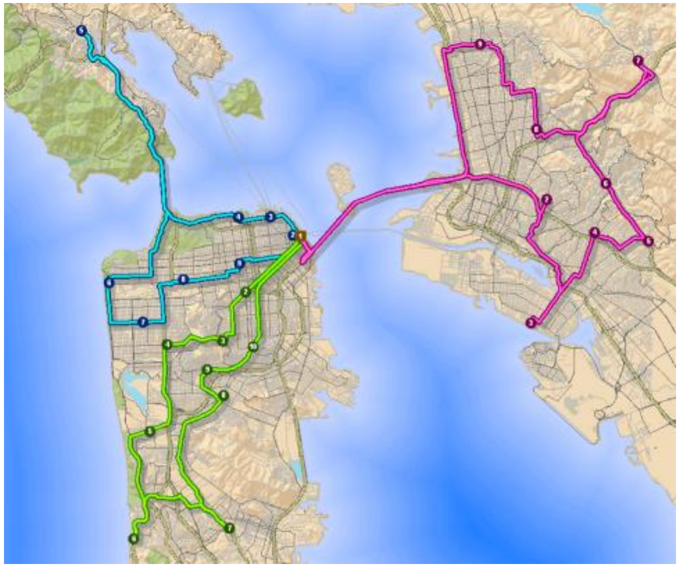
Optimisation and uncertainty: comparing stochastic and robust programming

THIBAUT CUVELIER

UNIVERSITÉ DE LIÈGE

Why uncertainty in optimisation?

- How to make the best decisions? Optimisation!
- Example: schedule deliveries



Source: ArcGIS documentation

- What if there is unexpected congestion?
  - Penalties when deliveries are late
  - Drivers working overtime

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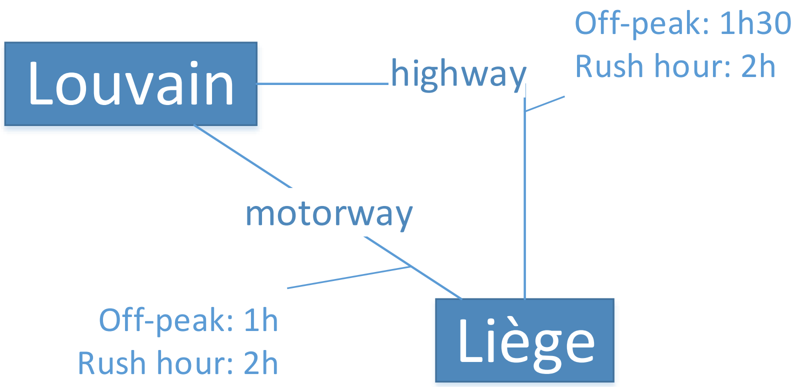
Modelling: stochastic approach

- First idea: probabilities, statistics
  - Common tool with uncertainty!

- Hence: **stochastic programming**
  - Best solution **on average**
  - Use probabilities and scenarios

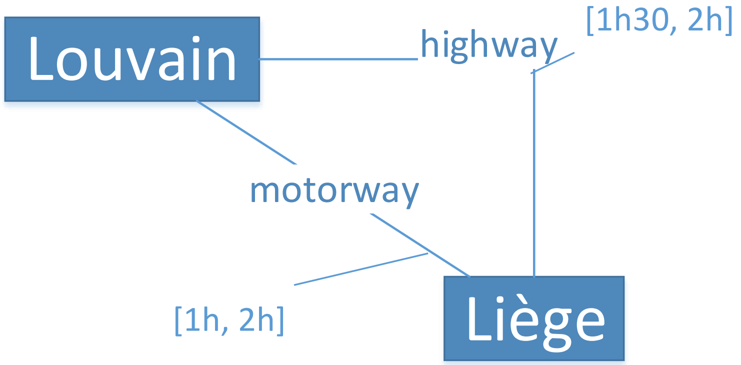


- Recourse action:
  - In the future, **adapt** your decisions to what you have seen
  - Uncertainty partly revealed (second stage)

Modelling: robust approach

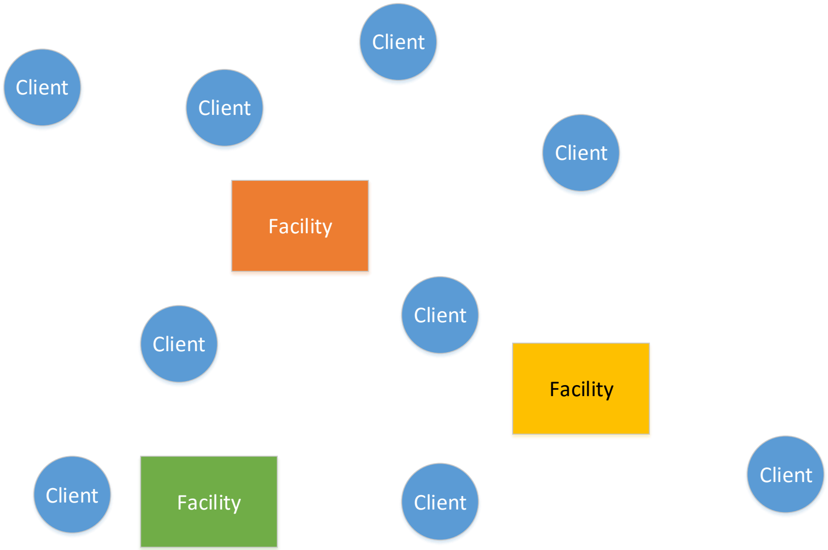
- How to deal with the curse of dimensionality?
- Second idea: have the best solution **in the worst case**

- Worst case defined through an **uncertainty set**
  - For example: confidence intervals
  - Optimise within a set of values
  - Ensure the solution works for all values



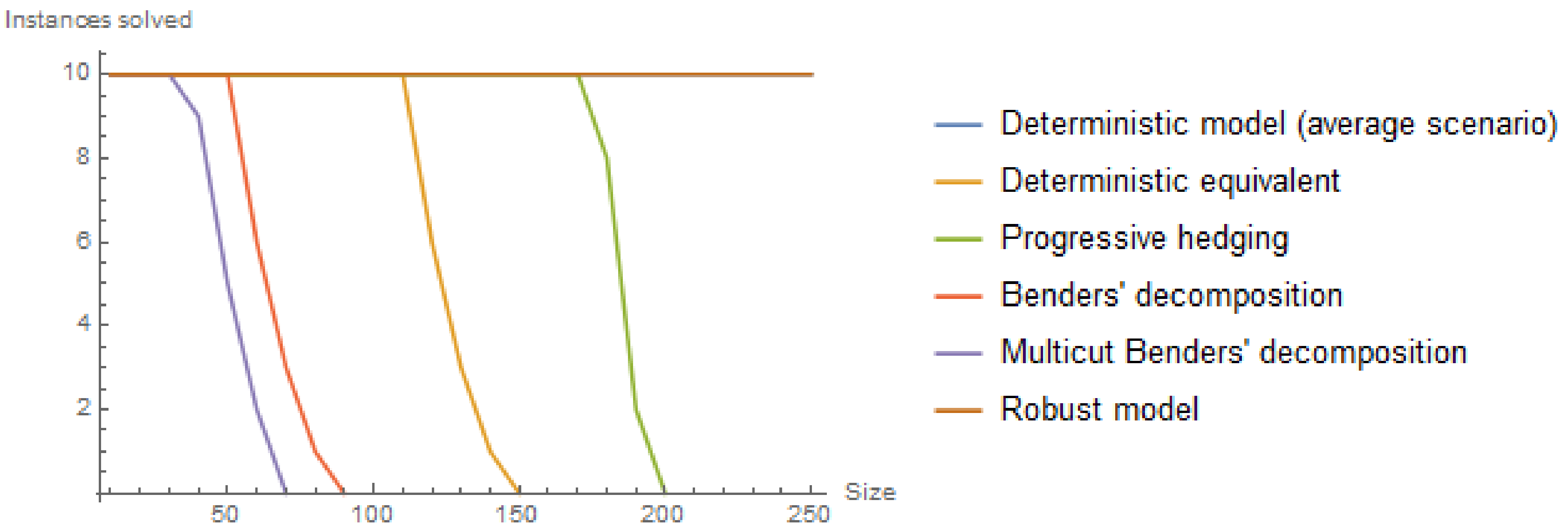
Application: Facility Location

- Where to place a hospital?
  - Demand to meet — with uncertainty!
  - Number of possible locations



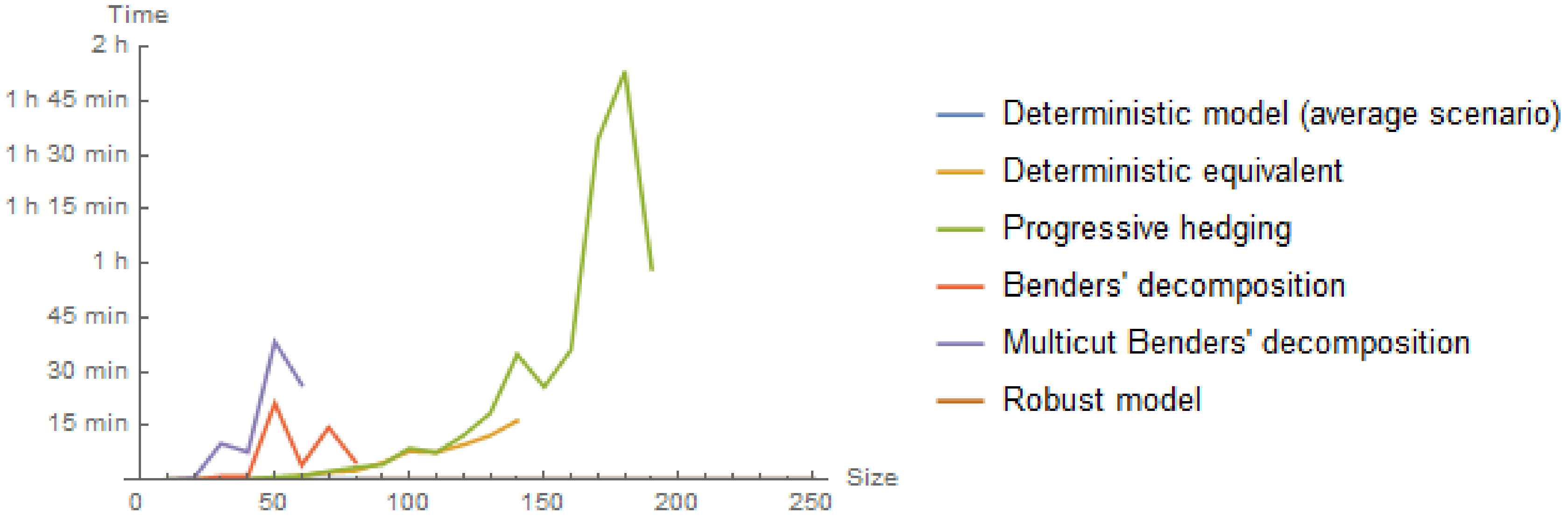
Maximum instance size

- Compare the various uncertainty models
  - And the way to solve them
- Maximum problem size under constraints?
  - Number of instances that could be solved
  - 10.5GB of RAM, 4 hours



Time to convergence

- Average time until solved on the same instances
- Results?
  - Best? Average or robust: a few seconds at most!
  - Different algorithms have various properties,
    but are not always able to solve large instances!

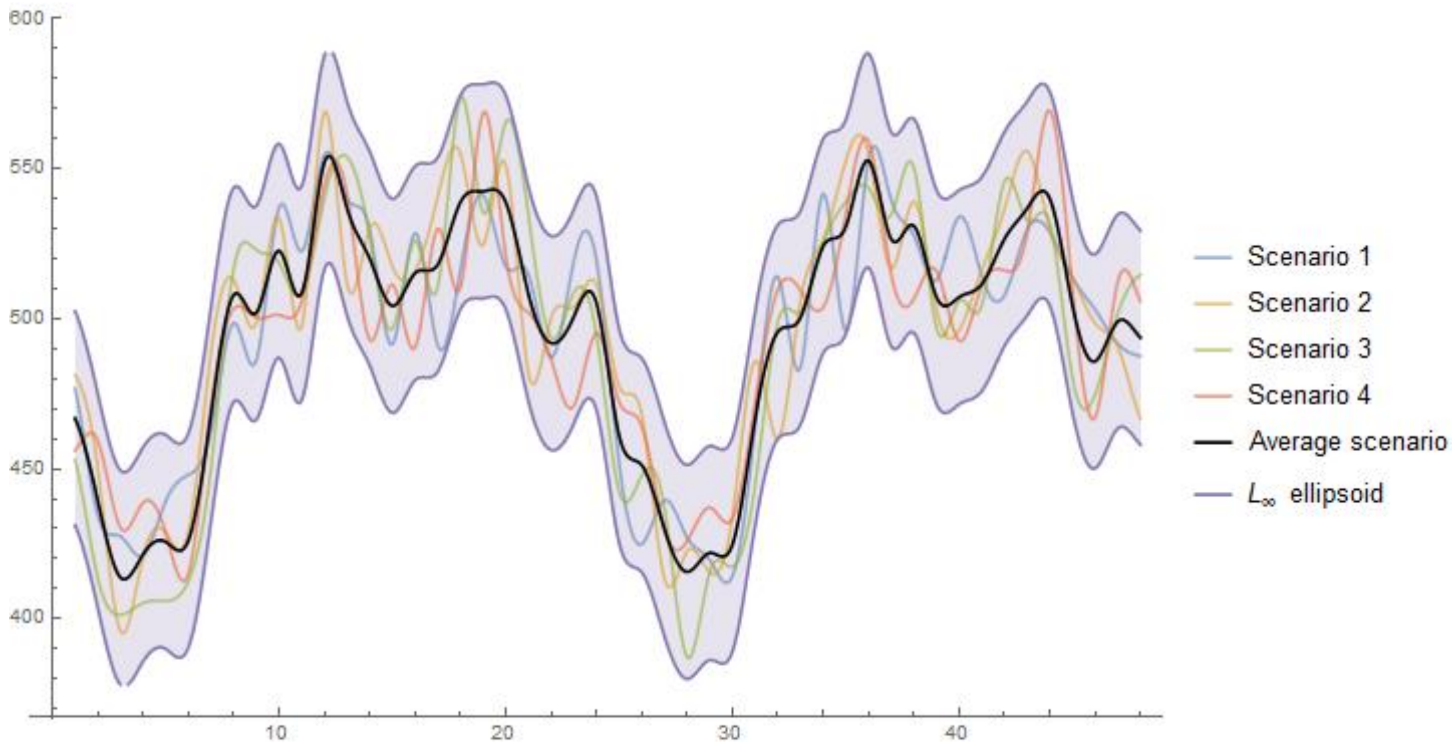


Application: Unit-Commitment

- How to meet some electrical demand?
  - Choose which power plants to use
  - Uncertainty lies in the demand
  - Under constraints:
    - Time to start, to stop
    - Minimum up/down time
- Two-stage model:
  - Plan today (first stage)
  - Prepare for tomorrow (second stage)

A look at uncertainty

- Stochastic scenarios and uncertainty set:
  - Intervals surround the scenarios

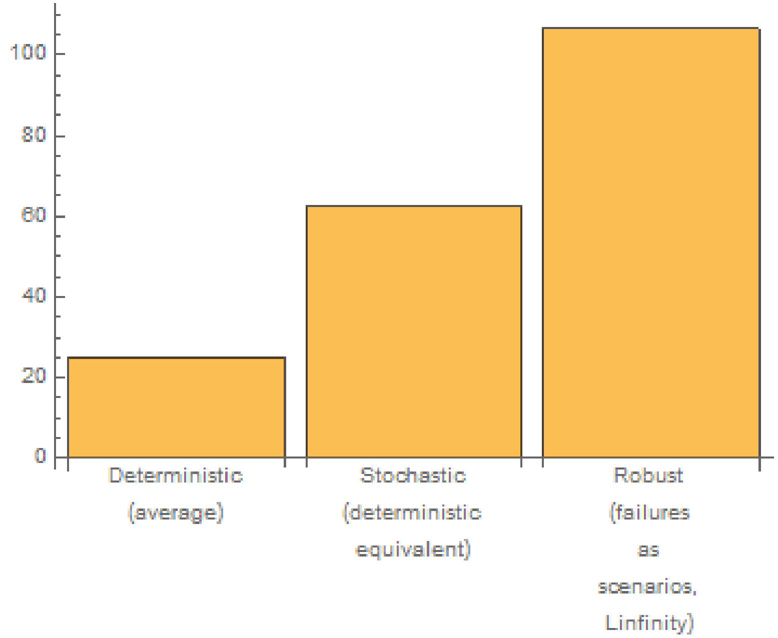


- Test case: 40 machines, 48-hour first stage, 48-hour second stage

Robustness comparison

- Compare solutions based on two criteria:
  - **Stochastic**: objective value (average on scenarios)
  - **Robust**: maximum interval size

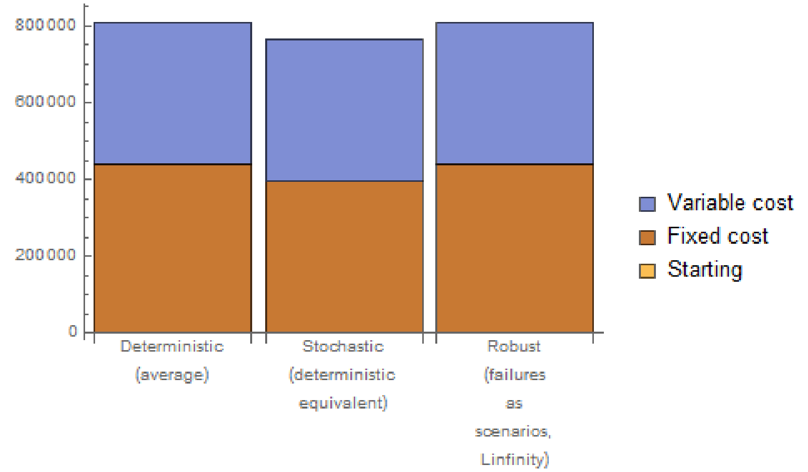
- First test: robust criterion
  - Robust: much better!
  - Interval size larger than asked



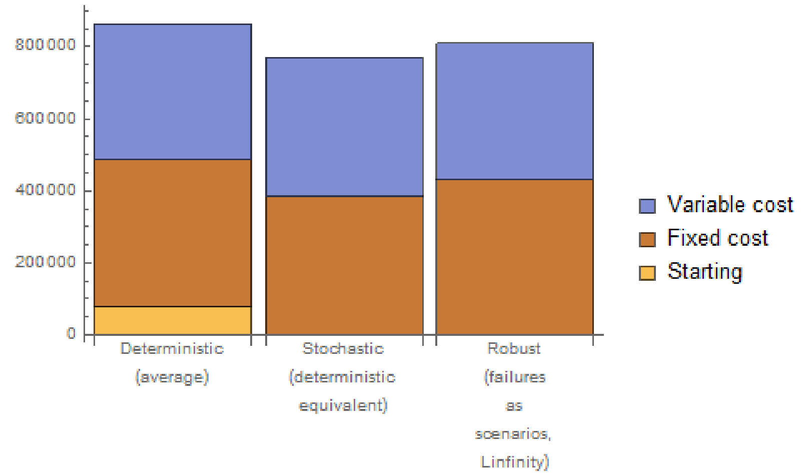
- What about cost?
  - Difference below one percent!

Robustness comparison

- What about failures?
  - Stochastic: great impact
    - Perceptibly better
  - Robust: similar to previous



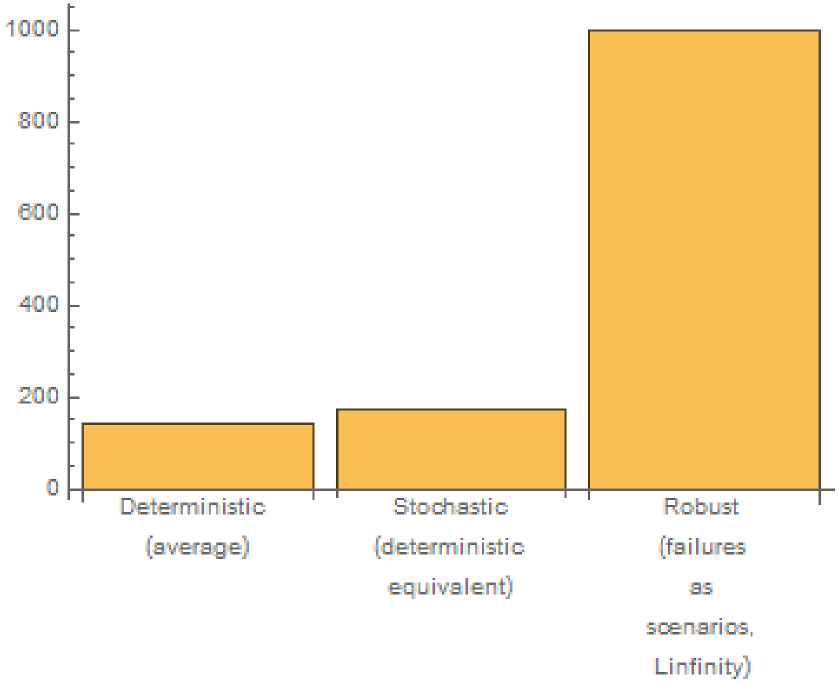
- Moving the recourse sooner?
  - Plan for six hours (instead of two days)
  - Same conclusions: stochastic fits better



Robustness comparison: Monte Carlo analysis

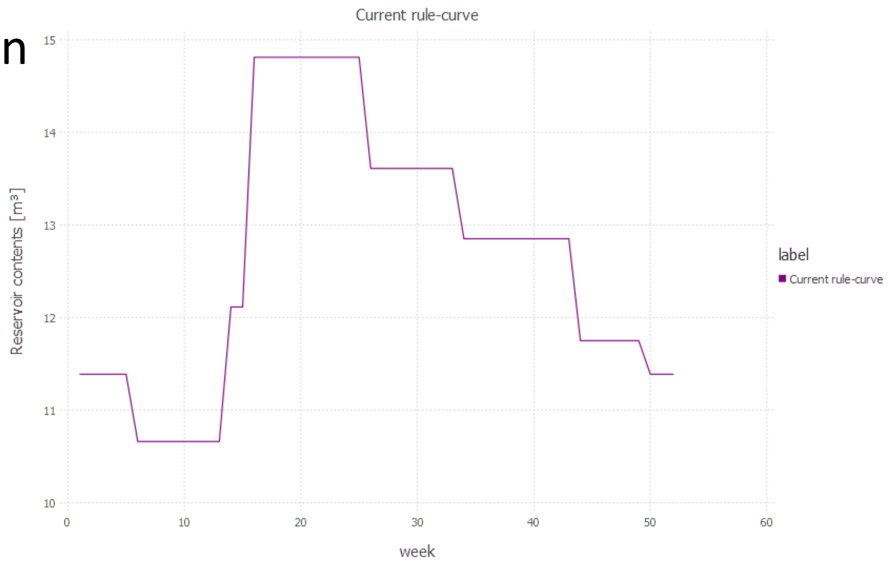
- Methodology to evaluate a solution:
  - Draw scenarios (95% chance within interval)
  - Evaluate the solution: check feasibility

- Robust feasible much more often!



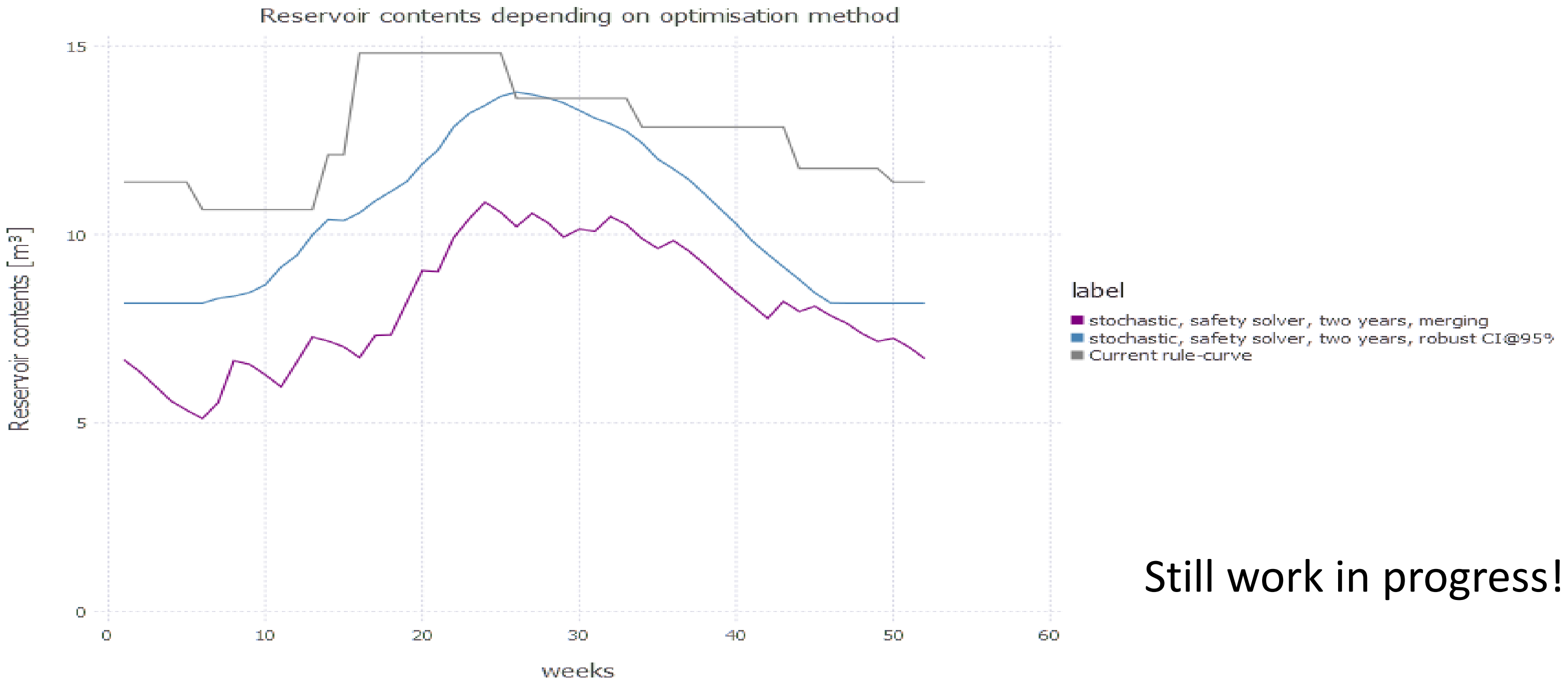
Application: Dam Management

- Belgian dams’ main role: provide drinking water
- Uncertainty in the inflows, i.e. rain
- How to ensure no shortage?
  - Lower bound for water depth
  - If lower than this:
    no more guaranteed



- Currently used:
  computed once with data from the 1970s

Current rule too conservative?



Still work in progress!

- Stochastic: not so smooth
- Robust: very conservative

Concluding remarks

- About robustness of solutions:
  - **Robust programming?** Most conservative
  - **Stochastic programming?** Risk of over-fitting

- About performance of solving:
  - Robust programming: little impact
  - Stochastic programming: quickly intractable
    - Dedicated algorithm: not to speed up!

And so?

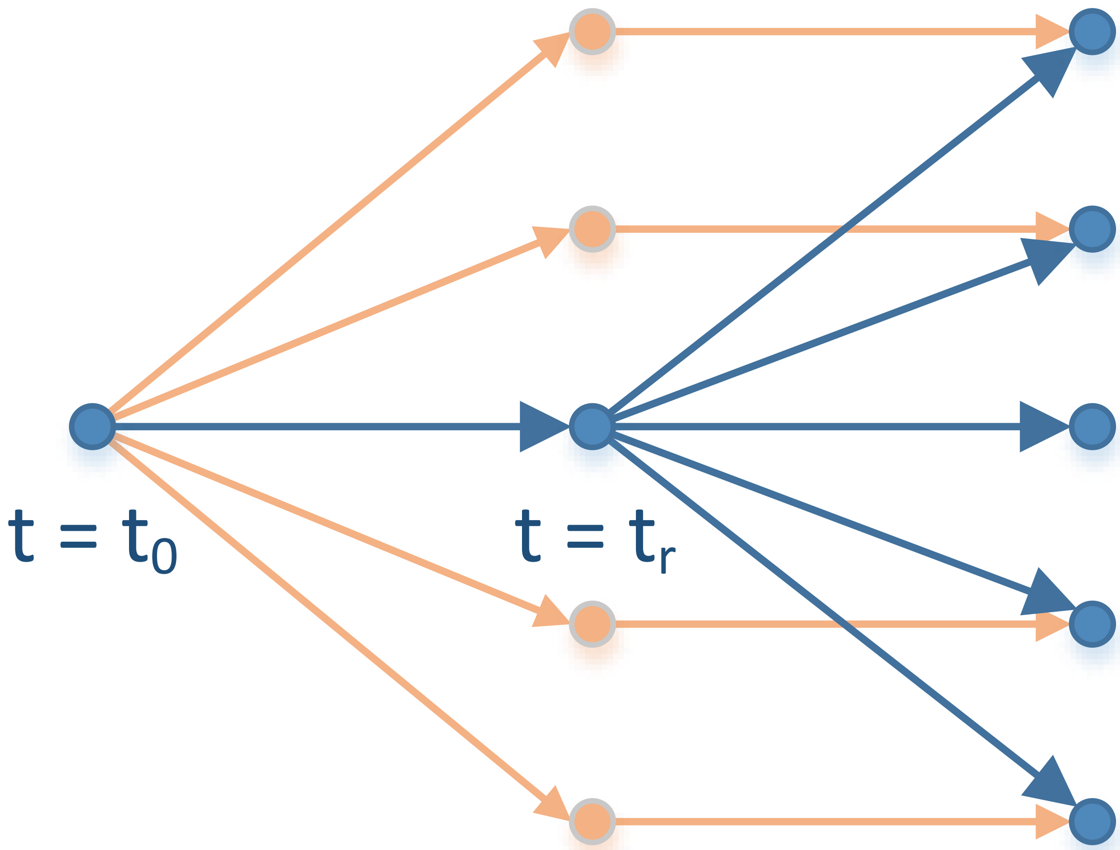
- Each has its strengths!
  - Provides different insights into the properties of the solution
  - Not incompatible within a given model

- Little research comparing those approaches
  - Nor a common modelling layer!

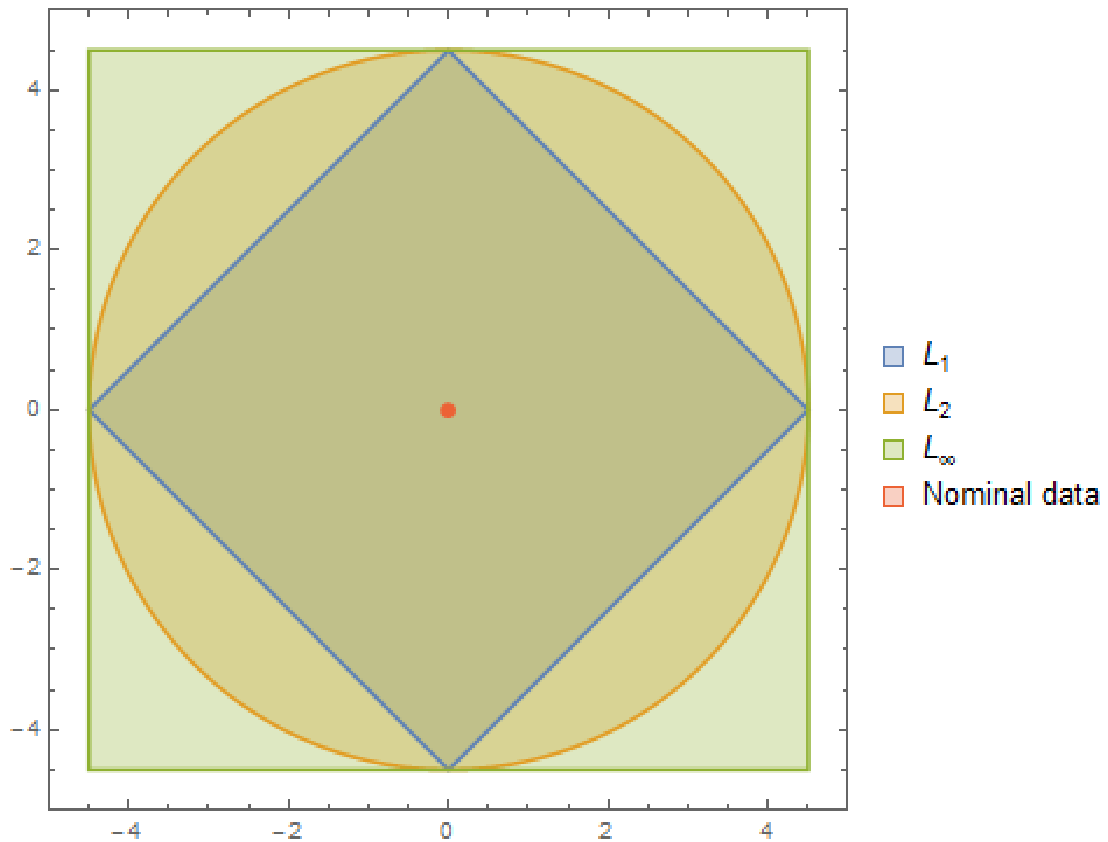
- Going further:
  - Look at other problems
  - Test different uncertainty sets

Back-up

Scenario tree

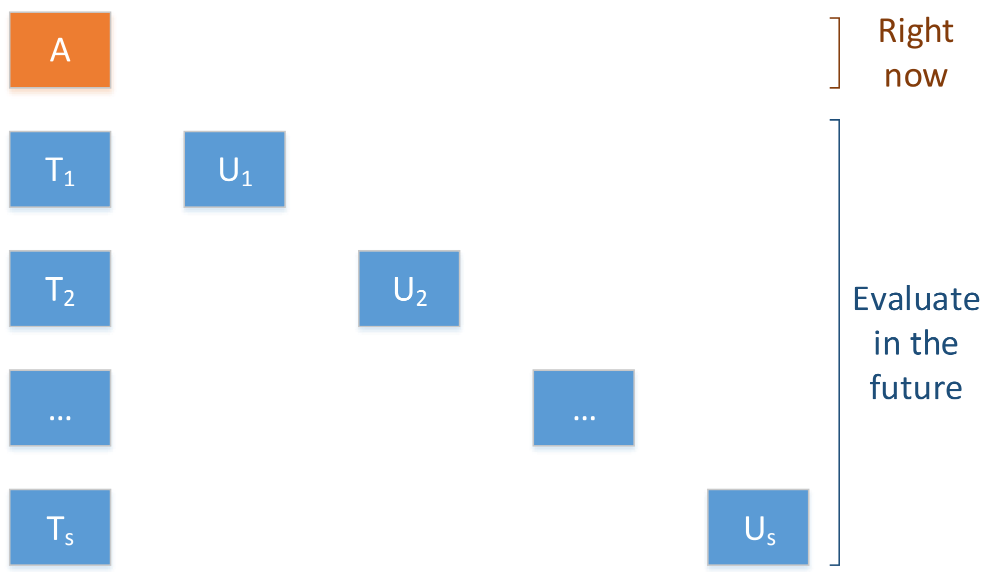


Ellipsoids



Solving stochastic programs

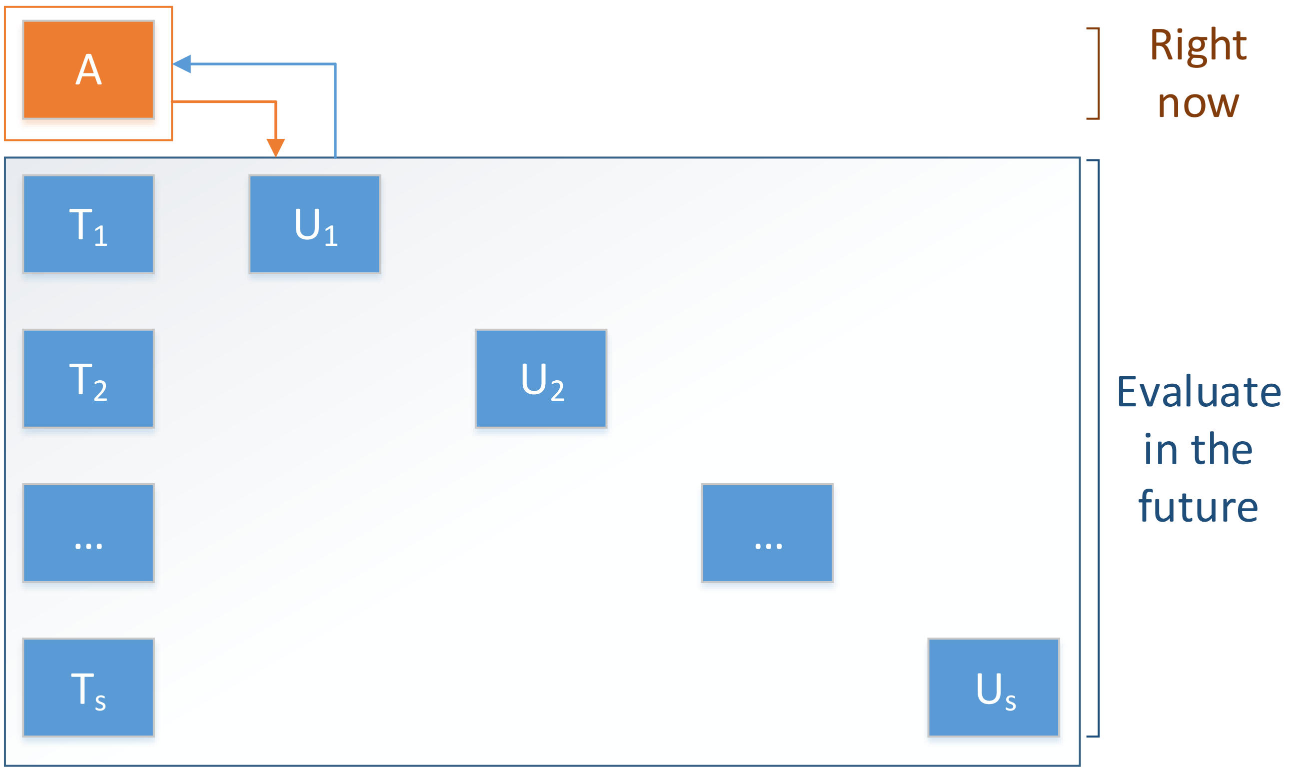
- Optimisation can be represented as a matrix
- Two-stage:



- Exploit this special structure!

Benders' decomposition

- Decompose this matrix along stages
- Consider only what happens now
  - And retrieve information from the future iteratively



Progressive hedging

- Decompose this matrix along scenarios
- Optimal solution for each scenario
  - Then bring them together