BLIND SOURCE SEPARATION TECHNIQUES — ANOTHER WAY OF DOING OPERATIONAL MODAL ANALYSIS

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Abstract

In many practical applications, it is difficult or even impossible to measure the force applied to a structure. This is especially the case in civil engineering applications. Therefore, this paper proposes to explore the utility of blind source separation (BSS) techniques for operational modal analysis. The basic idea of BSS is to recover unobserved source signals from their observed mixtures. The feasibility and practicality of the proposed method are demonstrated using an experimental application.

1 Introduction

For modal analysis of large structures, it is unpractical and expensive to use artificial excitation (e.g., shakers). However, engineering structures are most often subject to ambient loads (e.g., traffic and wind) that can be exploited for modal parameter estimation. One difficulty is that the actual loading conditions cannot generally be measured, and output-only measurements are available.

During the last few years, there have been several successful attempts to address this issue using operational modal analysis (OMA) techniques [1-2]. Recently, signal processing techniques have been used to perform OMA through the estimation of the modal coordinates. For instance, Lardies et al. [3] exploit the wavelet transform to determine the response of each mode and to subsequently compute the modal parameters. In [4], output-only data are processed using the empirical mode decomposition (also known as Hilbert-Huang transform) to identify the different modal contributions. Digital band-pass filters are considered by Kim et al. [5] for the same purpose. Although attractive in principle, these signal processing-based methods present several drawbacks such as edge effects and difficulty in identifying closely spaced modes.

In this paper, we propose a new OMA method by borrowing one technique from the statistical literature. The technique, second-order blind identification (SOBI), decomposes measured signals in terms of elemental components. When SOBI is applied to the response of engineering structures, the elemental components are directly related to the modal coordinates, as shown in [6]. This is also the case for other blind source separation (BSS) techniques such as independent component analysis (ICA, [8]), but this is not discussed herein. The reader can refer to [7] for further detail.

2 Blind Source Separation (BSS)

2.1 Theoretical Background

Recovering unobserved source signals from their observed mixtures is a generic problem in many domains and is referred to as blind source separation (BSS) in the literature. One well-known example is the cocktail-party problem, the objective of which is to retrieve the speech signals emitted by several persons speaking simultaneously in a room using only the signals recorded by a set of microphones located in the room.

The simplest BSS model assumes the existence of n source signals $s_1(t),...,s_n(t)$ and the observation of as many mixtures $x_1(t),...,x_n(t)$. Although convolutive and non-linear mixtures can be considered, we focus on linear and static mixtures for which BSS is well established. The noisy model can be expressed as

$$x_i(t) = \sum_{j=1}^{n} a_{ij} s_j(t) + \sigma_i(t), \quad i = 1,...,n \tag{1}$$

or, in matrix form,

$$\mathbf{x}(t) = \mathbf{y}(t) + \boldsymbol{\sigma}(t) = \mathbf{A}\mathbf{s}(t) + \boldsymbol{\sigma}(t) \tag{2}$$

where $\mathbf{A}$ is referred to as the mixing matrix, and $\boldsymbol{\sigma}(t)$ is the noise vector corrupting the data.

The basic idea of BSS is to recover the unobserved source signals $\mathbf{s}(t)$ from their observed mixtures $\mathbf{x}(t)$. Blind means that very little, if anything, is known about the mixing matrix, and that fairly general assumptions are made about the source signals. BSS has two inherent indeterminacies, because the mixing matrix is only identifiable up to scaling and permutation of its rows. As a result, it is not possible to determine the order and the variances of the identified sources. Generally, they are scaled to have a unit variance.

2.2 Second-Order Blind Identification (SOBI)

Most BSS approaches are based (explicitly or not) on a model in which the sources are independent and identically distributed variables. On the contrary, the objective of the SOBI algorithm is to take advantage, whenever possible, of the temporal structure of the sources for facilitating their separation [9]. SOBI is therefore an interesting technique for sources with different spectral contents, which is often the case in structural dynamics as explained in Section 3.

For the sake of conciseness, a detailed description of the SOBI method is not carried out herein. The reader can refer to [6,9] for further detail. However, we mention that SOBI is based on the joint diagonalization of time-lagged covariance matrices

$$\mathbf{R}(\tau) = E\left[\mathbf{x}(t+\tau)\mathbf{x}^*(t)\right] \tag{3}$$

These matrices are evaluated from the measured data, and the basic idea is to find a matrix $\mathbf{U}$, which jointly diagonalizes all the covariance matrices. It can be proven that the unitary matrix $\mathbf{U}$ corresponds to the mixing matrix $\mathbf{A}$. For illustration, SOBI is applied to a mixture of five sinusoids in Figure 1 and provides an accurate identification of the sources.

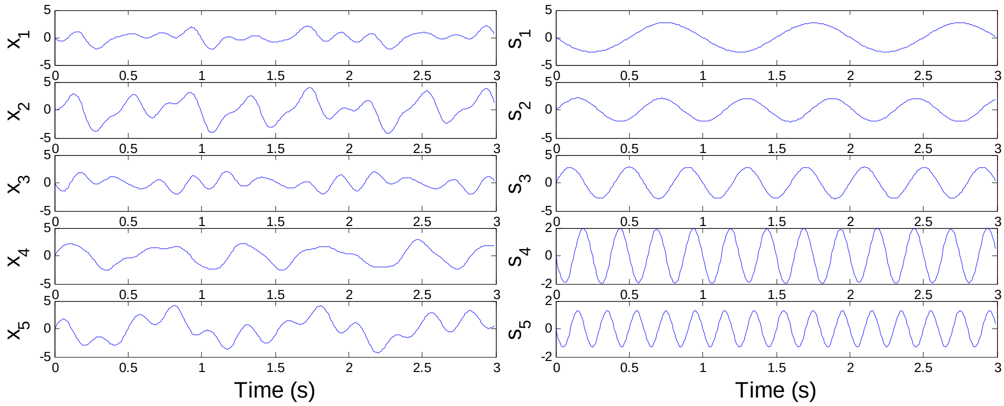

Fig. 1: Mixture of five sinusoids (left plots) and sources identified using SOBI (right plots)

3 Modal Coordinates: Virtual Sources

Consider a mechanical system governed by the equations of motion

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \tag{4}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively. The system can be seen as a dynamic mixture of sources, because its response $\mathbf{x}(t)$ is the convolution product of the impulse response function $\mathbf{h}(t)$ and the external force vector $\mathbf{f}(t)$. However, the application of BSS techniques to mechanical systems is difficult, because the problem of a convolutive mixture of sources is not yet completely solved.

Without loss of generality, the conservative system is considered now. Because the normal modes provide a complete set for the expansion of an arbitrary vector, the response of system (4) may be expressed through modal expansion

$$\mathbf{x}(t)=\sum_{i=1}^{m}\mathbf{n}_{(i)}(t)\eta_i(t)=\mathbf{N}\,\eta(t) \tag{5}$$

where $\eta_i(t)$ are the modal coordinates, that is the amplitude modulations of the corresponding normal modes $\mathbf{n}_{(i)}$, and m is the number of degrees of freedom of the system. Expression (5) shows that when expanding the system response in terms of the vibration modes, the modal coordinates may act as virtual sources regardless of the number and type of the physical excitation forces [6-7]. In addition, the time response can be interpreted as a static mixture of these virtual sources, which renders the application of BSS techniques possible. Finally, the modal coordinates are monochromatic (i.e., sources with different spectral contents) for the free response and mostly monochromatic for the random response, which is exactly what SOBI requires.

In summary, the key idea is to interpret the modal coordinates of a dynamic system as virtual sources with different spectral contents. In this context, SOBI provides a one-to-one mapping between the mixing matrix and the vibration modes of the structure, which forms the basis of a truly simple modal analysis procedure:

1. Perform experimental measurements of the response of the tested system to obtain time series at different sensing positions.
2. Apply SOBI to the measured time series to estimate the mixing matrix and the sources.
3. The mode shapes are contained in the mixing matrix $\mathbf{A}$.
4. It is then straightforward to identify the natural frequencies and damping ratios of the corresponding vibration modes. In the free response case, this is carried out by fitting the time series of the sources $\mathbf{s}(t)$ with exponentially damped harmonic functions

$$Y \exp(-\xi\omega t) \cos\left(\sqrt{1-\xi^2}\,\omega t + \alpha\right) \tag{6}$$

   where ω and ξ are the natural frequency and damping ratio of the considered mode, respectively. The amplitude Y and the phase α are constants depending on the initial conditions. In the case of random excitation, the same procedure can be applied, but the identified (random) sources are first transformed into free decaying responses using NExT (Natural Excitation Technique) algorithm [10].

An interesting feature of the proposed method is that it does not require the measurement of the applied force and can perform OMA. This is particularly convenient in practical applications for which the external force cannot be measured (e.g., vibrations of a bridge due to traffic and wind).

4 Experimental Demonstration

To support the previous theoretical findings, the proposed OMA technique was applied to the response of the truss structure depicted in Figure 2. For the free response, a hammer provided a short impulse to the system. For the random response, the structure was mounted on a 26kN electrodynamic shaker, as shown in Figure 2. 16 accelerometers were distributed on the structure (two at each corner, eight on each storey), measuring its response in a horizontal plane. The results were also compared with another OMA technique, the (covariance-driven) stochastic subspace identification (SSI) technique [1,11].

4.1 Free Response

The SOBI and SSI methods were first applied to the free response of the truss structure. The first 6000 samples of the measured time series were taken into account, the sampling frequency being set to 5120 Hz. For SSI, 20 block rows and columns were selected in the Hankel matrix. To build the time-lagged covariance matrices in SOBI (cf. Equation 3), 20 time lags τ were uniformly distributed between 0.0025 and 0.1 s, which covers the frequency range of interest.
Because there are 16 measurement locations, a total of 16 virtual sources can be considered. The identification of reliable virtual sources is greatly facilitated by computing the error realized during the fitting of the time series of the sources with exponentially damped harmonic functions. For instance, Figure 3 depicts the measured and fitted signals for two different sources. Clearly, the measured source in the top plot of Figure 3 can be considered as an actual source (fitting error=0.1%), whereas this is likely not the case for the source in the bottom plot (fitting error=69%). Figure 4 shows the fitting error of the 16 virtual sources. 11 sources have a fitting error below 7% and can be safely retained. The sum of their participation in the system response is above 97.7%.

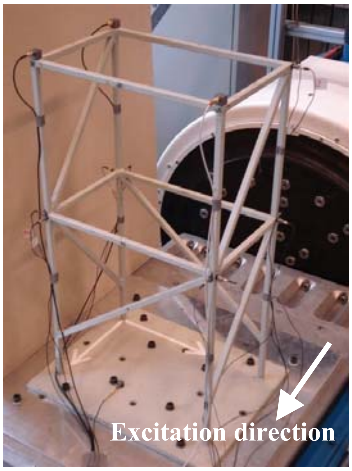

Fig. 2: Experimental fixture mounted on a 26kN electrodynamic shaker

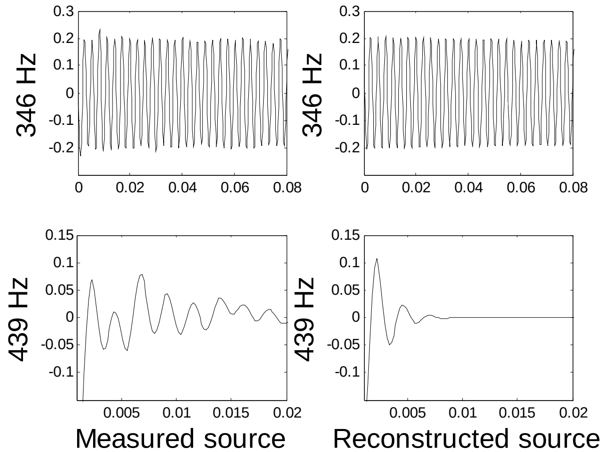

Fig. 3: Measured and reconstructed sources (free response)

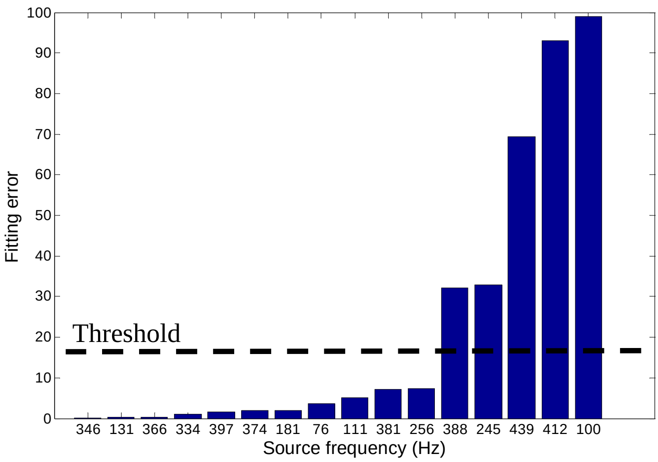

Fig. 4: Fitting error of the 16 virtual sources (free response, SOBI)

The identification results are listed in Table 1, which shows that the truss structure possesses closely spaced modes. For SSI, an interval rather than a well-definite value is given for the damping ratios, because their values change with the selected model order. The MAC matrix between the modes identified using SSI and SOBI is shown in Figure 5. One can see a complete correspondence of the results obtained with both methods, which confirms that an accurate and consistent identification is carried out using SOBI.

Frequency (Hz) (SOBI)	Frequency (Hz) (SSI)	Damping ratio (%) (SOBI)	Damping ratio (%) (SSI)
75.94	75.82	0.20	[0.05 – 0.12]
111.37	110.99	0.37	[0.40 – 0.60]
130.75	130.76	0.21	[0.20 – 0.28]
181.06	180.69	0.18	[0.20 – 0.28]
256.30	256.48	0.18	[0.10 – 0.15]
334.24	334.32	0.05	[0.02 – 0.05]
345.75	345.76	0.04	[0.04 – 0.05]
365.79	365.81	0.05	[0.05 – 0.06]
374.34	374.45	0.15	[0.10 – 0.30]
380.55	380.45	0.16	[0.20 – 0.40]
396.91	396.81	0.08	[0.07 – 0.10]

Table 1: Identified natural frequencies and damping ratios (free response)

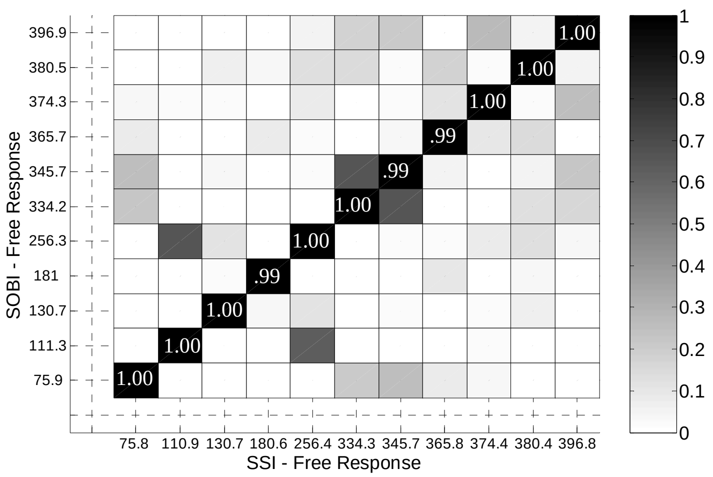

Fig. 5: MAC matrix between the modes identified using SOBI and SSI (free response)

4.2 Random Response

The SOBI and SSI methods were then applied to the random response of the truss structure. The random force imparted to the truss structure by the electrodynamic shaker was not measured. 160000 samples of the measured time series were taken into account, the sampling frequency being set to 5120 Hz. For SSI, 20 block rows and columns were selected in the Hankel matrix. For SOBI, 20 time lags τ were uniformly distributed between 0.0025 and 0.1 s.

Figure 6 shows the fitting error of the 16 virtual sources. 14 sources have a fitting error below 8%, and the two remaining sources have an error above 60%. Source selection is therefore trivial. The identification results are listed in Table 2. The MAC matrix between the modes identified using SSI and SOBI is shown in Figure 7. Apart from the first mode, there is an excellent agreement between the results obtained with both methods. If the results obtained using SSI in the free response case are taken as a reference, one can compare how well SOBI and SSI perform in the random case. Table 3 shows that they perform equally well. Another finding is that none of the methods seems able to accurately estimate the mode at 75 Hz using the random response. This is because the structural deformation of this mode takes place in a plane orthogonal to the excitation direction. The mode has therefore a very low participation in the system response.

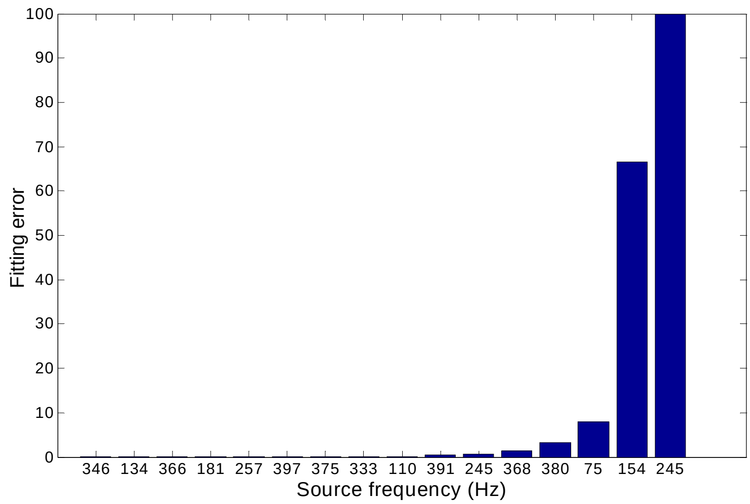

Fig. 6: Fitting error of the 16 virtual sources (random response, SOBI)

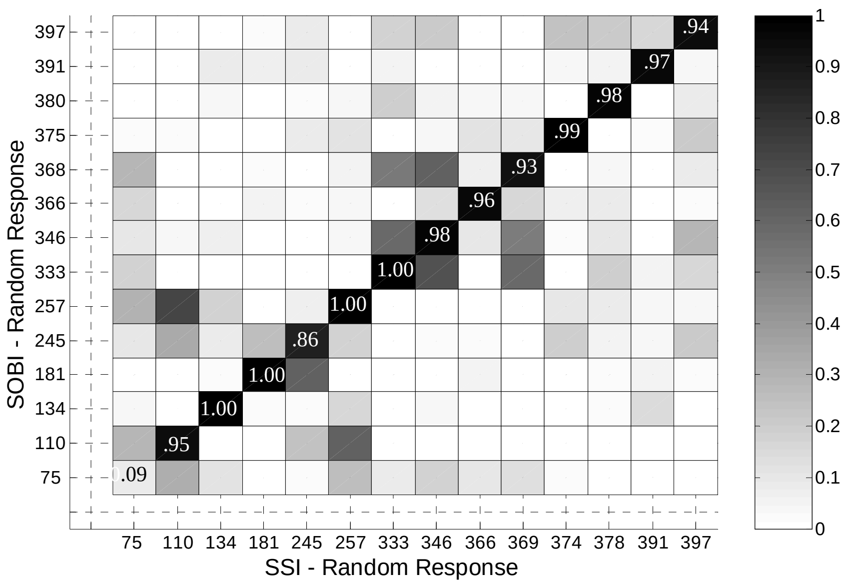

Fig. 7: MAC matrix between the modes identified using SOBI and SSI (random response)

Freq. (Hz-SOBI)	Freq. (Hz-SSI)	Damp. (%-SOBI)	Damp. (%-SSI)
74.75	74.68	2.15	[1.70 – 2.00]
110.06	110.28	2.03	[1.50 – 2.00]
133.59	133.77	0.85	[0.60 – 0.80]
180.87	180.98	0.23	[0.20 – 0.30]
245.29	245.38	0.16	[0.01 – 0.05]
257.47	257.47	0.11	[0.09 – 0.11]
333.21	333.34	0.12	[0.05 – 0.10]
345.64	345.51	0.09	[0.10 – 0.12]
365.60	365.76	0.12	[0.07 – 0.15]
368.19	369.53	0.33	[0.15 – 0.30]
374.34	374.69	0.16	[0.20 – 0.40]
380.06	378.34	0.71	[0.50 – 0.70]
390.81	390.95	0.33	[0.45 – 0.50]
396.83	397.20	0.17	[0.15 – 0.25]

Table 2: Identified natural frequencies and damping ratios (random response)

Frequency	MAC(SOBI random, SSI free)	MAC(SSI random, SSI free)
75	0.64	0.63
110	0.99	0.96
134	1.00	1.00
181	0.99	0.99
245	0.78	0.94
257	0.99	0.99
333	1.00	1.00
346	0.93	0.98
366	0.92	0.92
374	0.95	0.95
379	0.94	0.94
391	0.81	0.71
397	0.95	0.97

Table 3: Correlation of the modes identified using the random response (SOBI and SSI) with those identified using the free response (SSI)

5 Conclusion

In this paper, a new operational modal analysis method is introduced by extracting modal coordinates from structural responses through second-order blind identification. The experimental application shows that the method holds promise for identification of mechanical systems:

- A truly simple identification scheme is proposed, because the straightforward application of SOBI to the measured data yields the modal parameters.
- A seemingly robust criterion has been developed for the selection of reliable sources. The use of stabilization charts, which always require a great deal of expertise, is therefore avoided. In addition, the selection of a model order, a common issue for conventional modal analysis techniques such as SSI, is not necessary.
- Compared to SSI, the computation load is very reduced, which makes the method a potential candidate for online modal analysis.

A possible limitation of the method is that sensors should always be chosen in number greater or equal to the number of active modes. This will be addressed in subsequent studies.

6 Acknowledgments

The author F. Poncelet is supported by a grant from the Walloon Government (Contract No. 516108— VAMOSNL). The author G. Kerschen is supported by a grant from the Belgian National Fund for Scientific Research (FNRS).

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