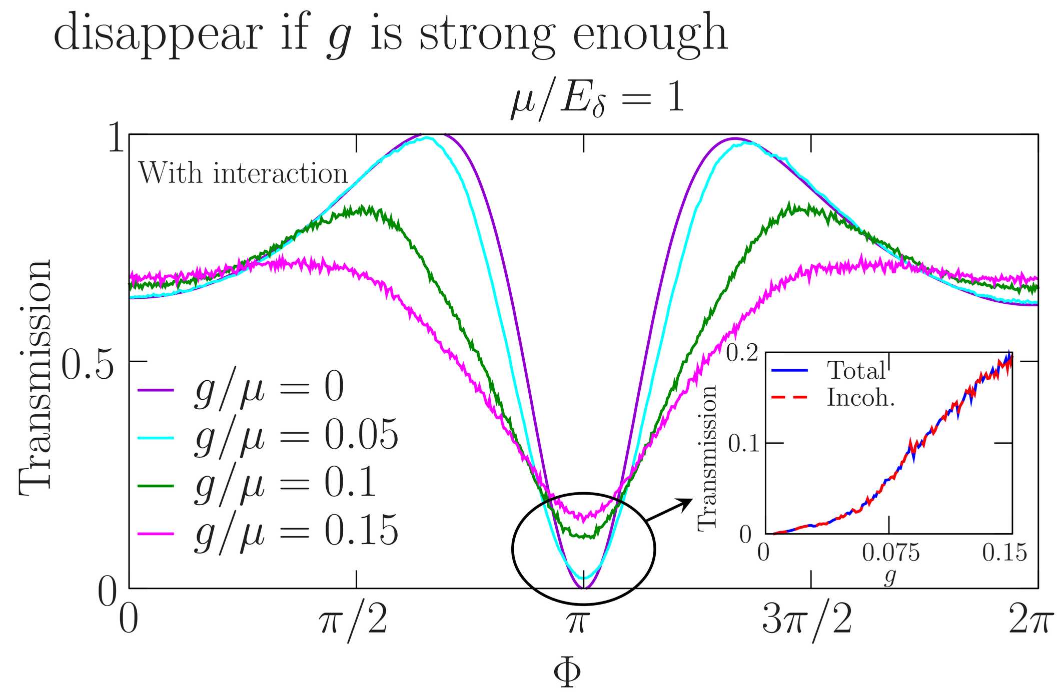
- Periodic transmission oscillation with ϕ.



- Clear signature of the AB effect
- Incoherent transmission when $g \neq 0$



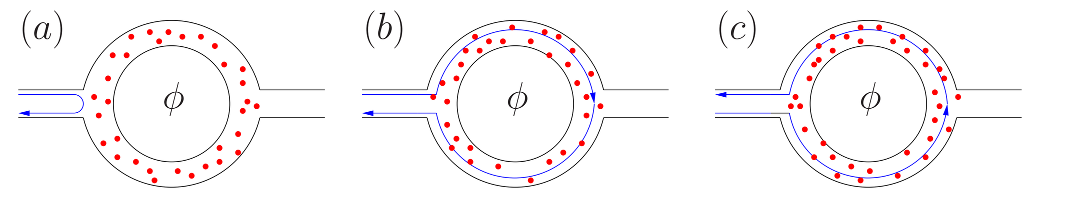
- Resonant transmission peaks move with g and disappear if g is strong enough



- More incoherent particles created as $g\uparrow$

Higher order interferences

- Presence of higher harmonics of weak intensity
- Diagrammatic approach of the problem



[Ihn T., *Semiconductor nanostructures*, Oxford (2010).]

The reflection probability is given by

$$\begin{aligned}\mathcal{R} &= \left|r_0 + r_1 e^{i\Phi} + r_1 e^{-i\Phi} + \ldots\right|^2 \\ &= |r_0|^2 + |r_1|^2 + \ldots & (1)\\ &\quad + 4|r_0|\cdot|r_1|\cos\Lambda\cos\Phi + \ldots & (2)\\ &\quad + 2|r_1|^2\cos(2\Phi) + \ldots & (3)\end{aligned}$$

with Λ the disorder-dependent phase accumulated after one turn with $\phi = 0$.

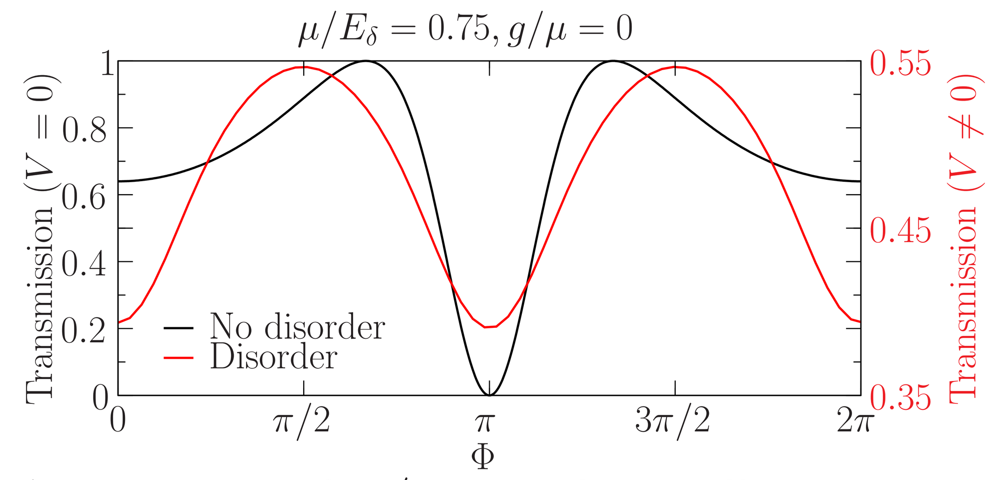
(1) no Φ-dependence, classical contributions

(2) Φ-periodicity, AB contribution, damped to zero when averaged over the disorder

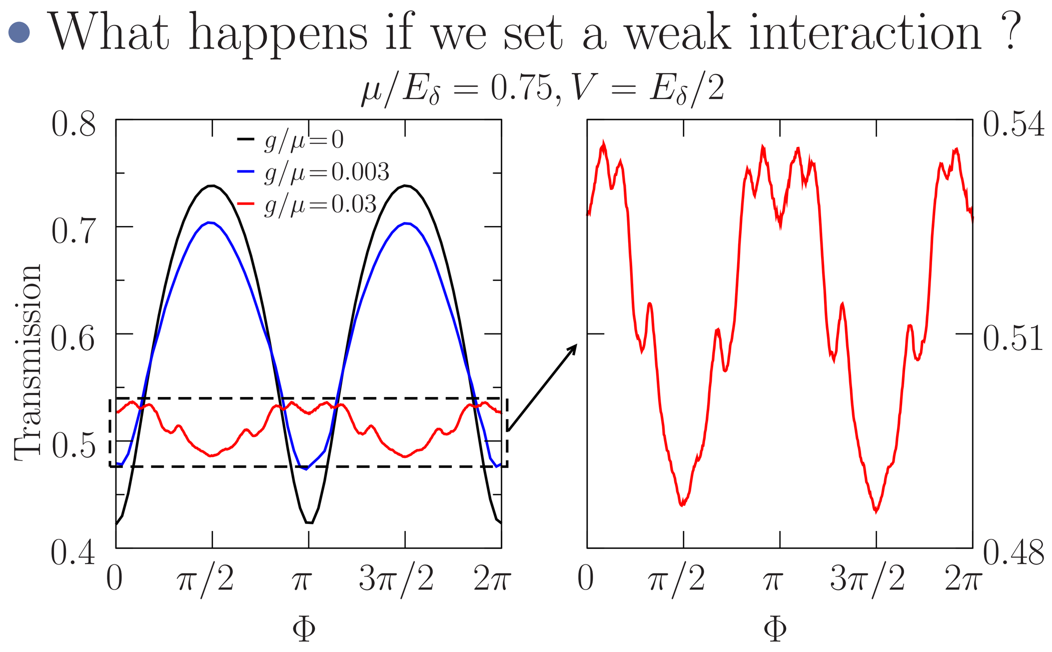
(3) $\Phi/2$-periodicity, AAS contribution, robust to averages over the disorder

AAS oscillations

- Averages over the disorder suppress Aharonov-Bohm oscillations



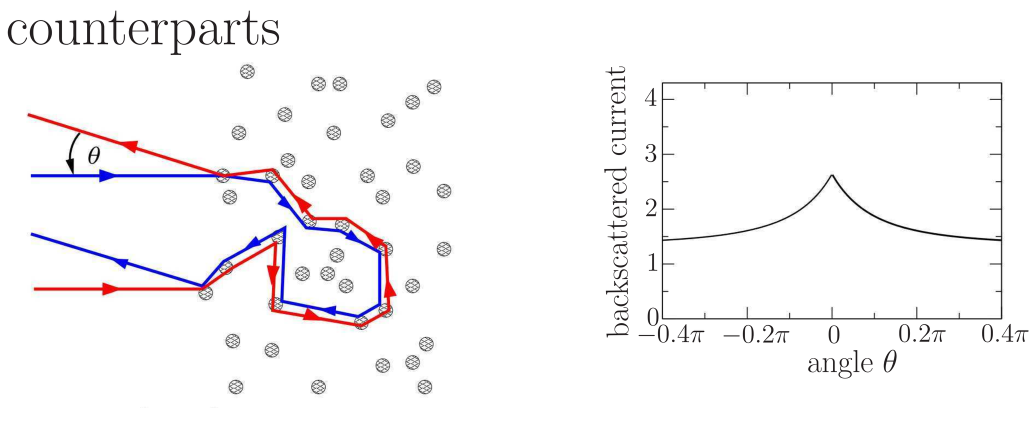
- Appearance of $\Phi/2$ periodic oscillations : Altshuler-Aronov-Spivak oscillations
- What happens if we set a weak interaction ?



- The oscillations amplitude is reduced
- The minimum at $\Phi = \pi$ becomes a maximum !

Towards coherent backscattering

- Same origin for CBS and AAS
  [E. Akkermans *et al.*, PRL **56**, 1471 (1986)]
- Constructive wave interference between reflected classical paths and their time-reversed counterparts



- Recent verification with BEC
  [F. Jendrzejewski, *et al.*, PRL **109**, 195302 (2012)]
- Inversion in the presence of nonlinearity (2D)
  [M. Hartung, *et al.*, PRL **101**, 020603 (2008)]
- Analytical calculations with our 1D model more feasible

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