Anticoherence of spin states with point-group symmetries

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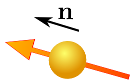
Coherent vs anticoherent spin-j states

Coherent states



$\langle \mathbf{J} \rangle = j\hbar\mathbf{n}$

$|\psi_j\rangle$ is a spin-j coherent state $|\mathbf{n}\rangle$ if it is eigenstate of $\mathbf{J}\cdot\mathbf{n}$ for some $\mathbf{n}$ with the highest eigenvalue $j\hbar$.

Anticoherent states

$\langle \mathbf{J} \rangle = 0$

$|\psi_j\rangle$ is a spin-j anticoherent state to order t if [1]

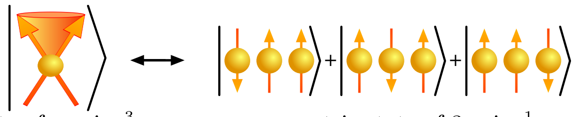
$$\langle\psi_j|\,(\mathbf{J}\cdot\mathbf{n})^k\,|\psi_j\rangle$$

does not depend on $\mathbf{n}$ for $k = 1, \ldots, t$.

One-to-one mapping

One-to-one mapping

single spin-j state ↔ symmetric state of $2j$ spin-$\frac{1}{2}$ ↔ symmetric state of $N = 2j$ qubits



state of a spin-$\frac{3}{2}$ ⟷ symmetric state of 3 spin-$\frac{1}{2}$

A single spin-j state can be seen as an N-qubit symmetric state

$$|\psi_S\rangle = \sum_{k=0}^{N} d_k |D_N^{(k)}\rangle, \qquad \sum_{k=0}^{N} |d_k|^2 = 1$$

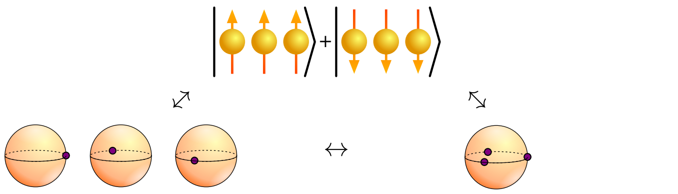
where $\{|D_N^{(k)}\rangle\}$ is the Dicke basis spanning the symmetric subspace, in formal correspondence with the standard basis $\{|j,m\rangle\}$ (common eigenstates of J_z and $\mathbf{J}^2$),

$$|D_N^{(k)}\rangle \leftrightarrow |\tfrac{N}{2}, \tfrac{N}{2} - k\rangle \qquad |j,m\rangle \leftrightarrow |D_{2j}^{(j-m)}\rangle$$

Geometrical representations

Majorana representation

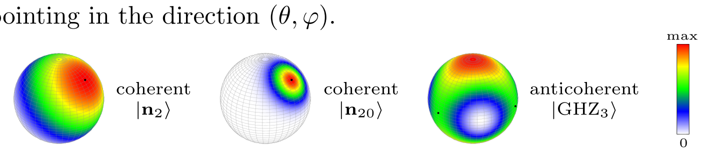
symmetric state of $N = 3$ qubits



$N = 3$ single-qubit states ↔ $N = 3$ points on the Bloch sphere

Husimi function

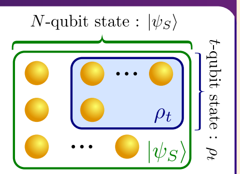
The Husimi function is the probability density to find the spin pointing in the direction (θ, φ).



For coherent states the Husimi function is localized around the Majorana points, all located at the same place.

Conditions for anticoherence

Results A multiqubit symmetric state $|\psi_S\rangle$ is **anticoherent** to order t iff all t-qubit states are maximally mixed [2,3] :



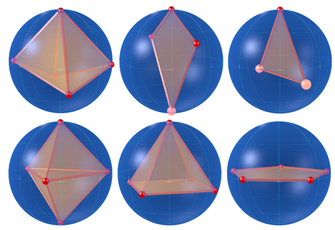
(1) $\rho_t = \dfrac{\mathbb{1}_{t+1}}{t+1}$ ⇔ maximal uncertainty about the state of $t < N$ qubits ⇔ maximal entanglement

Point groups for Majorana points

For anticoherent states, Majorana points are expected to be spread out over the Bloch sphere as evenly as possible, in complete opposition to coherent states. In this work, we have considered Majorana points arrangement with a point-group symmetry.

Results: link between geometry and entanglement [4].

- States whose Majorana points display C_n symmetry
  - have Dicke coefficients of the form
    $$\mathbf{d}_{C_n} = (\mathbf{0}_{n_S}, d_{n_S}, \mathbf{0}_{n-1}, d_{n_S+n}, \mathbf{0}_{n-1}, d_{n_S+2n}, \ldots, \mathbf{0}_{n_N})$$
  - have a diagonal $(n-1)$-qubit reduced density matrix in the Dicke basis.
- States whose Majorana points display C_{nh}, S_{2n} and D_n symmetries are necessarily anticoherent to order 1.
- If an entanglement class contains an anticoherent state which does not display C_n symmetry, then the class does not contain any state displaying C_n symmetry.

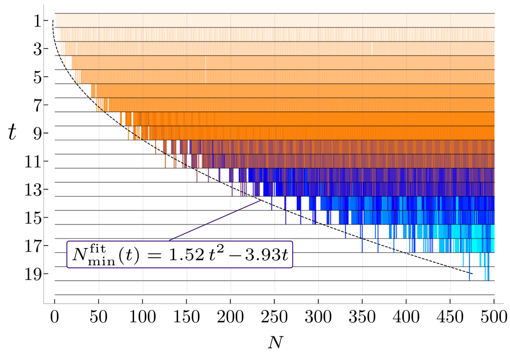


Majorana representation of 5-qubit states displaying C_n symmetry which are anticoherent to order 1 (from top left to bottom right $C_2, C_2, C_2, C_3, C_4, C_5$). The small (red) and large (pink) points on the spheres correspond to nondegenerate and twice-degenerate Majorana points, respectively.

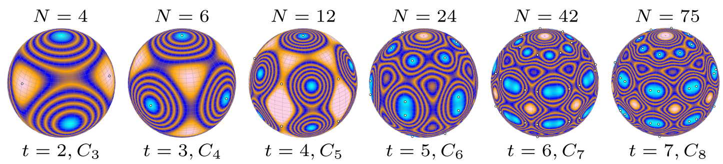
Higher order of anticoherence for C_n-symmetric states

Results The matrix equation (1) has been solved systematically for C_n symmetric states using linear programming [4].

Existence of anticoherent states



Each bar corresponds to a couple (t, N) for which a C_n-symmetric N-qubit t-anticoherent state with $t < n$ exists. Dashed line is a fit of the minimum number of qubits $N_{\min}$ for which such states exist.



Husimi function of C_n-symmetric states with the smallest possible number of qubits $N_{\min}$ for a given order of anticoherence t.

We have proved that anticoherent states (with cyclic symmetry) exist to any order (provided the number of qubits is large enough) [4].

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