The Fractal Nature of Mars Topography Analyzed via the Wavelet Leaders Method

Adrien Deliège, Thomas Kleyntssens, and Samuel Nicolay

Institute of Mathematics, University of Liège, Belgium
[adrien.deliege,tkleyntssens,s.nicolay]@ulg.ac.be

Abstract. This work studies the scaling properties of Mars topography based on Mars Orbiter Laser Altimeter (MOLA) data through the wavelet leaders method (WLM). This approach shows a scale break at ≈ 15 km. At small scales, these topographic profiles display a monofractal behavior while a multifractal nature is observed at large scales. The scaling exponents are greater at small scales. They also seem to be influenced by latitude and may indicate a slight anisotropy in topography.

1 Introduction

Previous works about the scaling properties of Mars topography revealed two distinct scaling regimes (at small scales and at large scales) while the scale break varies from one paper to another. These works do not analyze the whole surface of Mars. The power-law exponents, or Hurst exponents, associated with these scaling laws appear to differ according to the region considered but some common features can still be noted. Besides, the mono- or multifractality of Mars topography has been studied, although it may depend on the definition of fractality and of the tools used. Table 1 summarizes the main previous results.

Table 1. Some previous results

Methods	**small scales**	**large scales**
power spectral density (PSD) [1]	$H \approx 1.2$ (< 10 km)	$H \approx 0.2 - 0.5$
variance of a wavelet transform [2]	$H \approx 1.25$ (< 24 km)	$H \approx 0.5$
statistical moments [3]	$H \approx 0.76$ (< 10 km)	$H \approx 0.52$

These studies are all based on along-track measurements, which implies that the 2D part of the topographic field has not been taken into account. In this paper, we perform our analysis on the MOLA data, using the 128 pix/deg map (more details can be found in [4] and on http://pds-geosciences.wustl.edu).

The aim of this paper is twofold: perform a complete study of the surface roughness of Mars while taking both longitudinal and latitudinal topographic profiles into account and show that the WLM may be a suitable candidate for the study of scaling properties of planetray surfaces. The WLM is the only tool that is theoretically justified [5] and that has already been used to study the bifractality of a signal [6] (for details, see e.g. [7, 8, 5]).

In this work, we use the D3 Daubechies wavelet. Let $d_{\lambda_{j,k}}$ be the wavelet leader a function f associated to the cube $\lambda_{j,k} = k/2^j + [0, 1/2^{j+1})$. One sets

$$S(j,q) = 2^{-j} \sum_{\lambda} d_\lambda^q \quad \text{and} \quad \eta(q) = \liminf_{j \to +\infty} \frac{\log S(j,q)}{\log 2^{-j}}.$$

If λ contains a point with Hurst exponent H, then $d_\lambda \sim 2^{-Hj}$ and $\eta(q) = Hq$.

2 Results

For a fixed value of q (ranges from -2 to 2 in order to limit the influence of anomalously large coefficients in the computations), $\eta(q)$ is obtained by performing linear regressions of $j \mapsto \log_2 S(j,q)$. Fig. 1 reveals a scale break at ≈ 15 kilometers. We thus consider two scaling regimes: the first one at the small scales (< 15km) and the second one at the large scales (> 60km); the scales in between represent the transition from one regime to the other.

If η is "linear enough", i.e. the associated linear correlation coefficient c is greater than 0.98, then the signal is said monofractal at the scales used to build η and the slope of η corresponds to the Hurst exponent, which characterizes the irregularity at these scales; otherwise it is multifractal at those scales. In this case, the slope of the best-fit (in the least square sense) linear regression of η gives a scaling exponent which does not fully represent the irregularity of the signal; other notions have then to be used (see e.g. [5, 3]) but are beyond the scope of this work. Table 2 summarizes our results.

Table 2. Results obtained with the WLM for the longitudes (l) and latitudes (L)

scales	monofractality (l)	mean H (l)	monofractality (L)	mean H (L)
small	99.7%	1.15 ± 0.06	92.1%	1.05 ± 0.13
large	91.7%	0.78 ± 0.087	63.2%	0.65 ± 0.11

At small scales, our results are in agreement with previous studies for the longitudinal signals (e.g. [1, 2]). For the latitudinal signals, the drop in the proportion of monofractal signals may be explain by the crustal dichotomy of Mars, the presence of polar caps, among others. Moreover, if we only keep latitudes between $80°S$ and $80°N$, then more than 96.7% of the signals are monofractal. The influence of latitude can be clearly seen in Fig. 1. The difference of results between the longitudinal and latitudinal signals may indicate a slight anisotropy of the surface roughness at small scales, as mentioned in [9].

At large scales, it turns out that the longitudinal and the latitudinal analyses mostly display a multifractal behavior, as in [3]. A large percentage of latitudinal bands still has to be considered monofractal; however, there is a clear difference compared to the small scales case (Fig. 1).

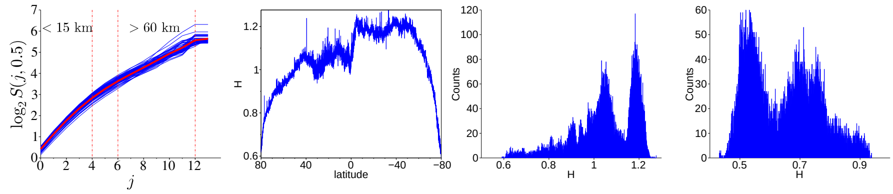

Fig. 1. Left: Blue lines: $\log_2 S(j, 0.5)$ versus j for several longitudinal bands. Red line: the mean of these functions. The scale j corresponds to $0.463 * 2^{j+1}$ kilometers (1 pixel $\approx$ 0.463 km). Scales 0 to 4 are used for the first scaling regime and scales 6 to 12 delimit the second scaling regime. Middle Left: Exponent H as a function of latitude at small scales. Middle Right: The corresponding histogram of the distribution of the exponents H. Right: The histogram of the distribution of H at large scales.

3 Conclusion

This work confirms that the WLM is well-suited for studying the irregularity of planetary bodies. Since the WLM can be adapted to 2D signals, we will examine the scaling properties of Mars topography using the 2D version of the WLM.

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