The Fractal Nature of Mars Topography Analyzed via the Wavelet Leaders Method

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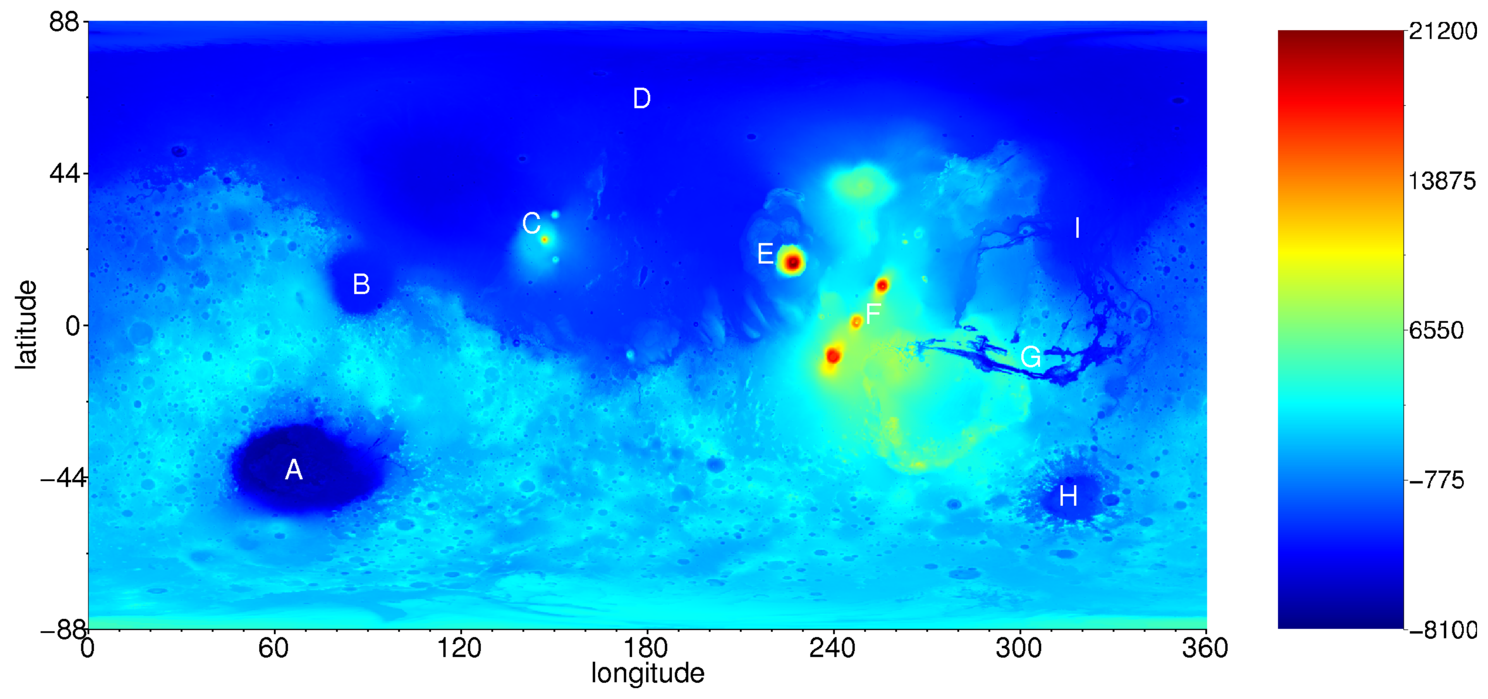
Introduction and Data

Previous works about the scaling properties of Mars topography revealed two distinct scaling regimes while the scale break varies from one paper to another. The next Table summarizes the main previous results:

Methods	small scales	large scales
power spectral density (PSD) [1]	$H \approx 1.2$ (< 10 km)	$H \approx 0.2 - 0.5$
variance of a wavelet transform [3]	$H \approx 1.25$ (< 24 km)	$H \approx 0.5$
statistical moments [4]	$H \approx 0.76$ (< 10 km)	$H \approx 0.52$

These studies are all based on along-track measurements, which implies that the 2D part of the topographic field has not been taken into account. We perform a complete study of the surface roughness of Mars while taking both longitudinal and latitudinal topographic profiles into account.

In this work, we use the MOLA data, using the 128 pix/deg map (http://pds-geosciences.wustl.edu).



Method: The Wavelet Leaders Method

For any signal f, let us denote by c_λ the wavelet coefficient associated to the dyadic interval $\lambda := \frac{k}{2^j} + [0, \frac{1}{2^{j+1}})$. The wavelet leader associated to the interval λ is the quantity

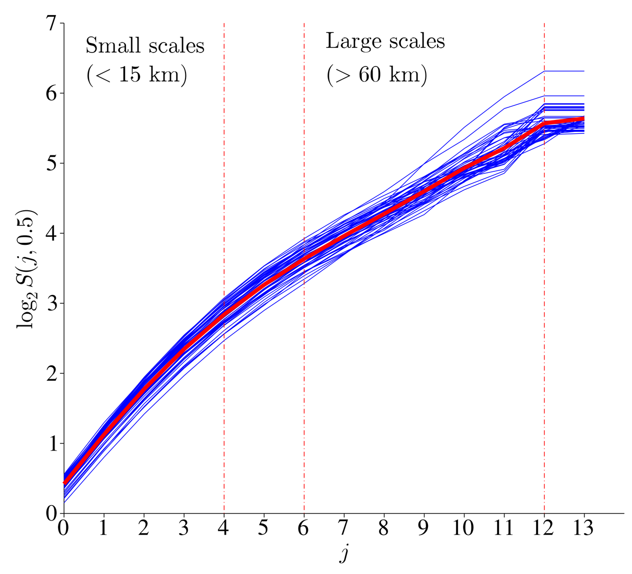
$$d_\lambda = \sup_{\lambda' \subset 3\lambda} |c_{\lambda'}|,$$

where 3λ is the set of intervals consisting of λ and the 2 intervals adjacent to λ. The method consists to compute the function η defined by

$$\eta(q) = \liminf_{j \to +\infty} \frac{\log S(j,q)}{\log 2^{-j}} \quad \text{where} \quad S(j,q) = 2^{-j} \sum_\lambda d_\lambda^q.$$

If λ contains a point with Hurst exponent H, then $d_\lambda \sim 2^{-Hj}$ and thus $\eta(q) = Hq$.
If η displays a linear behavior (i.e. the linear correlation coefficient associated is larger than 0.98), then the signal is said *monofractal* and the slope gives the Hurst exponent; else the signal is said *multifractal*.

Results



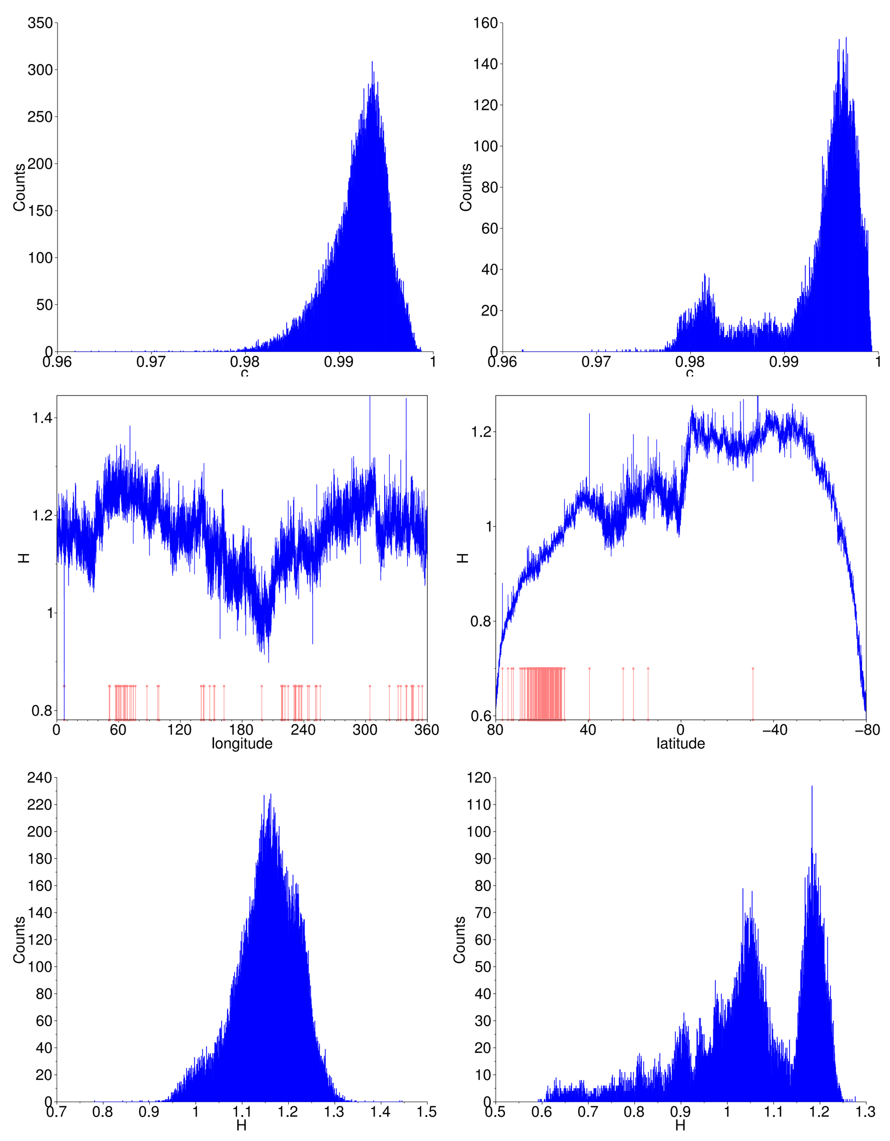
Function $j \mapsto \log_2 S(j, 0.5)$ for several longitudinal bands. The scale j corresponds to $0.463 * 2^{j+1}$ kilometers (1 pixel corresponds to 0.463 kilometers). The first vertical dashed line indicates that a scale break occurs at ≈ 15 kilometers.

Results obtained with the WLM for the longitudes (l) and latitudes (L).

scales	mono (l)	mean H (l)	mono (L)	mean H (L)
small	99.7%	1.15 ± 0.06	92.1%	1.05 ± 0.13
large	91.7%	0.78 ± 0.087	63.2%	0.65 ± 0.11

Small scales: For the latitudinal signals, the drop in the proportion of monofractal signals may be explain by the crustal dichotomy of Mars and the polar caps for example. The influence of latitude is clear. The difference of results between the latitude and longitude may indicate a slight anisotropy of the surface roughness.
Large scales: The signals mostly display a multifractal behavior. There is a clear difference compared to the small scales.

Results for the Small Scales



Top: Histograms of the distribution of the linear correlation coefficients c (related to the functions η, see text) for the longitudinal (left) and latitudinal (right) analyses at small scales (< 15 km). Since almost all the values of c exceed our threshold of 0.98, a monofractal behavior clearly emerges. The data are subdivided into 1000 equally spaced bins. Middle: Exponent H as a function of longitude (left) and latitude (right) at small scales (< 15 km). The lines indicate the topographic profiles that are considered multifractal. Bottom: the corresponding histograms of the distributions of H. The data are subdivided into 1000 equally spaced bins.

Conclusion and Further Research

This works confirms that the WLM is well-suited for studying the irregularity of planetary bodies. Since the WLM can be easily adapted to 2D signals and the fact that the MOLA data allow a such study, we have done the first complete study of Mars in 2D [2]. It allows, for example, to exhibit the link between the scaling exponents and several famous features of the Martian topography.

References :

[1] Aharonson, O., Zuber, M., Rothman, D.: Statistics of Mars' Topography from the Mars Orbiter Laser Altimeter: Slopes, Correlations, and Physical Models. Journal of Geophysical Research **106**(E10) (2001) 23723–23735

[2] Deliège A., Kleyntssens T., Nicolay S.: Mars Topography Investigated Through the Wavelet Leaders Method: a Multidimensional Study of its Fractal Structure. Submitted (2016)

[3] Malamud, B., Turcotte, D.: Wavelet analyses of Mars polar topography. Journal of Geophysical Research **106**(E8) (2001) 17497–17504

[4] Landais, F., Schmidt, F., Lovejoy, S.: Universal multifractal Martian topography. Nonlinear Processes in Geophysics Discussions **2** (2015) 1007–1031