Using a polynomial decoupling algorithm for state-space identification of a Bouc-Wen system

Alireza Fakhrizadeh Esfahani, Philippe Dreesen, Koen Tiels, Jean-Philippe Noël⋆, Johan Schoukens
firstname.lastname@vub.ac.be ⋆jp.noel@ulg.ac.be
Vrije Universiteit Brussel, Dept. ELEC ⋆University of Liege

1 Introduction

The polynomial nonlinear state space (PNLSS) approach [1] is a powerful tool for modeling nonlinear systems. A PNLSS model consists of a discrete-time linear state space model, extended with polynomials in the state and the output equation:

$$x(t+1) = Ax(t)+Bu(t)+E\zeta(t) \quad (1)$$
$$y(t) = Cx(t)+Du(t)+F\eta(t) \quad (2)$$

where $\zeta(x(t),u(t))$ and $\eta(x(t),u(t))$ are both vectors with monomials in the states $x(t)$ and the inputs $u(t)$. The matrices E and F contain the polynomial coefficients. The PNLSS model is very flexible as it can capture many different types of nonlinear behavior, such as nonlinear feedback and hysteresis. This flexibility generally comes at the cost of a large number of parameters. Increasing the order of the polynomials for example leads to a combinatorial increase of the number of parameters due to the multivariate nature of the polynomials $E\zeta(x(t),u(t))$ and $F\eta(x(t),u(t))$. In this study, the PNLSS approach is used to model a Bouc-Wen hysteretic system [2]. The multivariate polynomials $E\zeta(x(t),u(t))$ and $F\eta(x(t),u(t))$ are decoupled using the method in [3]. Like this, the nonlinearity in the PNLSS model is described in terms of univariate polynomials for which increasing their order is not so parameter expensive.

2 Methodology

In a first step, we estimate the best linear approximation (BLA) [4] of the system. A linear state-space model estimated on the BLA serves as an initial guess for the PNLSS model in (1) and (2), which is optimized using a Levenberg-Marquardt approach. In a second step, the multivariate polynomials $E\zeta(t)$ and $F\eta(t)$ are decoupled using the decomposition method in [3]:

$$E\zeta(x(t),u(t)) \approx W_x g\left(V^T \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}\right) \quad (3)$$

$$F\eta(x(t),u(t)) \approx W_y g\left(V^T \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}\right) \quad (4)$$

where the matrix V transforms the states and inputs in new variables $\xi = V^T \begin{bmatrix} x(t) & u(t) \end{bmatrix}^T$. The function g is a collection of univariate polynomials $g_i(\xi_i)$ for $i = 1,2,\cdots,r$: $g(\xi) = \begin{bmatrix} g_1(\xi_1) & g_2(\xi_2) & \cdots & g_r(\xi_r) \end{bmatrix}^T$ that act as basis functions for the decoupled state-space model. The matrices W_x and W_y contain the corresponding basis function coefficients.

3 Results

The Bouc-Wen model is excited with a random-phase multisine of 8192 samples, once with a standard deviation (std) of 6.8130 N and once with a std of 4.6419 N. Twenty realizations and 5 steady-state periods are used to estimate the BLA, a 3^{rd} order linear model, and the full and decoupled PNLSS model. The results on the validation data (using a multisine with a std of 4.6419 N) are plotted in Figure 1. The rms error on the test data are 1.8703×10^{-5} (multisine) and 1.2024×10^{-5} (swept sine) for the full PNLSS model and 3.7406×10^{-4} (multisine) and 3.8806×10^{-4} (swept sine) for the decoupled model. The decoupled model has an rms error higher than that of the linear model on the test data.

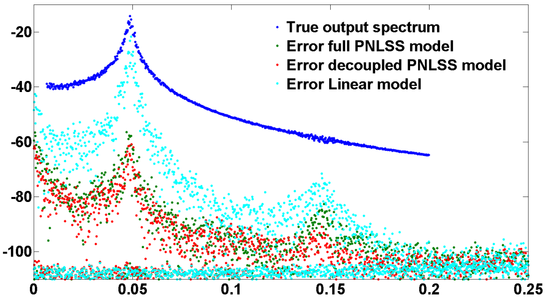

Figure 1: The validation output spectrum (in blue), the error of linear model (cyan) and the error of PNLSS (green) for 2nd and 3rd degree monomials of states and inputs in state updates ($F = 0$), and the error for the decoupled model with 11th degree polynomials (in red).

4 Conclusion

A PNLSS model can capture the behavior of a Bouc-Wen system. On the lower amplitude data, a decoupled PNLSS model reaches a similar accuracy, but has less than two third of the number of parameters (67 instead of 106). The order of the polynomials in the decoupled model can also be increased without blowing up the number of parameters, as it is the case for the full PNLSS model. On the higher amplitude benchmark data, the decoupling fails.

Acknowledgement

This work has been supported by the ERC Advanced grant SNLSID, under contract 320378.

References

[1] J. Paduart, L. Lauwers, J. Swevers, K. Smolders, J. Schoukens, and R. Pintelon, "Identification of nonlinear systems using Polynomial Nonlinear State Space models," *Automatica* 46(4) 2010: 647-656.

[2] J.P. Noël, and M. Schoukens, "Hysteretic benchmark with a dynamic nonlinearity," Technical report, http://homepages.vub.ac.be/∼mschouke/benchmark2016.html, 2015

[3] Ph. Dreesen, M. Ishteva, and J. Schoukens, "Decoupling multivariate polynomials using first-order information and tensor decoupling," *SIAM Journal on Matrix Analysis and Applications* 36, pp. 864-879, 2015.

[4] R. Pintelon, and J. Schoukens, "System Identification: a Frequency Domain Approach," John Wiley & Sons, 2012.