A nonlinear state-space solution to a hysteretic benchmark in system identification

J.P. Noël, G. Kerschen
Space Structures and Systems Laboratory
University of Liège, Liège, Belgium
jp.noel, g.kerschen@ulg.ac.be

A.F. Esfahani, J. Schoukens
ELEC Department
Vrije Universiteit Brussel, Brussels, Belgium
alireza.fakhrizadeh.esfahani, johan.schoukens@vub.ac.be

1 Introduction and model structure

The present contribution addresses the identification of the hysteretic benchmark introduced in Ref. [1]. To this end, a discrete-time nonlinear state-space model is built considering multivariate polynomial terms in the state equation, *i.e.*

$$\left\{ \begin{array}{rcl} \mathbf{x}(t+1) & = & \mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\,\mathbf{u}(t)+\mathbf{E}\,\mathbf{e}\,(\mathbf{x}(t)) \\ \mathbf{y}(t) & = & \mathbf{C}\,\mathbf{x}(t)+\mathbf{D}\,\mathbf{u}(t), \end{array} \right. \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^q$ the input vector, $\mathbf{y} \in \mathbb{R}^l$ the output vector, n the model order, and where $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times q}$, $\mathbf{C} \in \mathbb{R}^{l\times n}$ and $\mathbf{D} \in \mathbb{R}^{l\times q}$ are the linear state, input, output and feedthrough matrices, respectively. The vector $\mathbf{e} \in \mathbb{R}^{n_e}$ includes all monomial combinations of the state variables up to degree d. The associated coefficients are arranged in matrix $\mathbf{E} \in \mathbb{R}^{n\times n_e}$.

2 Identification methodology

A two-step methodology was proposed in Ref. [2] to identify the parameters of the model structure in Eqs. (1). First, initial estimates of the linear system matrices $(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D})$ are calculated by measuring and fitting the best linear approximation (BLA) of the system under test. Second, assuming zero initial values for the nonlinear coefficients in $\mathbf{E}$, a full nonlinear model is obtained using optimisation.

3 Identification results and discussion

The frequency-domain behaviour of the validation modelling error is studied in Fig. 1, where the output spectrum in grey is compared with linear and nonlinear fitting error levels in orange and blue, respectively. Details regarding the excitation signals and noise assumptions underlying the construction of this figure are to be found in Ref. [3]. Using monomials of degree 3, 5 and 7 is found to reduce the linear modelling error by a factor of 30 dB, whilst requiring 217 model parameters. It should be remarked that an exact polynomial description of the nonlinearities in the system demands, in principle, an infinite series of terms, preventing the nonlinear error in the figure from reaching the noise level depicted in black.

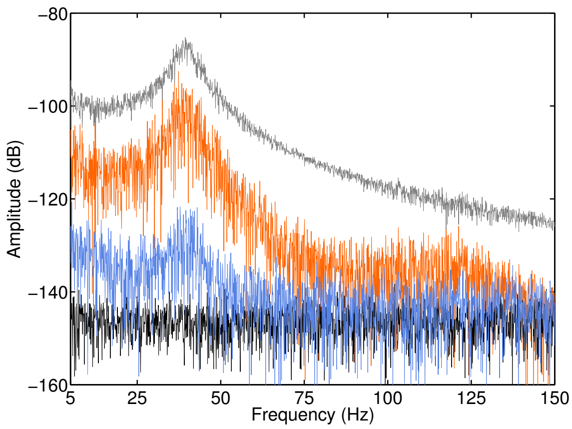

Figure 1: Validation modelling error, featuring the output spectrum (in grey), the linear (in orange) and nonlinear (in blue) fitting error levels, and the noise level (in black) considering monomials of degree 3, 5 and 7.

Acknowledgement

This work was supported by the ERC Advanced grant SNL-SID, under contract 320378.

References

[1] J.P. Noël and M. Schoukens, Hysteretic benchmark with a dynamic nonlinearity, unpublished document, 2016.

[2] J. Paduart, L. Lauwers, J. Swevers, K. Smolders, J. Schoukens, R. Pintelon, Identification of nonlinear systems using Polynomial Nonlinear State Space models, 2010, Automatica 46, 647-656.

[3] J.P. Noël, A.F. Esfahani, G. Kerschen, J. Schoukens, A nonlinear state-space approach to hysteresis identification, 2016, Mechanical Systems and Signal Processing, in review.