A New Wavelet-Based Mode Decomposition for Oscillating Signals and Comparison with the Empirical Mode Decomposition

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- Decomposing time series into several modes has become more and more popular and useful in signal analysis.
- Methods such as EMD or SSA (among others) have been successfully applied in medicine, finance, climatology, ...
- Old but gold: Fourier transform allows to decompose a signal as

$$f(t) \approx \sum_{k=1}^{K} c_k \cos(\omega_k t + \phi_k).$$

- Problem: often too many components in the decomposition.
- Idea: Considering the amplitudes and frequencies as functions of t to decrease the number of terms.
- Development of an empirical wavelet mode decomposition (EWMD) method which can be used in numerous ways.

Edit: EWMD becomes WIME (wavelet-induced mode extraction).

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Empirical Mode Decomposition (EMD) in a nutshell

1. Given a signal f, compute m the mean of its upper and lower envelopes.
2. Compute $h = f - m$ and repeat steps 1. and 2. with h instead of f until h is "stable enough" and becomes an "intrinsic mode component" (IMF), f_1.
3. Repeat the whole process with $f - f_1$ instead of f.
4. The procedure always stops and the IMFs successively extracted reconstruct f accurately.

Main drawbacks include: lack of solid mathematical theoretical background, high sensitivity to noise, mode-mixing problems.

Improvements made are computationally expensive, not intuitive.

Empirical Wavelet Mode Decomposition (EWMD) – WIME

- The wavelet used in this study is the function

$$\psi(t) = \frac{e^{i\Omega t}}{2\sqrt{2\pi}} e^{-\frac{(2\Omega t+\pi)^2}{8\Omega^2}} \left(e^{\frac{\pi t}{\Omega}} + 1\right)$$

  with $\Omega = \pi\sqrt{2/\ln 2}$, which is similar to the Morlet wavelet. The Fourier transform of ψ is given by

$$\hat{\psi}(\omega) = \sin\left(\frac{\pi\omega}{2\Omega}\right) e^{\frac{-(\omega-\Omega)^2}{2}} .$$

- The wavelet transform of the signal is computed as:

$$Wf(a,t) = \int f(x)\bar{\psi}\left(\frac{x-t}{a}\right)\frac{dx}{a},$$

  where $\bar{\psi}$ is the complex conjugate of ψ, $t \in \mathbb{R}$ stands for the location/time parameter and $a > 0$ denotes the scale parameter.

Empirical Wavelet Mode Decomposition (EWMD) – WIME

a) Perform the continuous wavelet transform $Wf(a,t)$ of f.

b) Compute the wavelet spectrum Λ associated to f:

$$\Lambda(a) = E\,|Wf(a,.)|$$

where E denotes the mean over time. Then look for the scale a^* at which Λ reaches its global maximum.

c) Extract the component related to a^*:

$$c_1 = R_\psi^{-1}|Wf(a^*,t)|\cos(\arg Wf(a^*,t))$$

where $R_\psi \approx 1.25$ is a reconstruction constant.

d) Repeat steps (a) to (d) with $f - c_1$ instead of f.

e) Stop the process when $\Lambda(a^*) < \varepsilon$ for a threshold ε or at your convenience. The components successively extracted reconstruct f accurately.

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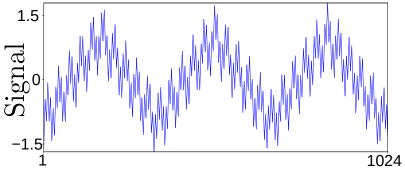
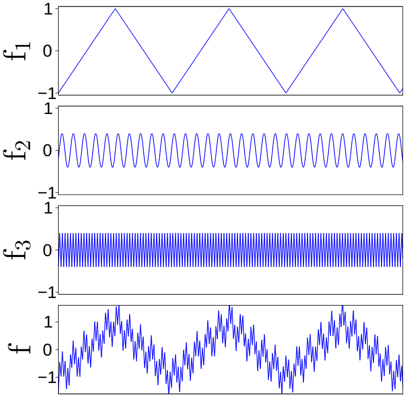
Reconstruction skills

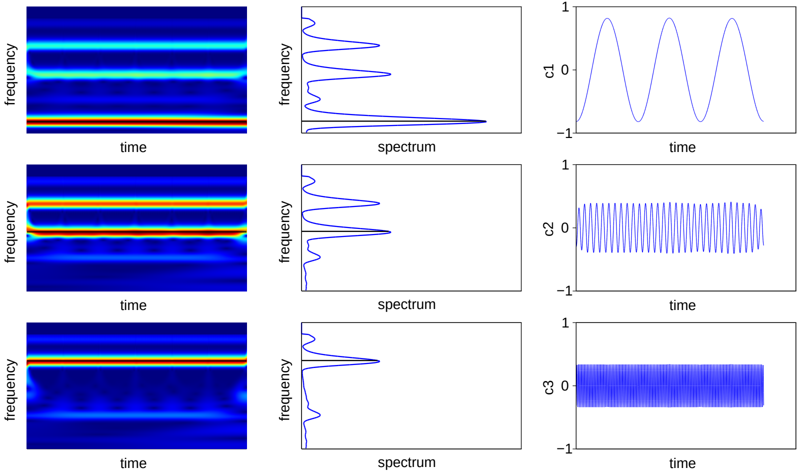
First example:





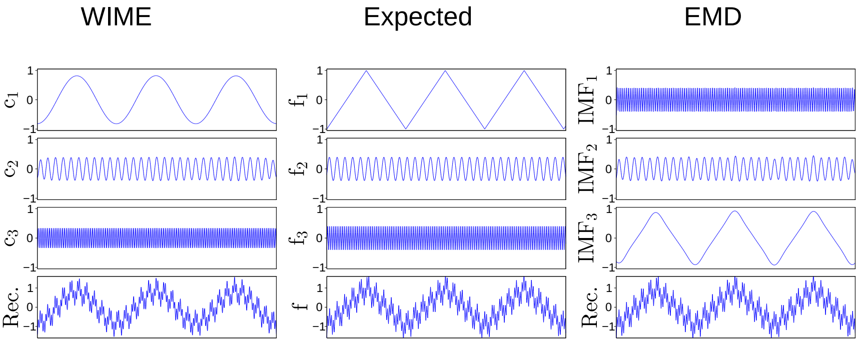
Sum of 2 triangular waveforms and a sine wave (classic example of the EMD).

Reconstruction skills



First row: $|Wf(a,t)|$, spectrum of f, first extracted component c_1. Second and third rows: the same for $f - c_1$ and $f - c_1 - c_2$.

Reconstruction skills



First column: components extracted with WIME. Second column: the real (expected) components. Third column: the IMFs extracted with the EMD. The reconstruction $c_1 + c_2 + c_3$ has a correlation of 0.992 with the original signal and a RMSE of 0.085.

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Period detection

We consider the AM-FM signal $f(x) = \sum_{i=1}^{4} f_i(x)$ where

$$f_1(x) = \left(1 + 0.5\cos\left(\frac{2\pi}{200}x\right)\right)\cos\left(\frac{2\pi}{47}x\right) \quad (1)$$

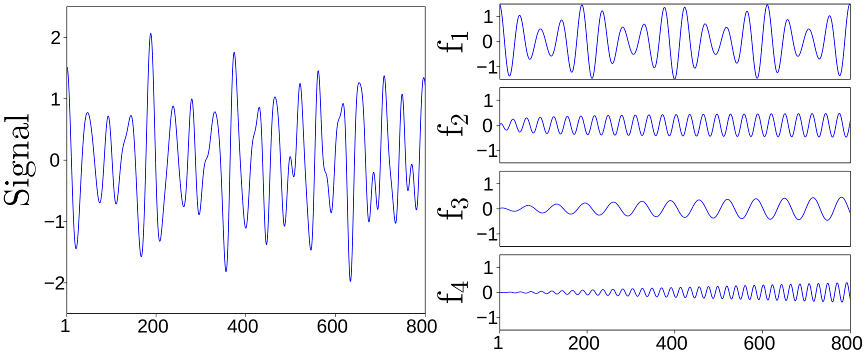
$$f_2(x) = \frac{\ln(x)}{14}\cos\left(\frac{2\pi}{31}x\right) \quad (2)$$

$$f_3(x) = \frac{\sqrt{x}}{60}\cos\left(\frac{2\pi}{65}x\right) \quad (3)$$

$$f_4(x) = \frac{x}{2000}\cos\left(\frac{2\pi}{23 + \cos\left(\frac{2\pi}{1600}x\right)}x\right) . \quad (4)$$

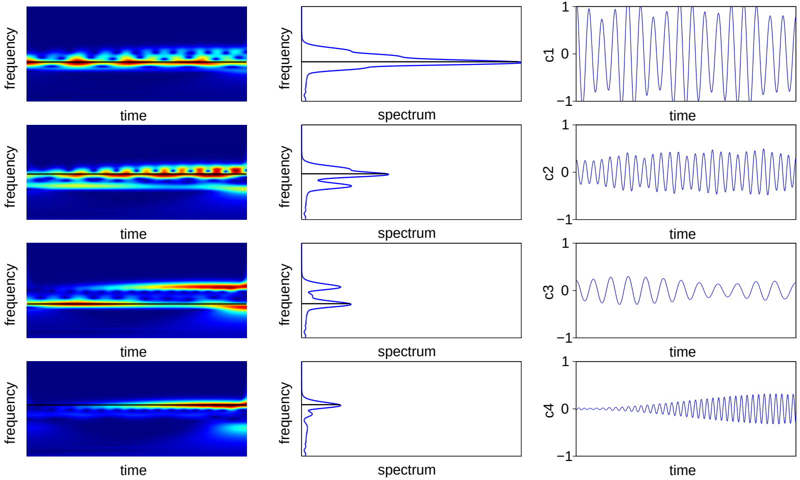
Target periods: $\approx$ 23, 31, 47, 65 units.

Period detection



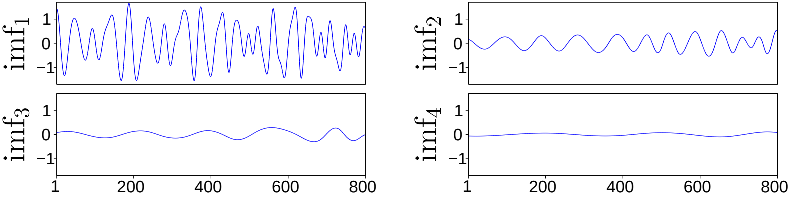
Target periods: $\approx$ 23, 31, 47, 65 units.

Period detection - WIME



Target periods: $\approx$ 23, 31, 47, 65 units. Detected periods corresponding to a^*: 46.4, 30.6, 65.5 and 21.6 units. Correlation: 0.996. RMSE: 0.069.

Period detection - EMD



Target periods: $\approx$ 23, 31, 47, 65 units. Periods extracted from the Hilbert-Huang transform: 41, 75, 165, 284 units.

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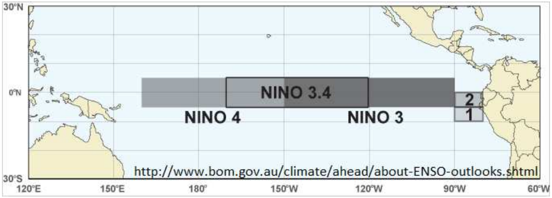
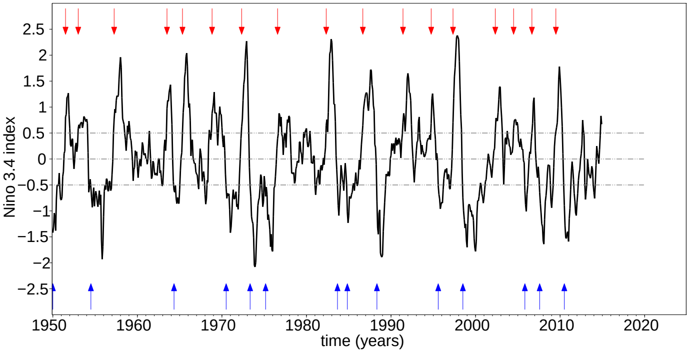
ENSO index

- Analyzed data: Niño 3.4 time series, i.e. monthly-sampled sea surface temperature anomalies in the Equatorial Pacific Ocean from Jan 1950 to Dec 2014 (`http://www.cpc.ncep.noaa.gov/`).



ENSO index

- Niño 3.4 index:

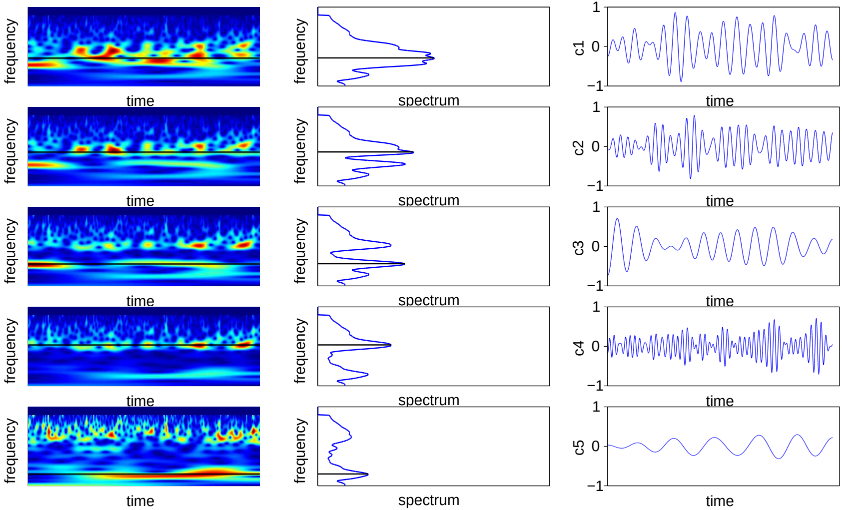


- 17 El Niño events: SST anomaly above $+0.5^{\circ}$C during 5 consecutive months.
- 14 La Niña events: SST anomaly below -0.5°C during 5 consecutive months.

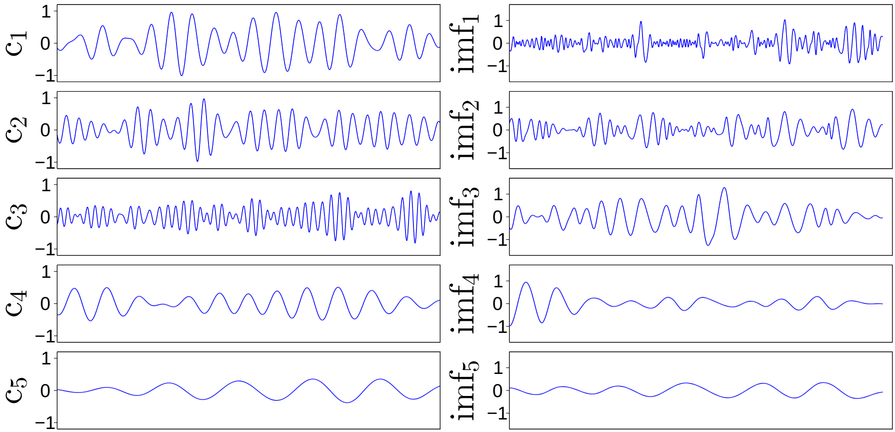
ENSO index

- Flooding in the West coast of South America
- Droughts in Asia and Australia
- Fish kills or shifts in locations and types of fish, having economic impacts in Peru and Chile
- Impact on snowfalls and monsoons, drier/hotter/wetter/cooler than normal conditions
- Impact on hurricanes/typhoons occurrences
- Links with famines, increase in mosquito-borne diseases (malaria, dengue, ...), civil conflicts
- In Los Angeles, increase in the number of some species of mosquitoes (in 1997 notably).
- ...

ENSO index



ENSO index



Components extracted with WIME and the IMFs from the EMD. Periods: 44.8, 28.6, 17, 65.6, 140.6 months (WIME) and 9.8, 21, 38.6, 75.9, 138.4 (EMD). Those from WIME are more in agreement with some previous works.

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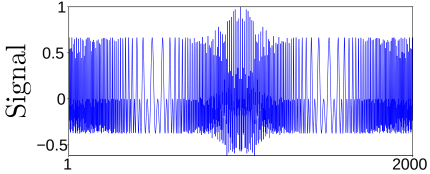
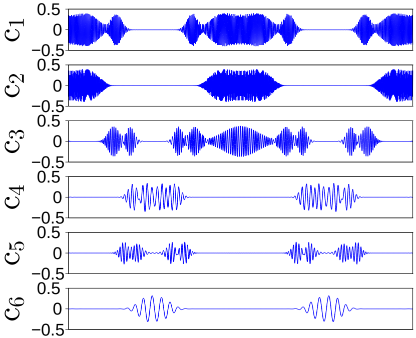
Recent improvements

Problems with “highly” non-stationary components (classic example of the EMD):

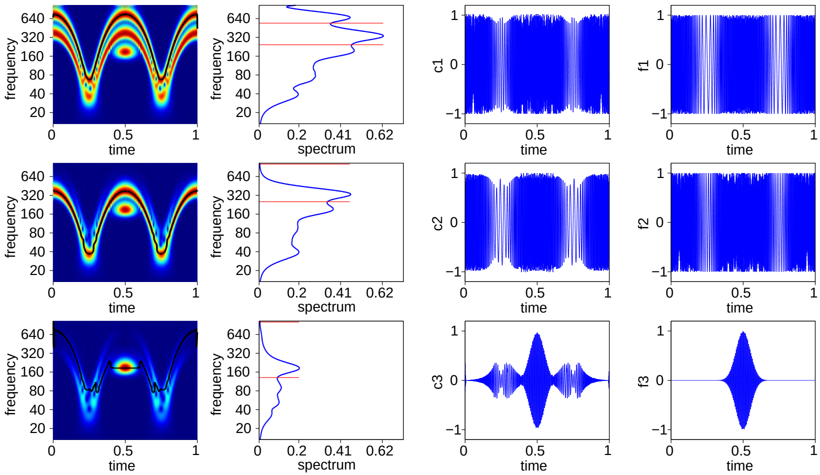




Reconstruction is excellent (RMSE $= 0.08$, PCC $= 0.97$) but components are not correct.

Recent improvements

Solution: add flexibility, follow ridges of maxima.



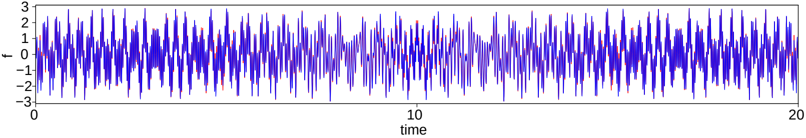
Recent improvements

Crossings in the TF plane:

$$f_1(t) = 1.25\cos\left(-(t-3)^3 + 180t\right) \quad (5)$$

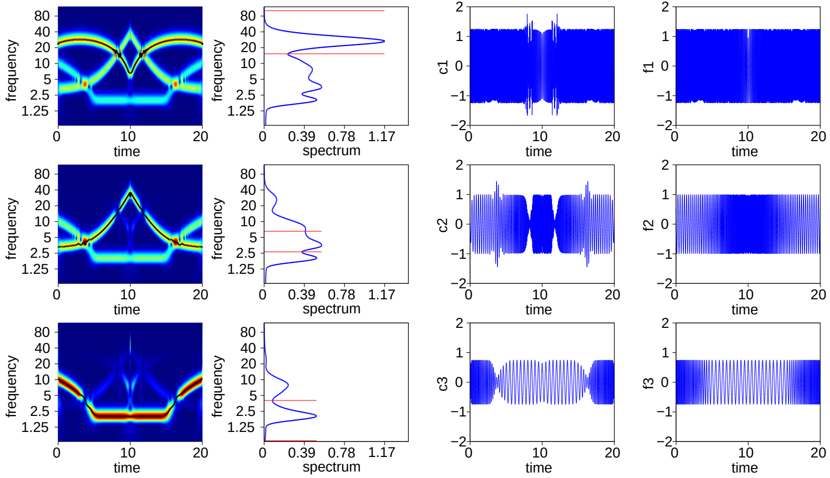
$$f_2(t) = \cos\left(1.8^t + 20t\right) \quad (6)$$

$$f_3(t) = \begin{cases} 0.75\cos(-1.6\pi t^2 + 20\pi t) & (t < 5) \\ 0.75\cos(4\pi t) & (t \geq 5) \end{cases} \quad (7)$$



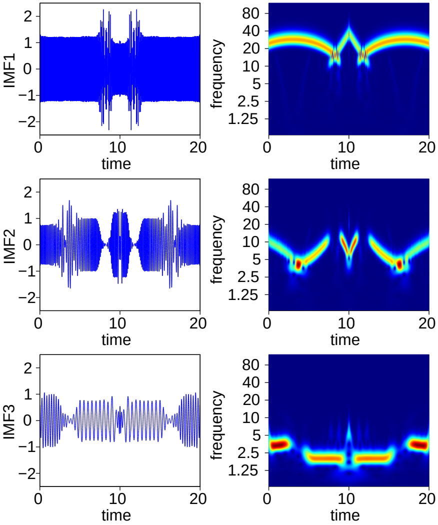
Recent improvements

WIME



Recent improvements

EMD



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Conclusion

WIME

- is a wavelet-based decomposition method
- has reconstruction skills comparable to the EMD
- has period detection skills better than the EMD in mode-mixing problems
- handles crossing patterns in the TF plane
- gives accurate results with real-life signals

Future works: study the mathematical properties of the method, test its tolerance to noise, limit edge effects,...

Thanks for your attention.

Please refer to the paper for relevant references.