Public Education Expenditures, Growth and Income Inequality

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Abstract

This paper analyzes the relationship between public education spending, long-run growth and income inequality. We propose an endogenous growth model with occupational choice and an endogenous supply of teachers and education quality. We show that endogenous school quality alters the shape of those relationships in a way that has new policy implications. First, growth depends on the level of public education expenditures and on the shape of the human capital distribution. Second, the relationship between public education and inequality can be either positive or negative. Calibrating our model to US state data, we find that a significant share of states faces a trade-off between increasing growth and decreasing inequality through public education spending. We find that this trade-off is overall more likely in states with higher public education expenditures, teacher employment share and relative wage, and intergenerational mobility. Finally, the existence of such a trade-off depends on how public education spending is financed.

Keywords: Endogenous growth, human capital, inequality, occupational choice, public education.
JEL Classification: E24, O11, O41, I24, I25

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1 Introduction

Public education is one of the largest items of public spending worldwide. Governments in high-income countries spent an average of 4.9% of GDP on all levels of education in 2017 (UNESCO UIS database).[1] In primary and secondary education, student enrollment rate in public institutions in 2013 exceeded 80% on average across OECD countries (92% in the U.S.). In tertiary education, on average, 69% of students (72% in the U.S.) were enrolled in public institutions in 2013 (OECD (2015)).

In theory, there are good economic reasons for governments to invest in public education. Public education can be an instrument to support economic growth and correct market inefficiencies arising from human capital externalities (Lucas (1988), Azariadis and Drazen (1990) and Romer (1990)) or credit market imperfections (Galor and Zeira (1993)). It may also reduce inequality (Glomm and Ravikumar (1992), Saint-Paul and Verdier (1993), Eckstein and Zilcha (1994) and Zhang (1996)).

These predictions, however, lack strong empirical support. Empirical work on the link between public education spending and economic growth is rather scarce and inconclusive (see for instance Levine and Renelt (1992), Easterly and Rebelo (1993), Sylwester (2000) and Blankenau, Simpson, and Tomljanovich (2007)). Empirical results on the relationship between public education spending and income inequality are no more conclusive. For instance, Keller (2010) finds a negative association between the level of public education expenditures and income inequality while Sylwester (2002) finds no statistically significant correlations and Braun (1988) and Barro (2000) even observe a positive relationship. These results suggest that, if there is a connection between public education expenditures, growth, and inequality, it is more nuanced than assumed in existing theoretical frameworks. In particular, this relationship should be modulated by other economic

[1] https://data.worldbank.org/indicator/SE.XPD.TOTL.GD.ZS.

conditions.

In this paper, we take the stance that a relevant modulating factor between public education expenditures, economic growth and inequality is the human capital of agents who choose to become teachers. In particular, we argue that the effects of public education expenditures depend on the relative average human capital of teachers, which is the result of occupational decisions along the human capital distribution. We show that this affects the relationship between public education expenditures, growth and inequality in several ways that are relevant to the design of education policies. The importance of teacher quality for student achievement and individual returns to education has been largely documented in the literature (see, among many others, Rockoff (2004), Rivkin, Hanushek, and Kain (2005) and Hanushek, Piopiunik, and Wiederhold (2019) for evidence of the central role of teacher quality on student performance). In an extensive review of the literature, Glewwe, Hanushek, Humpage, and Ravina (2011) find that teacher education and knowledge positively contribute to student learning. Hanushek (2011) finds that better teacher quality is associated with higher student earnings. Card and Krueger (1992) show that the return to education is positively associated with education quality (proxied by teacher salary) and teacher education level while DeCicca and Krashinsky (2020) find that the return (in terms of earnings) to education policies crucially depends on teacher quality measured by teacher relative salary. Figlio (1997), Loeb and Page (2000), Figlio and Kenny (2007), Dolton and Marcenaro-Gutierrez (2011), Hendricks (2014) and Britton and Propper (2016) provide further evidence for a positive relationship between teacher pay, education quality and student performance.

We build an overlapping-generations general equilibrium growth model which crucially departs from the existing literature by featuring occupational choice and endogenous education quality. In the model, agents can choose between three occupations: worker, teacher and manager. Managers' span of control determines their demand for workers and leads to a wage function that is convex in human capital for managers. An increase in the relative wage of teachers, financed by higher public education expenditures, raises the number of teachers and education quality. This, in turn, accelerates human capital accumulation and economic growth. We provide several suggestive empirical facts that relate to these predictions of our model. In particular, we show that higher public education expenditures are associated with higher teacher salaries, a larger teacher employment share and a higher average level of teacher education.

Our model produces new predictions regarding public education expenditures and growth: economies with fatter right tails of their human capital distribution attract better teachers for a given level of public education spending and have a higher elasticity of growth to public education expenditures. In other words, the effectiveness of public education policies at raising income growth depends critically on the distribution of human capital in the economy and might be higher in economies with higher inequality *ceteris paribus*. From a theoretical point of view, our model also allows for new growth decompositions i.e. decomposing worker income growth into human capital and wage rate per unit of human capital growth as well as decomposition aggregate growth into a measure of aggregate manager human capital and worker human capital growth.

Our model also sheds light on the link between public education expenditures and income inequality. In particular, it shows that these two variables are linked via two main channels that relate to occupational choice and the shape of the human capital distribution, as a change in public education affects both the bottom and top of the income distribution differently. First, raising public education expenditures alters relative wages across occupations, the human capital distribution and ultimately occupational decision. At the bottom of the distribution, higher public education investment affects labor supply. As the supply of human capital by workers decreases, their wage rate goes up. As this increases the cost of labor for managers, this reduces profits and wages at the top of the income distribution. These two forces tend to decrease income inequality. Second, higher public education expenditures also affect the top of the income distribution more directly. Managers benefit the most from an increase in their human capital as their wage function is convex. Profits and manager wages become more concentrated at the very top of the income distribution, which tends to increase income inequality. Overall, whether public education decreases income inequality depends on which of these two forces dominates. We show that, in theory, both effects can dominate depending on parameter values. These results have policy implications. In particular, they show that economies could face a trade-off between income inequality and economic growth through public education.

We calibrate our model and find that a significant share of US states faces such a trade-off. In addition, we show that whether an increase in public education spending raises income inequality crucially depends on the way increased spending is financed. An increase in public education spending financed by an increase in tax progressivity, rather than by an overall increase in the tax level, is more likely to lead to an increase in (before-tax) income inequality. Our results also suggest that states with higher public education expenditures, higher teacher employment share and relative wage and higher intergenerational mobility are overall more likely to face a tradeoff between growth and income inequality through public education *ceteris paribus*. Finally, our calibrations reveal that raising public education spending through an overall increase in the level of the tax achieves better results than through increased tax progressivity in terms of both increased growth and smaller increase (or larger decrease) in income inequality.

Related literature Robust evidence shows that investment in education has positive effects on individual earnings.[2] Empirical evidence also tends to demonstrate that edu-

[2] See for instance Card (1999) and Krueger and Lindahl (2001) for a discussion of empirical evidence.

cation attainment is positively correlated with aggregate income growth.[3]

Our theoretical model is related to the literature on public education, economic growth and inequality. Glomm and Ravikumar (1992), Saint-Paul and Verdier (1993), Eckstein and Zilcha (1994), Zhang (1996), Glomm and Ravikumar (2003) and Blankenau, Simpson, and Tomljanovich (2007) among others show that public education should raise long-run economic growth and lower income inequality.[4] Our model departs from this literature by allowing for a non-degenerate distribution of human capital in a balanced growth path and by modeling occupational choice. Agents in our model decide whether to become teachers, which endogenously determines the quality of education. This implies that both public education expenditures and the distribution of human capital in the economy matter for the quality of education. In turn, the quality of education affects the shape of the human capital distribution, growth and inequality. These two features of our model (occupational choice and endogenous teacher quality) imply that the direction of relationship between public education and inequality in the long run is *a priori* ambiguous.

The notion that education quality matters for economic outcomes, in particular economic growth, has a long tradition. Hanushek and Kimko (2000), Hanushek and Woessmann (2012) and Hanushek and Woessmann (2015) show that differences in the quality of education can explain variations in economic growth rates across countries. This suggests that factors affecting the quality of education (and not only years of schooling) should be taken into consideration when analyzing the role of education on economic growth. Manuelli and Seshadri (2014) show that a large share of TFP differences across countries can actually be explained by differences in human capital when agents can choose both the number of years of schooling but also the amount of human capital acquired per

[3] Evidence of the role of human capital and schooling on economic growth can be found among others in Barro (1991), Mankiw, Romer, and Weil (1992), Benhabib and Spiegel (1994), Barro (2001), Cohen and Soto (2007) and Sunde and Vischer (2015).

[4] Glomm and Ravikumar (2003) show that the effect of higher public education expenditures on income inequality may be positive in the short run. They nevertheless obtain a negative association in the long run.

year of schooling. Schoellman (2012) also concludes that education quality differences can explain a significant share of cross-country income differences using data on foreign-educated immigrants. We add to this literature by considering the supply side of education in a general equilibrium model of growth and occupational choice. To the best of our knowledge, this is the first paper that endogenizes the supply side of education in a model of endogenous growth.[5] In particular, we show that the outcome of public education policies crucially depends on the occupational choice response of agents (in particular teachers).

The results of our paper also have policy implications. First, they highlight that the return in terms of economic growth from education expenditures depends on the human capital distribution. Second, the increase in income inequality since the late 1970s has recently attracted a lot of attention both in academic and policy circles.[6] The role of several public policies in shaping the evolution of income inequality has been investigated in the literature.[7] Our results complement this literature by showing that public education expenditures do not necessarily reduce inequality. Our joint results regarding growth and inequality have important implications for the optimal design of public policies and, in particular, regarding the factors that are relevant to the success of such policies.

We also relate to the empirical literature on public education, growth and inequality. Empirical work on the link between public education spending and economic growth is rather scarce and inconclusive (see for instance Levine and Renelt (1992), Easterly and

[5]Our model shares some features with the recent growth literature with heterogeneous agents and occupational choice developed by Lucas (2009), Eeckhout and Jovanovic (2012), Alvarez, Buera, and Lucas (2013), Lucas and Moll (2014), Perla and Tonetti (2014) and Luttmer (2014). They develop growth mechanisms based on knowledge diffusion but do not consider educators as a distinct occupation group.

[6]See for instance Katz and Autor (1999) for a discussion of recent changes in wage dispersion and Piketty and Saez (2003) and Alvaredo, Atkinson, Piketty, and Saez (2013) for the evolution of top income inequality.

[7]See for instance Piketty, Saez, and Stantcheva (2014) and Guvenen, Kuruscu, and Ozkan (2014) for a discussion of the role of income taxation and David, Manning, and Smith (2016) and Card and DiNardo (2002) for minimum wage.

Rebelo (1993), Sylwester (2000) and Blankenau, Simpson, and Tomljanovich (2007)). The empirical literature on public education and income inequality also finds mixed results. For instance, Keller (2010) finds a negative association between the level of public education expenditures and income inequality, Sylwester (2002) finds no statistically significant correlation and Braun (1988) and Barro (2000) even observe a positive relationship.[8] We contribute to this literature by showing that the relationship between public education expenditures, growth and income inequality can be non-linear and potentially ambiguous.

The remainder of the paper is organized as follows. Section 2 introduces a static version of the model with occupational choice between three occupations (worker, teacher and manager). Section 3 embeds this static occupational choice in a dynamic general equilibrium model of endogenous growth. It also derives the theoretical properties of our model in a balanced growth path and shows how both economic growth and income inequality are affected by changes in public education expenditures. Section 4 presents some suggestive empirical evidence about the relationship between public education expenditures, teacher employment share, wage and quality, which are in line with the predictions of our model. Section 5 presents calibrations of our model to US states. Section 6 concludes.

2 A Static Model of Occupational Choice

For expositional purposes, we start with a static (one-period) model of occupational choice. We embed this static model in an overlapping generations structure in Section 3.

Agents can choose between three different jobs: worker, teacher and manager. There is a measure one of agents in the economy and they are heterogeneous in their level of human capital (h) with cdf $F : \mathbb{R}^+ \to [0, 1]$. There is a single consumption good produced by a continuum of firms.

[8] Sylwester (2002) finds a negative association between the change in income inequality and public education spending.

2.1 Agents' Problem

The choice of occupation is driven by the return to the three potential jobs which is itself a function of human capital. We assume that agents cannot perform more than one job. If an agent decides to become a worker, she receives a wage rate w per unit of human capital which is determined endogenously. A teacher receives a wage $w^T h$. Teachers do not directly participate in production. Firms produce a homogeneous final good by combining one manager and the human capital of workers. A firm's production is determined by the manager's span of control as in Lucas (1978) and Eeckhout and Jovanovic (2012). The production function of a manager with a level of human capital z is given by:

$$y(z) = z^{\varphi} H^{\alpha} \tag{1}$$

where $\alpha \in (0,1)$, $\varphi > 0$ and H is the aggregate human capital of workers of the firm.

We assume that the profit of the firm ($\pi(z)$) is entirely paid to its manager as a wage.[9] In equilibrium, firms maximize profit and $\pi(z) = \left[\frac{\alpha}{w}\right]^{\frac{\alpha}{1-\alpha}} (1-\alpha) z^{\frac{\varphi}{1-\alpha}}$.

The government raises progressive income tax. This tax is used to finance public education spending and, in particular, to pay teacher wages. The (average) tax rate ($\bar{\tau}$) of an agent with income $\hat{w}$ is:[10]

$$\bar{\tau}(\hat{w}) = 1 - (1-\tau)\left[\frac{\hat{w}}{Y}\right]^{-\lambda} \tag{2}$$

[9]This is obtained if we assume that there is free-entry of firms competing for managers. In this case, any manager with human capital z works for a firm which offers a wage equal to the maximum profit that a manager with human capital z can generate. Alternatively, managers receiving a fixed share of profits would not change our results qualitatively.

[10]The same log-linear tax function has been used in the literature and shown to be a good approximation to the US tax system. See, for instance, Benabou (2002), Guner, Kaygusuz, and Ventura (2014), Ferriere and Navarro (2014) and Heathcote, Storesletten, and Violante (2020). Note that Equation (2) allows for negative tax rates (positive transfers) for low levels of income.

where $Y = \int_{\mathcal{M}} y(z)dF(z)$ is aggregate income in the economy, $\mathcal{M}$ is the set of managers, $\tau \in (0, 1)$ denotes the tax level and $\lambda \in [0, 1)$ measures the degree of tax progressivity.[11] We assume that the government budget is balanced.

From now on, we make the assumption that $\varphi > \frac{1-\alpha}{1-\lambda}$ which ensures that the after-tax wage of managers is strictly convex in their level of human capital.

The utility function is given by:

$$u(c) = c - \mathbb{1}_{T,M}\ \gamma \tag{3}$$

where c is consumption, $\mathbb{1}_{T,M}$ takes the value one if the agent is a teacher or a manager and zero otherwise and γ is a cost in terms of the final good of working as a teacher or a manager.[12]

The utility of an agent with human capital h under each of the three occupations is given by:

$$\begin{aligned}
&u^W(h) = [wh]^{1-\lambda}(1-\tau)Y^\lambda \ \textit{as a worker} \\
&u^T(h) = \left[w^T h\right]^{1-\lambda}(1-\tau)Y^\lambda - \gamma \ \textit{as a teacher} \\
&u^M(h) = (1-\tau)\left[\frac{\alpha}{w}\right]^{\frac{(1-\lambda)\alpha}{1-\alpha}}(1-\alpha)^{1-\lambda} z^{\frac{(1-\lambda)\varphi}{1-\alpha}} Y^\lambda - \gamma \ \textit{as a manager}
\end{aligned}$$

[11] The normalization of wages by total income in the economy is needed to obtain a balanced growth path equilibrium in the dynamic version of the model described in Section 3.

[12] This reflects, for instance, the cost of obtaining post-secondary degrees required for most teaching and managerial occupations. $\gamma > 0$ implies that the average teacher wage is higher than the average worker wage, in line with what is observed in the data.

2.2 Equilibrium Conditions

We first show that the distribution of agents over occupations for given wage rates for workers (w) and teachers (w^T) can be summarized by two cutoffs h_W and h_M ($h_W \leq h_M$).

Proposition 1 *Given wage rates ($w > 0$ and $w^T > 0$), $\tau \in (0,1)$, $\lambda \in [0,1)$ and a continuous human capital distribution with support $\mathbb{R}^+$, the optimal occupational choice of agents is defined by two cutoffs ($h_M \geq h_W$). Agents with human capital below h_W become workers and agents with human capital between h_W and h_M become teachers. Agents with human capital above h_M work as managers.*

Proof: See Appendix A.1.

Proposition 1 shows that teachers, provided that $u^T(h_W) > u^M(h_W)$, are to be found in the middle of the human capital distribution. We now prove that $u^T(h_W) > u^M(h_W)$ holds in equilibrium.

Proposition 2 *Given a continuous human capital distribution with support $\mathbb{R}^+$ and a tax system ($\tau \in (0,1)$ and $\lambda \in [0,1)$), there is a positive mass of agents working in each occupation in equilibrium i.e. $h_W < h_M$.*

Proof: See Appendix A.2.

To determine the equilibrium conditions of the static model, we combine the indifference conditions, the government budget constraint and the market clearing condition:

$$
\begin{aligned}
[wh_W]^{1-\lambda}(1-\tau)Y^{\lambda} &= \left[w^T h_W\right]^{1-\lambda}(1-\tau)Y^{\lambda} - \gamma && (4)\\
w^T h_M &= \left[\frac{\alpha}{w}\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)h_M^{\frac{\varphi}{1-\alpha}} && (5)\\
w^T \int_{h_W}^{h_M} h\ dF(h) &= \int_0^{h_W} wh\left[1-(1-\tau)\left(\frac{wh}{Y}\right)^{-\lambda}\right]\ dF(h)\\
&+ \int_{h_W}^{h_M} w^T h\left[1-(1-\tau)\left(\frac{w^T h}{Y}\right)^{-\lambda}\right] dF(h)\\
&+ \int_{h_M}^{\infty} \pi(z)\left[1-(1-\tau)\left(\frac{\pi(z)}{Y}\right)^{-\lambda}\right]\ dF(z) && (6)\\
\int_0^{h_W} h\ dF(h) &= \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}}\int_{h_M}^{\infty} z^{\frac{\varphi}{1-\alpha}}\ dF(z) && (7)
\end{aligned}
$$

where Equations (4) and (5) come from agents' indifference between occupations at the cutoffs h_W and h_M, Equation (6) corresponds to the balanced government budget and Equation (7) is the labor market clearing condition.

Figure 1 shows an example of an equilibrium occupational choice. The different lines represent utility as a function of human capital under the three occupations. Workers get a wage after tax which is concave in their level of human capital. Teachers receive a concave after-tax wage in human capital and pay the cost γ. The utility of managers is increasing and convex in human capital. The solid line represents the equilibrium utility of agents as a function of human capital i.e., the highest utility level across the three occupations.[13]

[13] The full definition of the equilibrium of the static model can be found in Appendix B.

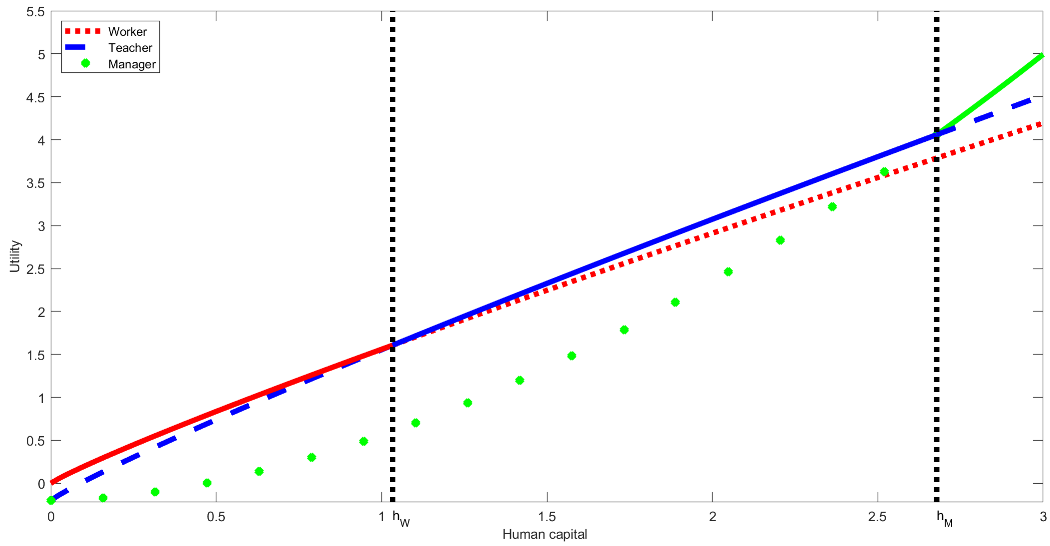

Figure 1: Occupational choice.
Notes: In this figure, we use $\gamma = 0.2$, $\alpha = 0.1$, $\tau = 0.05$, $\lambda = 0.1$ $\varphi = 1.75$ and F is a log-normal distribution with parameter values equal to zero and 1 respectively. Lines represent utility under the three different occupations. The solid line represents the equilibrium utility derived by agents as a function of their level of human capital.

2.3 Comparative Statics

In this section, we study how the equilibrium changes as we change the tax system (τ and λ). In a static one-period model, education plays no role in the economy as there is no human capital accumulation. We can nevertheless study how the quality of education changes as public education spending increases. We measure the quality of education as the aggregate human capital of teachers:[14]

$$S = \int_{h_W}^{h_M} h \; dF(h) \tag{8}$$

Figure 2 shows comparative statics for the level of the tax (τ). Increasing τ raises the incentive for agents to become teachers as it increases the relative wage rate of teachers. As a consequence, the mass of teachers increases and the mass of workers and managers

[14]This measure can be interpreted as the human capital of teacher per student. It captures two important dimensions of education quality: human capital of teachers and the student-teacher ratio.

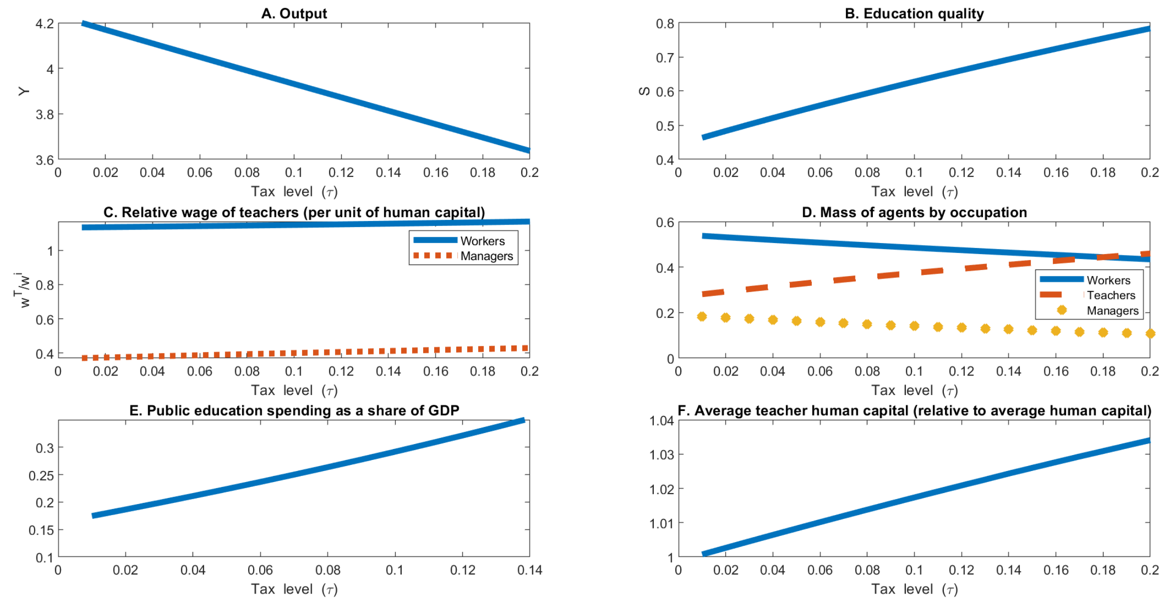

Figure 2: Comparative statics for the tax level (τ).
Notes: In this figure, we use the following parameter values: $\gamma = 0.2$, $\lambda = 0.1$ $\alpha = 0.1$, $\varphi = 1.75$ and F is a log-normal distribution with parameter values equal to zero and 1 respectively

decreases, as some workers and managers switch to teaching. As teaching attracts more agents, the quality of education (S) also improves. It increases for two reasons. First, higher wages attract more teachers and, second, the average human capital of teachers increases. However, there is a trade-off between production and quality of education, as higher public education spending diverts agents away from the productive sector of the economy. Figure C.1 in Appendix C.1 shows that similar results are obtained when the tax is raised through an increase in tax progressivity (λ).

3 The Dynamic Model

3.1 Environment and Equilibrium

We embed the static model from Section 2 into an overlapping generations framework. We assume that there is, at any time, a mass one of families composed of one young and one old agent. Agents live for two periods and only consume when old with preferences similar to those described in the one-period model in Section 2.1, i.e.:

$$u_t^W(h) = [w_t h]^{1-\lambda} (1-\tau) Y_t^{\lambda} \textit{ as a worker}$$
$$u_t^T(h) = \left[w_t^T h\right]^{1-\lambda} (1-\tau) Y_t^{\lambda} - \gamma_t \textit{ as a teacher}$$
$$u_t^E(h) = (1-\tau) \left[\frac{\alpha}{w_t}\right]^{\frac{(1-\lambda)\alpha}{1-\alpha}} (1-\alpha)^{1-\lambda} h^{\frac{(1-\lambda)\varphi}{1-\alpha}} Y_t^{\lambda} - \gamma_t \textit{ as a manager}$$

When young, agents go to school and build their human capital:

$$h_{t+1} = a_t h_t^{\beta_1} S_t^{\beta_2} \tag{9}$$

where h_t is the human capital level of the old agent in the family at time t, a_t is an idiosyncratic shock to the transmission of human capital to the child with distribution $G_t(a)$ and $S_t = \int_{h_{W,t}}^{h_{M,t}} h \; dF_t(h)$ is the quality of education.

An agent's human capital is thus a function of her parent's human capital, the quality of the educational system when she is young and a random shock to her ability to absorb the knowledge from her parent and teachers. This shock allows for social mobility across generations. The relative importance of parents and public education in the formation of human capital is captured by β_1 and β_2. We assume that $\beta_1 + \beta_2 = 1$ and $\beta_1 \in (0,1)$.

Dynamic Equilibrium Definition: *Given an initial continuous distribution of human capital with cdf $F_0 : \mathbb{R}^+ \to [0,1]$, a distribution for the shock with cdf $G : \mathbb{R}^+ \to [0,1]$ and a tax system ($\tau \in (0,1)$ and $\lambda \in [0,1)$), a dynamic equilibrium is a sequence of wages ($w_t h$, $w_t^T h$), profits ($\pi_t(z)$), cutoffs ($h_{W,t}$, $h_{M,t}$), demand for human capital ($H_t(z)$), education quality (S_t) and final good production ($Y_t = \int_{h_{M,t}}^{\infty} y_t(h)$) such that, at every period:*

1. *Given wages, firms maximize profit.*

2. *Given wages and a tax system, agents maximize utility by following a cutoff strategy in which agents with human capital in* $[0, h_{W,t})$ *become workers, agents with human capital in* $[h_{W,t}, h_{M,t})$ *are teachers and agents with human capital above* $h_{M,t}$ *work as managers.*

3. *Labor market clears:* $\int_{h_{M,t}}^{\infty} H_t(z)\ dF_t(z) = \int_0^{h_{W,t}} h\ dF_t(h)$

4. *Government budget is balanced:* $\int_{h_{W,t}}^{h_{M,t}} w_t^T h\ dF_t(h) = \int_0^{h_{W,t}} w_t h \left[1 - (1-\tau)\left(\frac{w_t h}{Y_t}\right)^{-\lambda}\right] dF_t(h) + \int_{h_{W,t}}^{h_{M,t}} w_t^T h \left[1 - (1-\tau)\left(\frac{w_t^T h}{Y_t}\right)^{-\lambda}\right] dF_t(h) + \int_{h_{M,t}}^{\infty} \pi_t(z) \left[1 - (1-\tau)\left(\frac{\pi_t(z)}{Y_t}\right)^{-\lambda}\right] dF_t(z)$

5. *Education quality is given by:* $S_t = \int_{h_{W,t}}^{h_{M,t}} h\ dF_t(h)$

6. *Human capital evolves as:* $h_{t+1} = a_t h_t^{\beta_1} S_t^{\beta_2}$

In the remainder of the paper, we make the following assumptions regarding initial conditions, γ_t and the distribution of a_t:

$$\gamma_t = \gamma Y_t \tag{10}$$

$$log(h_0) \sim \mathcal{N}(\mu_0, \sigma_0^2) \tag{11}$$

$$log(a_t) \sim \mathcal{N}(\mu_a, \sigma_a^2) \tag{12}$$

The fact that γ_t scales with the size of the economy is required for the existence of a balanced growth path. The distribution of the shock is assumed to be the same across agents and across time. The distributional assumptions lead to the existence of a balanced growth path. Given these assumptions, we can show that the distribution of human capital at any time t follows a log-normal distribution with parameters μ_t and σ_t^2. In particular,

$$log(h_{t+1}) \sim \mathcal{N}\left(\mu_a + \beta_1 \mu_t + \beta_2 log(S_t), \sigma_a^2 + \beta_1^2 \sigma_t^2\right). \tag{13}$$

From now on, we focus on balanced growth path equilibria. In the long run, the variance of $log(h_t)$ converges to $\frac{\sigma_a^2}{1-\beta_1^2}$. If $\sigma_a^2 = 0$, the distribution of human capital converges to a degenerate distribution, in which case there is no income inequality in the long run.

Balanced Growth Path Definition: *A balanced growth path equilibrium is a dynamic equilibrium in which:*

1. w_t *and* w_t^T *grow at a constant rate* g_w*;* $h_{W,t}$*,* $h_{M,t}$*,* e^{μ_t} *and* S_t *grow at a constant rate* g_h *such that* $(1+g_h) = (1+g_w)^{\frac{1}{\alpha+\varphi-1}}$*; and* $Y_t = \int_{h_{M,t}}^{\infty} y_t(z)\ dF_t(z)$ *grows at a constant rate* g *such that* $(1+g) = (1+g_w)(1+g_h) = (1+g_w)^{\frac{\alpha+\varphi}{\alpha+\varphi-1}}$.

2. $\sigma^2 = \frac{\sigma_a^2}{1-\beta_1^2}$

3. *The mass of workers, teachers and managers remains constant.*

3.2 Public Education and Long-Run Growth

In this section, we derive the growth rate of the economy in a balanced growth path equilibrium and see how it relates to the level of public education expenditures.

Proposition 3 *In a balanced growth path, the growth rate of the economy (g) remains constant and is given by:*

$$(1+g)^{\frac{1}{\varphi+\alpha}} = e^{\mu_a}\left(\frac{S_t}{e^{\mu_t}}\right)^{1-\beta_1} \tag{14}$$

$$g \approx (\varphi+\alpha)\left\{\mu_a + (1-\beta_1)\left[log(S_t) - \mu_t\right]\right\} \tag{15}$$

Proof: See Appendix A.3.

Our model predicts that workers' income growth comes from two different sources: (i) growth of workers' human capital ($\int_{h_{M,t}}^{\infty} H_t(z)\ dF_t(z)$) and (ii) growth in the wage rate per unit of human capital (w_t).

In addition, long-term growth in the economy is increasing in the quality of education (S_t) relative to the level of human capital in the economy. This has two main consequences. First, raising teacher relative wage leads to an increase in education quality and faster growth. Second, the whole shape of the human capital distribution matters for how public education spending translates into economic growth. To see this, we can look at how a change in the variance of the (log) shock (σ_a^2) affects long-run growth. σ_a is linked to the variance and the thickness of the right tail of the human capital distribution but does not directly affect economic growth (Equation (14)). This enables us to identify the effect of the shape of the human capital distribution on the endogenously determined quality of teachers and economic growth.

Figure 3 shows comparative statics with respect to σ_a for a fixed progressive tax system (i.e. constant $\tau > 0$ and $\lambda > 0$). We can notice that human capital distributions with fatter right tails (larger σ_a) are associated with faster growth and higher inequality. Faster growth is obtained as a result of an increase in the relative quality of teachers. We should note that part of the increase in the quality of teachers is due to the fact that a larger σ_a is associated with higher tax revenues with progressive taxation. For a given progressive tax system, economies with a larger share of high human capital agents can raise higher revenues and increase relative teachers' wage and education quality.

The previous experiment combines two forces i.e. (i) higher σ_a allows to raise more tax revenues for a given tax system and (ii) for a given level of public education expenditures, economies with a fatter right tail of the human capital distribution can attract better teachers. To isolate the second effect, we run another experiment. In an economy with proportional income tax (i.e. $\lambda = 0$), the ratio of public education expenditures to GDP

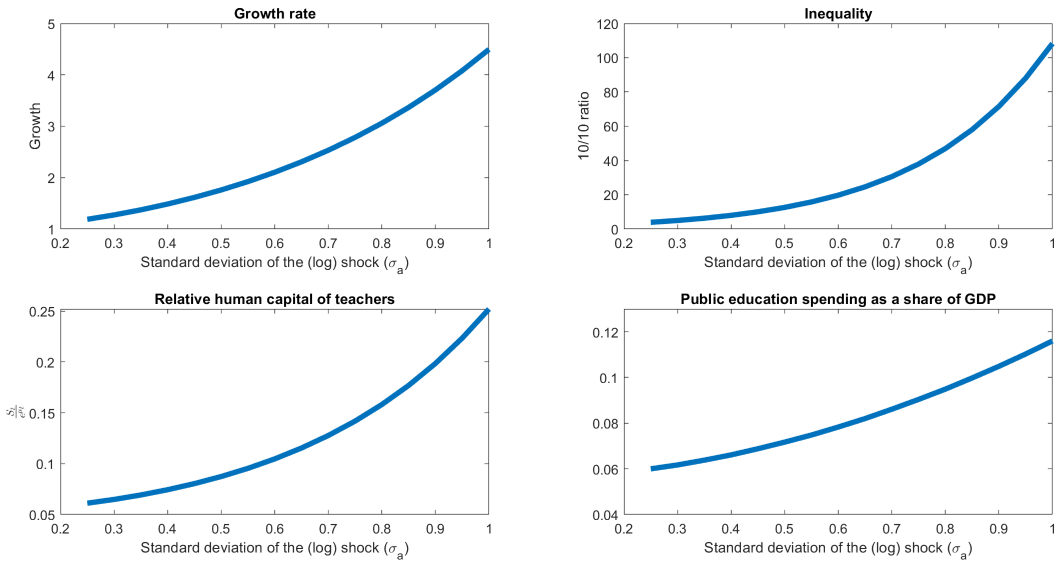

Figure 3: Comparative statics in a balanced growth path with fixed progressive tax system: standard deviation of the (logarithm of the) shock (σ_a).
Notes: In this figure, we use the following parameter values: $\alpha = 0.3$, $\beta_1 = 0.5$, $\gamma = 0.1$, $\varphi = 1$, $\mu_a = 2$, $\tau = 0.05$ and $\lambda = 0.05$. The figure shows growth, inequality, teachers' human capital (relative to the average human capital in the economy) and public education spending for different values of the standard deviation of the (log) shock to human capital (σ_a) for a fixed tax system i.e. for fixed tax level (τ) and tax progressivity (λ).

is independent of σ_a and is equal to $\frac{\tau}{1-\tau}$. Figure 4 shows the effect of varying σ_a in such an economy. Economies with a fatter right tail of the human capital distribution exhibit faster growth even though public education expenditures (as a share of GDP) remain constant, as they attract teachers with higher human capital. This implies that economic growth is not only a function of the average level of human capital in the economy but also of higher-order moments of the human capital distribution through their effect on the quality of teachers.[15]

Similar results are obtained in economies with progressive taxation ($\lambda > 0$) (see Figure C.2 in Appendix C.2).[16] Figure C.3 in Appendix C.2 further shows the increase in growth resulting from a one percentage point increase in public education expenditures to GDP for different values of σ_a. Raising public education expenditures is more effective at

[15]In a different context, Perla and Tonetti (2014) find a similar positive correlation between the thickness of the tail of the productivity distribution and economic growth.

[16]Since higher σ_a allows the government to raise more taxes for a given tax system, we need to decrease the level of tax progressivity as we increase σ_a to keep the ratio of public education to GDP constant.

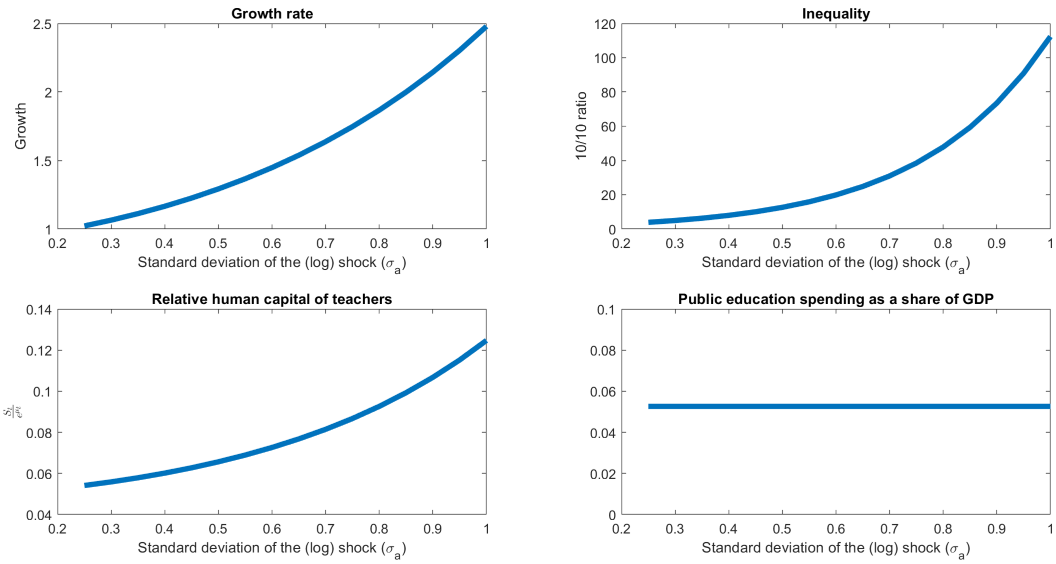

Figure 4: Comparative statics in a balanced growth path with proportional tax system: standard deviation of the (logarithm of the) shock (σ_a). In this figure, we use $\alpha = 0.3$, $\beta_1 = 0.5$, $\gamma = 0.1$, $\varphi = 1$, $\mu_a = 2$, $\tau = 0.05$ and $\lambda = 0$. The figure shows growth, inequality, teachers' human capital (relative to the average human capital in the economy) and public education spending for different values of the standard deviation of the (log) shock to human capital (σ_a) with a fixed proportional tax system i.e with tax progressivity $\lambda = 0$.

increasing economic growth when the human capital distribution exhibits a fatter right tail.

Finally, we can derive an aggregate production function that depends on an aggregate of managers' human capital and the aggregate human capital of workers.

Proposition 4 *The aggregate production function can be written as* $Y_t = Z_t HC_t^{\alpha}$, *where* $Z_t = \left[\int_{h_{M,t}}^{\infty} z^{\frac{\varphi}{1-\alpha}}\, dF_t(z)\right]^{1-\alpha}$ *is an aggregate of managers' human capital and* $HC_t = \int_0^{h_{W,t}} h\, dF_t(h)$ *is the aggregate human capital of workers. In a balanced growth path,* HC_t *grows at rate* g_h *and* Z_t *grows at rate* $g_Z = (1 + g_h)^{\varphi} - 1$.

Proof: See Appendix A.4.

This result can be linked to the general decomposition in Jones (2014) in which the productivity of unskilled workers depends on the human capital of other types of workers in the economy.[17]

3.3 Education Spending and Inequality

In this section, we compare balanced growth paths for different values of τ and λ and see how this translates into changes in income inequality. We measure inequality using the 10/10 ratio i.e. the ratio of (before-tax) income of the top 10% to the bottom 10% of the income distribution. This ratio allows us to derive a clear decomposition regarding the mechanisms at play in the model related to income inequality. However, we also show that similar results regarding the relationship between public education and income inequality are obtained using alternative measures of inequality (see Appendix C.4).

The effect of public education on inequality is the result of how a tax change affects equilibrium occupational choice, human capital demand and supply, and concentration of income at both the bottom and top of the distribution. Assuming that there is no teacher in either the top or bottom 10% of the income distribution (over the relevant range of taxes) and using the labor market clearing condition in Equation (7), the 10/10 ratio (before tax) can be written as:

$$\begin{aligned} 10/10\ ratio_t \quad & \propto \frac{\int_0^{h_{W,t}} h\ dF_t(h)}{\int_0^{F_t^{-1}(0.1)} h\ dF_t(h)} \frac{\int_{F_t^{-1}(0.9)}^{\infty} \pi(z)\ dF_t(z)}{\int_{h_{M,t}}^{\infty} \pi(z)\ dF_t(z)} && (16) \\ & \propto \Omega_t \Psi_t && (17) \end{aligned}$$

where $\Omega_t = \frac{\int_0^{h_{W,t}} h\ dF_t(h)}{\int_0^{F_t^{-1}(0.1)} h\ dF_t(h)}$ and $\Psi_t = \frac{\int_{F_t^{-1}(0.9)}^{\infty} \pi(z)\ dF_t(z)}{\int_{h_{M,t}}^{\infty} \pi(z)\ dF_t(z)}$.

[17] This decomposition is also related to the literature on the role of managerial skills for worker productivity, see for instance Gibbons and Henderson (2012), Bloom, Lemos, Sadun, Scur, and Van Reenen (2014), Bloom, Sadun, and Van Reenen (2016) and Bender, Bloom, Card, Van Reenen, and Wolter (2018).

The first term relates to labor supply at the bottom of the human capital distribution while the second term is related to profit concentration at the top.[18] The change in these two terms is itself a function of changes in occupational choice, relative wages and the human capital distribution. Using Equations (4) and (5) and $\gamma_t = \gamma Y_t$, we can notice that the thresholds $h_{W,t}$ and $h_{M,t}$ can be rewritten as:

$$h_{W,t} = \frac{\gamma^{\frac{1}{1-\lambda}} Y_t}{(1-\tau)^{\frac{1}{1-\lambda}} \left[w_t^{T\,1-\lambda} - w_t^{1-\lambda} \right]^{\frac{1}{1-\lambda}}} \tag{18}$$

$$h_{M,t} = \frac{\left[\frac{\alpha}{w_t} \right]^{\frac{\alpha}{1-\alpha-\varphi}} (1-\alpha)^{\frac{1-\alpha}{1-\alpha-\varphi}}}{w_t^{T\,\frac{1-\alpha}{1-\alpha-\varphi}}} \tag{19}$$

which shows how tax parameters (τ and λ) directly and indirectly (through equilibrium wages) affect occupational choice.

In addition, the human capital distribution $F_t(h)$ is also affected by changes in public education spending as higher spending leads to higher education quality and faster growth.

Increasing τ and λ leads to a decrease in Ω and an increase in Ψ. In other words, the share of labor supply coming from the bottom 10 percent of the human capital distribution increases (and so does their share of the labor share of income). At the same time, profits become more concentrated at the top of the income distribution. These two forces go in opposite direction, so that the net effect on income inequality of an increase in public education spending depends on which force dominates and can result in a negative or positive relationship between public education and income inequality, as illustrated in Figures C.4 and C.5 in Appendix C.3

These results contrast with the idea that public education should in theory decrease income inequality in the long run. In particular, our model highlights the endogenous

[18]Both terms are constant in a balanced growth path.

response of occupational choice (and teaching decisions) and human capital distribution following an increase in public education spending and how those can potentially translate into higher levels of inequality. Our results can also potentially reconcile the ambiguous results obtained in the empirical literature which find both positive and negative effects of public education on income inequality.

Whether a country faces a positive or negative relationship between public education and inequality has potentially important policy implications. Countries with an upward-sloping curve face a trade-off between increasing growth and reducing inequality through public education.

4 Empirical Analysis

Our model delivers several testable predictions. In particular, it predicts that higher public education spending should result in higher teacher wage, employment share and quality. In this section, we provide some suggestive empirical evidence regarding these relationships. We use data for contiguous US states (excluding the District of Columbia) for the years 1960, 1970, 1980, 1990, 2000 and 2010. A detailed description of the data can be found in Appendix D.

Figure 5 displays scatter plots for the year 2010 (the last year in our sample) and shows an overall positive association between public education spending and teacher relative wage, employment share and education across US states. In what follows, we further provide regression results using our full panel with additional control variables that confirm the significance of these positive correlations. Even though those correlations cannot be interpreted in a causal sense, combined with the existing empirical evidence in the literature regarding the role of teacher relative wage and quality on student performance (see the discussion in the Introduction), they offer some suggestive evidence and support regarding

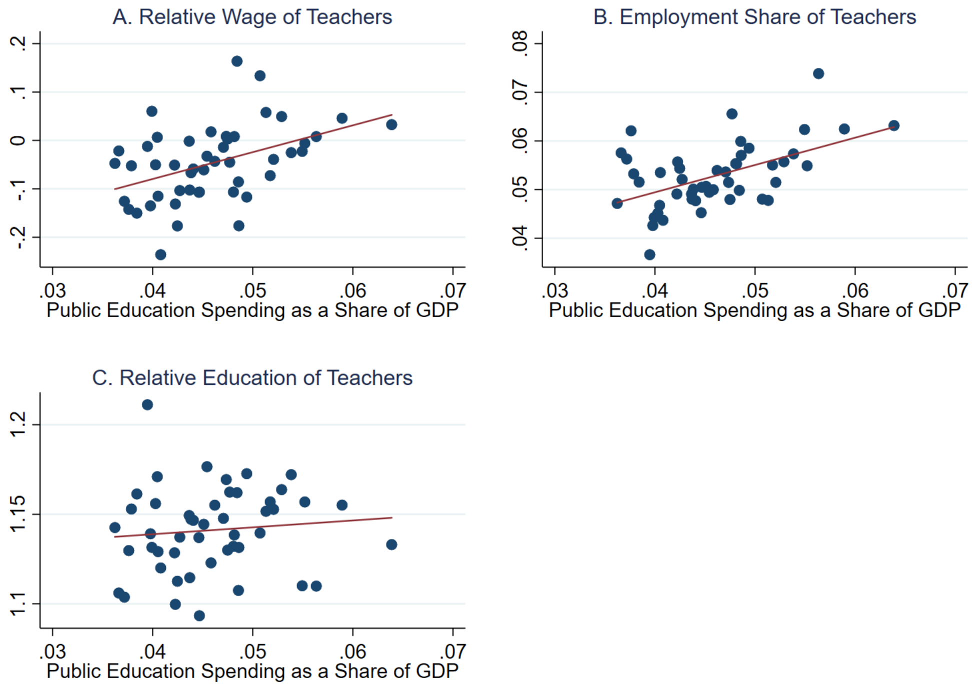

Figure 5: Cross-sectional relationships in 2010.
Notes: This figure shows the cross-state relationship between respectively the (log) difference in the average wage of teachers and the average wage in the state, the teacher employment share and teacher years of schooling (relative to average years of schooling in the state), and the ratio of public education expenditures to GDP for the year 2010. The solid line represents the line of best fit.

the predictions of our model.

Public education spending and teacher salary: We first provide some evidence that higher levels of public education spending are associated with higher teacher salaries. In Table 1, we report the results of a regression of average (log) teacher salary on public education expenditures (as a share of total state personal income).[19] We control for the average (log) wage in the state, teacher average age, teacher education (years of schooling), (log) total enrollment and the (log) number of teachers in the state as well as year and state fixed effects. In a second set of regressions, we also control for lagged values of the dependent variable using the instrumental variable methodology proposed by Arellano

[19]State-level GDP data is not available before 1997 from the Bureau of Economic Analysis.

and Bond (1991) and Blundell and Bond (1998) for dynamic panels.

	(1) Teacher (log) wage	(2) Teacher (log) wage	(3) Teacher (log) wage	(4) Teacher (log) wage
Pub. Educ.	2.853***	1.949***	4.498***	2.164**
	(0.840)	(0.755)	(0.937)	(0.927)
Time FE	Y	Y	Y	Y
State FE	Y	Y	Y	Y
Observations	288	240	288	240

Table 1: Public education spending and teacher salary
Notes: US (contiguous, excluding the District of Columbia) state regressions of teacher (log) wage on public education expenditures (as a share of personal income - *Pub. Educ.*). Columns (1) and (2) use Census data estimates of teacher wages and average log wages. Columns (3) and (4) use data on teacher wages from the US Census Bureau Statistical Abstracts of the United States and the National Center for Education Statistics Digest of Education Statistics (log of average wage). Columns (1) and (3) report results from OLS regressions. Columns (2) and (4) report results from Arellano and Bond (1991) and Blundell and Bond (1998) estimator for dynamic panels. Controls include the average (log) wage in the state, teacher average age, teacher education (years of schooling), (log) total enrollment and the (log) number of teachers in the state as well as year and state fixed effects. Standard errors are clustered at the state level for OLS regressions (in parenthesis). For Columns (2) and (4), we report GMM standard errors. Significance level: * 10%; ** 5% ; *** 1%.

We use teacher wage estimates from both Census data as well as from the US Census Bureau Statistical Abstracts of the United States and the National Center for Education Statistics Digest of Education Statistics. In all cases, we find a (statistically and economically) significant and positive correlation between public education spending and teacher salary.

Public education spending and teacher share of employment: Another prediction of our model is that higher education spending is associated with a larger employment share of teachers.

Table 2 shows that higher public education spending is associated with a larger share of employment in teaching occupations. This holds when controlling for student enrollment and the share of college-educated workers in the state as well as year and state fixed effects. Our results suggest that a one percentage point increase in the public education spending to income ratio raises teacher employment share by around 0.29 to 0.47 percentage point.

	(1) Teacher employment share	(2) Teacher employment share
Pub. Educ.	0.465***	0.292***
	(0.0920)	(0.0781)
Time FE	Y	Y
State FE	Y	Y
Observations	288	240

Table 2: Public education spending and teacher employment share
Notes: US (contiguous, excluding the District of Columbia) state regressions of teacher employment share on public education expenditures (as a share of personal income - *Pub. Educ.*). Column (1) reports results from OLS regressions. Column (2) reports results from Arellano and Bond (1991) and Blundell and Bond (1998) estimator for dynamic panels. Controls include (log) student enrollment and the share of college-educated workers in the state as well as year and state fixed effects. Standard errors are clustered at the state level for OLS regressions (in parenthesis). For Column (2), we report GMM standard errors. Significance level: * 10%; ** 5% ; *** 1%.

Public education spending and teacher education: Besides predicting that an increase in public education spending raises the employment share of teachers, our model also implies that the average teacher quality (human capital) goes up with education spending.

	(1) Teacher education	(2) Teacher education
Pub. Educ.	10.46***	5.538**
	(3.417)	(2.416)
Time FE	Y	Y
State FE	Y	Y
Observations	288	240

Table 3: Public education spending and teacher education
Notes: US (contiguous, excluding the District of Columbia) state regressions of teacher education (years of schooling) on public education expenditures (as a share of personal income - *Pub. Educ.*). Column (1) reports results from OLS regressions. Column (2) reports results from Arellano and Bond (1991) and Blundell and Bond (1998) estimator for dynamic panels. Controls include the average level of education in the state and the share of college-educated workers as well as year and state fixed effects. Standard errors are clustered at the state level for OLS regressions (in parenthesis). For Column (2), we report GMM standard errors. Significance level: * 10%; ** 5% ; *** 1%.

Regressing average teacher years of schooling (as a proxy for average human capital) on public education expenditures, we find that a one percentage point increase in education spending raises the average teacher education by around 0.05 to 0.1 year of schooling, controlling for the average level of education in the state.[20] Results are reported in Table 3 in which regressions also control for the share of college-educated workers as well as time and state fixed effects.

5 Quantitative Analysis

In this section, we calibrate the model to US state data and study whether they face a trade-off between growth and inequality through public education. These quantitative exercises reveal that a positive relationship between public education and income inequality is more than a theoretical possibility and that some US states might face such a trade-off.

For each state, we have to set the value of 8 parameters: φ, α, γ, β_1, μ_a, σ_a, τ and λ. We use estimates for tax progressivity (λ) at the state level from Fleck, Heathcote, Storesletten, and Violante (2021) who estimate a similar log-linear tax function. For the remaining 7 parameters, we target moments related to long-run growth, occupational choice, relative wages across occupations, intergenerational income mobility and public education spending. In particular, we match the following seven moments: the real income per capita growth rate, the employment shares of teachers and managers, the relative (log) wage of teachers to workers and of managers to teachers, intergenerational income mobility (rank-rank slope) and the ratio of public education expenditures to state income. We focus on the years 1990, 2000 and 2010 and on 47 US states: contiguous US states excluding the District of Columbia as well New Hampshire for which Fleck, Heathcote, Storesletten, and Violante (2021) estimate a regressive tax system.[21] See Appendix D for a more detailed description of the data.

[20]Note that due to degree requirements for education occupations, there is limited variation in the average years of schooling of teachers. In our sample, it ranges from 14.7 to 16.7 years. The effect of public education spending on teacher education is nevertheless positive and significant.

[21]We make the assumption that one period in the model is equivalent to 20 years.

5.1 Calibration

The calibrated model matches the targeted moments very closely for all states. The full set of parameter values can be found in Table E.1 in Appendix E. Table E.2 in Appendix E compares the targeted moments in the model and in the data.[22]

Using our calibrated model, we can first determine whether an increase in public education (from an increase in the tax level, τ, or in tax progressivity, λ) would lead to a decrease or an increase in income inequality (measured by the Gini coefficient) for each state. Our results display significant heterogeneity in terms of the relationship between public education expenditures and income inequality across states. Starting with changes in the level of taxation (τ), we find that most states display a negative relationship between public education expenditures and inequality. The derivative of the Gini coefficient with respect to τ is negative in 35 out of 47 states (74.5%). This suggests that most states could increase growth and decrease income inequality through higher public education expenditures raised through an increase in the level of taxes.

Another way to increase public education spending is to raise proceeds from the tax through tax progressivity (λ). Repeating the same experiment, we find significantly different results. For most states (31 out of 47), raising tax progressivity leads to an increase in income inequality. Figure 6 shows the percentage change in the Gini coefficient following a one percentage point increase in public education spending to GDP for each state. We should note that those results relate to before-tax income inequality. Consequently, they do not directly capture the redistributive effect of increasing tax progressivity (which would tend to compress the after-tax income distribution). Instead, they show how changing tax progressivity alters occupational choice (which depends on after-tax income) and

[22]Figure C.7 in Appendix C.5 shows the level of heterogeneity in terms of public education spending, growth and employment shares of teachers across states in the data as well as the distribution of tax level (τ) from our calibration and tax progressivity λ from Fleck, Heathcote, Storesletten, and Violante (2021).

how this translates into changes in the distribution of income before tax.

Overall, these results suggest that there exists a lot of heterogeneity in terms of how increased public education expenditures affect income inequality. Not only do different states face different relationships between public education and inequality (some positive and some negative) but the direction of this relationship also depends on the way increased public education is financed (through an increase in the level or in the progressivity of the tax). We can further notice from Figure 6 that states with larger public education expenditures tend to experience a larger increase (or smaller decrease) in income inequality for a given increase in public education spending. As the derivative of income inequality with respect to public education is an increasing function of public education expenditures and turns from negative to positive, those results are overall consistent with a U-shape relationship between public education and inequality across US states. Raising public education expenditures tends to decrease (respectively increase) inequality at low (high) levels of public education spending.

Figure 7 further compares the change in occupational choice and growth after a one percentage point increase in public education spending coming from an increase in λ (tax progressivity) as opposed to τ (tax level). More specifically, it shows the difference (as a percentage of the moment value) between the change in the variable when λ is increased minus the change of the same variable when τ is raised. We can see that the increase in teacher employment share for a given increase in public education spending is larger when changes in the tax level (τ) are used. This also leads to a larger decrease in the share of workers but a smaller reduction in manager employment share. In other words, the two dimensions of the tax system (level and progressivity) affect occupational choice differently. Increasing public education spending through τ attracts relatively more teachers, but relatively more of those new teachers are coming from the worker side of the distribution. Overall, education quality increases by more when τ rises which leads

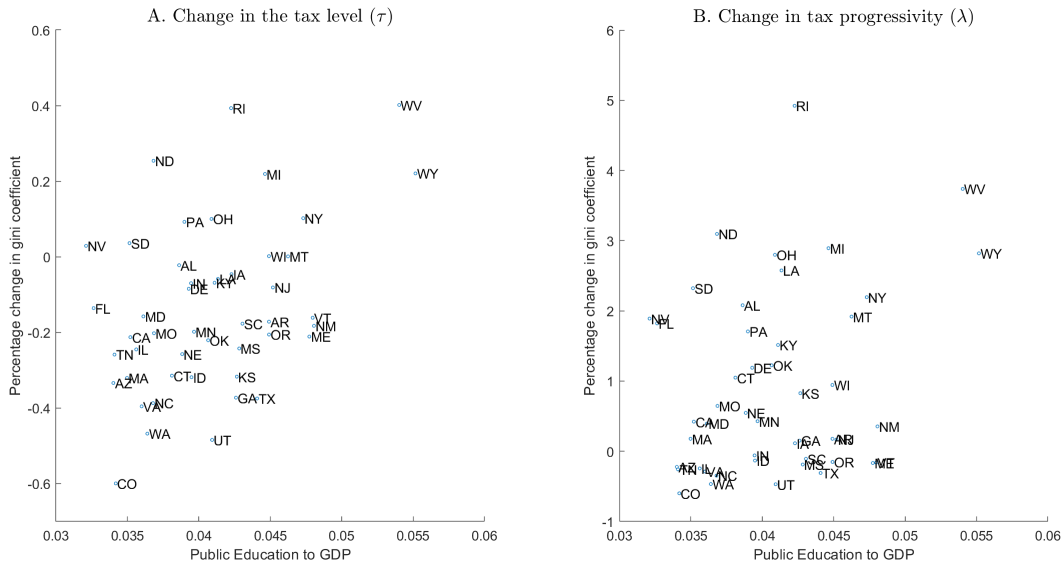

Figure 6: Percentage change in the Gini coefficient following a one percentage point increase in public education expenditures to GDP by state.

Notes: This figure shows the predicted percentage change in income inequality (Gini) following a one percentage point increase in public education to GDP for each of our state-level calibrations. The horizontal axis refers to the level of public education in the state. Increase in public education spending is obtained through an increase in the level of the tax (τ) on the left panel and from an increase in tax progressivity (λ) on the right panel.

to larger gains in growth. These results further highlight the role played by occupational choice and the endogenous quality of education in determining how increases in public education spending lead to changes in growth and inequality. In addition, we find that the "semi-elasticity" of inequality to public education (reported in Figure 6) is larger when tax progressivity is raised (rather than the tax level). In other words, raising public education through an increase in τ is consistently found to lead to a larger increase in growth and a smaller increase (or larger decrease) in income inequality. This seems to suggest that, despite the potential existence of a trade-off between growth and inequality, increasing the level of the tax is preferable to raising tax progressivity both in terms of growth and income inequality.

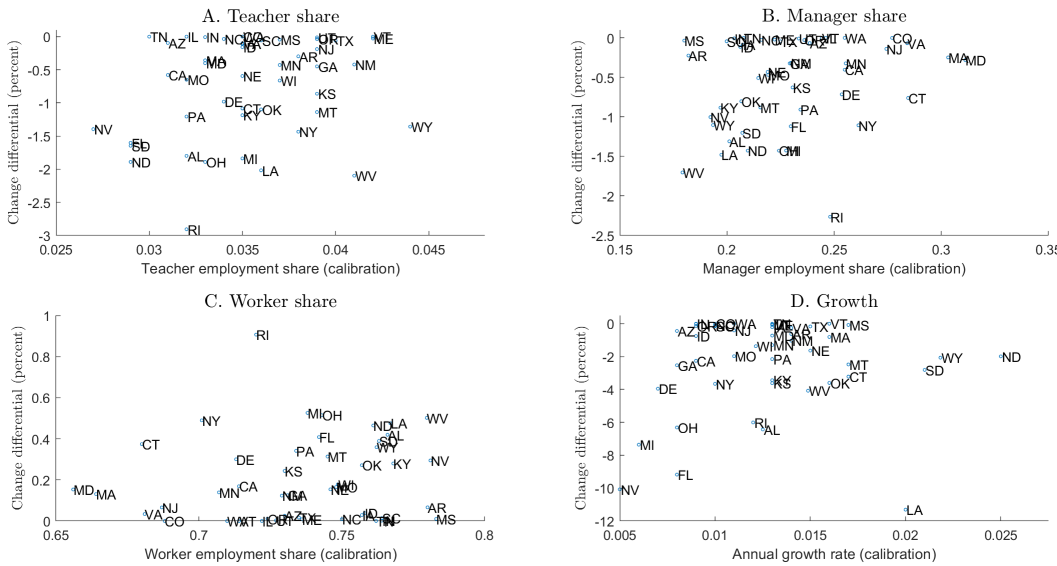

Figure 7: Differential change (percent) following a one percentage point increase in public education expenditures to GDP by state: tax progressivity (λ) vs. tax level (τ).
Notes: This figure shows, for each state, the difference between the percentage change in employment shares and growth resulting from a one percentage point increase in public education to GDP through increased tax progressivity (λ) minus the percentage change in the same variables obtained through an increase in the tax level (τ) instead. A positive value means that the percentage change in the relevant variable is larger when public education spending is raised through a change in tax progressivity compared to the tax level.

5.2 Parameter Values and the Trade-Off Between Growth and Inequality

This section identifies the role of the parameters of the model in determining whether there is a trade-off between growth and inequality. We verify how each of the parameters affects the slope of the inequality-public education relationship for each state. In particular, we compute the second-order partial derivative of inequality with respect to respectively τ and λ and the six parameters of our model, i.e. $\frac{\partial^2 Gini}{\partial i \partial \tau}$ and $\frac{\partial^2 Gini}{\partial i \partial \lambda}$ with $i \in \{\varphi, \alpha, \beta_1, \gamma, \mu_a, \sigma_a\}$. A positive (respectively negative) second-order partial derivative implies that an increase in parameter i raises (decreases) the derivative of the Gini coefficient with respect to public education spending. Economies with higher (lower) values of that parameter are more likely to be facing a trade-off between growth and income inequality. We report the full set of second-order partial derivatives in Tables F.1 and F.2 in Appendix F.

We find that every parameter with the exception of μ_a directly affects whether there exists a trade-off between growth and inequality through public education expenditures.[23] We can also relate those parameters to observable characteristics, as each parameter is tightly related to one of the targeted moments. Even though there is some heterogeneity across states, we find in the vast majority of cases that, *ceteris paribus*, lower values of γ (lower relative wage of teachers), and higher values of α (higher employment share of workers), β_1 (lower intergenerational income mobility), σ_a (higher employment share of managers) and φ (higher relative wage of managers) are all associated with lower (potentially negative) slopes of the inequality-public education relationship. Hence, those states are less likely to face a trade-off between growth and income inequality through public education. Those results hold for changes in public education expenditures coming from changes in both the level τ and the progressivity λ of the tax.

6 Conclusion

This paper theoretically shows that the relationship between public education, growth and inequality is critically shaped by education quality, which depends on occupational decisions and on the human capital distribution in the economy. To study these effects, we propose an endogenous growth model of occupational choice with an endogenous supply of teachers. We show that the elasticity of growth to public education expenditures depends positively on the thickness of the right tail of the human capital distribution. In addition, we show that the relationship between public education and income inequality can be either positive or negative. Both of these results have important policy implications as

[23] μ_a directly affects the growth rate of the economy but plays no role in determining the existence of a trade-off between growth and inequality in our model.

there might exist a trade-off between economic growth and income inequality through public education.

Our calibrations highlight a significant degree of heterogeneity across US states and show that some of them might face a trade-off between growth and inequality through public education. The presence of this trade-off also depends on the way public education expenditures are financed and is more likely to occur when tax progressivity (rather than the tax level) is increased.

This paper focuses on the role of the endogenous quality of education that arises from occupational decisions coupled with public education expenditure levels. In doing so, we have left out some other dimensions that might have relevant implications. In particular, while a novel feature of our model is to focus on the supply side of public education, the heterogeneity in returns to education is exogenous in our model. There might be heterogeneity in individual investment in education, for instance through differential effort, time spent in school or spending on private education, that could interact with the mechanisms that we highlight in this paper. In addition, we have focused on the role of public education spending on teacher salaries. Even though instructors' salaries represent a major share of public education expenditures in the US, there might be other types of expenses or investments in public education that would affect education quality and that may be worthy of further investigation. We leave these questions for future research.

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Appendix

A Proofs

A.1 Proof of Proposition 1

The proof starts from the limit behavior of the utility function under the three different occupations and the fact that all (after-tax) wage functions are continuous in human capital. For workers, the utility function is (strictly) increasing and concave in human capital with intercept at zero. For teachers and managers, we have:

$$\lim_{h \to 0} u^T(h) = -\gamma \text{ and } \lim_{h \to \infty} u^T(h) = \infty$$
$$\lim_{h \to 0} u^M(h) = -\gamma \text{ and } \lim_{h \to \infty} u^M(h) = \infty$$

This implies:

$$\lim_{h \to 0} u^W(h) > \lim_{h \to 0} u^T(h) \text{ and } \lim_{h \to 0} u^W(h) > \lim_{h \to 0} u^M(h)$$

In addition, since $u^M(h)$ is strictly increasing and strictly convex in h, we get:

$$\lim_{h \to \infty} \{u^M(h) - u^W(h)\} = \infty \text{ and } \lim_{h \to \infty} \{u^M(h) - u^T(h)\} = \infty$$

This, combined with the fact that all wage functions are continuous in human capital, proves the existence of two cutoffs. First, there exists a cutoff ($h_W > 0$) up to which agents find it optimal to become workers. There also exists another cutoff ($h_M \geq h_W$) above which agents decide to become managers. This implies that only agents in the middle of the human capital distribution (between h_W and h_M) want to become teachers. To have a strictly positive mass of agents working as teachers, two conditions are needed: first, the wage rate of teachers (w^T) must be strictly greater than that of workers since teachers face an additional positive cost and, second, the utility of working as a teacher with human capital h_W must be strictly greater than the utility of a manager with the same level of human capital i.e., we need the teacher utility function to intersect $u^W(h)$ before the manager utility function (both teacher and manager utility intersects the worker utility function only once over $\mathbb{R}_{>0}$):

$$h^* = \{h : u^W(h) = u^T(h)\} < h^{**} = \{h : u^W(h) = u^M(h)\} \tag{A.1}$$

If Equation (A.1) is satisfied, $h_M > h_W = h^*$ and there is a positive mass of teachers in the economy. Otherwise, $h_W = h_M$ and there are only workers and managers in the economy. $\square$

A.2 Proof of Proposition 2

First, we can show that the wage of workers is strictly positive in equilibrium. If it was not positive, managers' demand for human capital would be infinite and, hence, the labor market condition could not be satisfied.[24] Having proved that $w > 0$ in any equilibrium, we know from Proposition 1 that a strictly positive mass of agents finds it optimal to become workers and managers.

To prove that there must also be a positive mass of teachers in equilibrium, let's assume that there are no teachers in the economy. Given a tax function with $\tau \in (0, 1)$ and

[24]By a similar argument, we can easily show that w has to be finite.

$\lambda \in [0, 1)$, proceeds from the tax (net of transfers) are strictly positive.[25]

Using the government budget constraint, we get:

$$\int_{h_W}^{h_M} (1-\tau)\left(w^T h\right)^{1-\lambda} Y^{\lambda}\, dF(h) = \int_0^{h_W} wh \left[1-(1-\tau)\left(\frac{wh}{Y}\right)^{-\lambda}\right] dF(h) + \int_{h_M}^{\infty} \pi(z) \left[1-(1-\tau)\left(\frac{\pi(z)}{Y}\right)^{-\lambda}\right] dF(z) \quad \text{(A.3)}$$

where the right-hand side of Equation (A.3) is the net proceed of the tax.

If there are no teachers, $h_W = h_M$ and Equation (A.3) imply that the teacher wage rate (w^T) is infinite. This would imply that no agent finds it optimal to work as a teacher at an infinite wage rate, which is a contradiction. Hence, there must be a positive mass of teachers in the economy in equilibrium. $\square$

[25] Given the tax function in Equation (2), we can show that, when there are no teachers, the net proceed of the tax is positive. We can show that it is true for any income function ($\tilde{w}(h)$). We should notice that tax rates at the bottom of the income distribution are negative and that the net proceed from the tax after redistribution can be written as:

$$Net\ Tax\ Proceed = \int \tilde{w}(h) \left[1-(1-\tau)\left(\frac{\tilde{w}(h)}{Y}\right)^{-\lambda}\right] dF(h)$$

where total income $Y = \int \tilde{w}(h) dF(h)$.
We can then rewrite:

$$Net\ Tax\ Proceed \quad = Y\left(1-(1-\tau)\int\left(\frac{\tilde{w}(h)}{Y}\right)^{1-\lambda} dF(h)\right)$$

As a result, net tax proceeds are strictly positive if:

$$1-(1-\tau)\int\left(\frac{\tilde{w}(h)}{Y}\right)^{1-\lambda} dF(h) > 0 \quad \text{(A.2)}$$

We can next show that $0 < \int\left(\frac{\tilde{w}(h)}{Y}\right)^{1-\lambda} dF(h) \leq 1$. Note that $Y = E[\tilde{w}(h)]$ and rewrite $\int\left(\frac{\tilde{w}(h)}{Y}\right)^{1-\lambda} dF(h) = \frac{E\left[\tilde{w}(h)^{1-\lambda}\right]}{E[\tilde{w}(h)]^{1-\lambda}} \leq 1$ by Jensen's inequality with $\lambda \in [0, 1)$. Using this result in Equation (A.2) with $\tau \in (0, 1)$ proves that net tax proceeds are positive.

A.3 Proof of Proposition 3

In a balanced growth path, the mean of the human capital distribution ($e^{\mu_t + \frac{\sigma_t^2}{2}}$), the thresholds ($h_{W,t}$ and $h_{M,t}$) and total human capital of workers grow at the constant rate g_h:

$$1 + g_h = \frac{e^{\mu_{t+1} + \frac{\sigma_{t+1}^2}{2}}}{e^{\mu_t + \frac{\sigma_t^2}{2}}}$$

In the long run, σ_t converges to a constant and we can rewrite:

$$1 + g_h = \frac{e^{\mu_{t+1}}}{e^{\mu_t}}$$

Using Equation (13), we get:

$$1 + g_h = e^{\mu_a} \left(\frac{S_t}{e^{\mu_t}} \right)^{1-\beta_1}$$

Using the first order condition from managers' problem, we can write total output (Y_t) as:

$$Y_t = \frac{w_t}{\alpha} \int_{h_{M,t}}^{\infty} H_t(z)\ dF_t(z)$$

Computing the growth rate of $HC_t = \int_{h_{M,t}}^{\infty} H_t(z)\ dF_t(z)$, we get in a balanced growth path:

$$\frac{HC_{t+1}}{HC_t} = \frac{\int_0^{h_{W,t+1}} h \; dF_{t+1}(h)}{\int_0^{h_{W,t}} h \; dF_t(h)}$$

Given that human capital is log-normally distributed, the partial expectation can be written as $\int_0^{h_{W,t}} h \; dF_t(h) = e^{\mu_t + \frac{\sigma_t^2}{2}} \Phi \left[\frac{log\left(h_{W,t}\right) - \mu_t - \sigma_t^2}{\sigma_t} \right]$, where $\Phi(x)$ is the standard normal cdf. In a balanced growth path, we can then rewrite:

$$\begin{aligned}
\frac{HC_{t+1}}{HC_t} &= \frac{e^{\mu_{t+1} + \frac{\sigma_{t+1}^2}{2}} \Phi \left[\frac{log\left(h_{W,t+1}\right) - \mu_{t+1} - \sigma_{t+1}^2}{\sigma_{t+1}} \right]}{e^{\mu_t + \frac{\sigma_t^2}{2}} \Phi \left[\frac{log\left(h_{W,t}\right) - \mu_t - \sigma_t^2}{\sigma_t} \right]} \\
&= \frac{e^{\mu_{t+1}}}{e^{\mu_t}} \frac{\Phi \left[\frac{log\left(h_{W,t}(1+g_h)\right) - \mu_t - log(1+g_h) - \sigma_t^2}{\sigma_t} \right]}{\Phi \left[\frac{log\left(h_{W,t}\right) - \mu_t - \sigma_t^2}{\sigma_t} \right]} \\
&= 1 + g_h
\end{aligned}$$

so that the growth rate of Y_t (g) is given by:

$$(1+g) = (1+g_w)(1+g_h)$$

Indifference condition at $h_{M,t}$ implies:

$$h_{M,t} \propto w_t^{\frac{\alpha}{\alpha+\varphi-1}} {w_t^T}^{\frac{1-\alpha}{\alpha+\varphi-1}} Y_t^{\lambda}$$

It follows that, in a balanced growth path, $(1+g_h) = (1+g_w)^{\frac{1}{\alpha+\varphi-1}}$ and, hence,

$$\begin{aligned} (1+g) &= (1+g_h)^{\varphi+\alpha} \qquad \text{(A.4)} \\ g &\approx (\varphi+\alpha)\left\{\mu_a + (1-\beta_1)\left[log(S_t) - \mu_t\right]\right\}. \end{aligned}$$

□

A.4 Proof of Proposition 4

Total production in the economy is given by $Y_t = \int_{h_{M,t}}^{\infty} z^{\varphi} H_t(z)^{\alpha}\ dF_t(z)$. Using labor demand, we can write:

$$Y_t = \left(\frac{\alpha}{w_t}\right)^{\frac{\alpha}{1-\alpha}} \int_{h_{M,t}}^{\infty} z^{\frac{\varphi}{1-\alpha}}\ dF_t(z)$$

Further using the labor market clearing condition in Equation (7), we obtain:

$$\begin{aligned} Y_t &= \left[\int_0^{h_{W,t}} h\ dF_t(h)\right]^{\alpha} \left[\int_{h_{M,t}}^{\infty} z^{\frac{\varphi}{1-\alpha}}\ dF_t(z)\right]^{1-\alpha} \\ &= Z_t HC_t^{\alpha} \end{aligned}$$

Using Equation (A.4) and the fact that HC_t grows at rate g_h, we can rewrite:

$$
\begin{aligned}
&(1+g) = (1+g_h)^{\alpha}(1+g_Z) \\
&(1+g_h)^{\alpha+\varphi} = (1+g_h)^{\alpha}(1+g_Z) \\
&g_Z = (1+g_h)^{\varphi} - 1.
\end{aligned}
$$

□

B Static Equilibrium Definition

Static Equilibrium Definition: *Given a continuous distribution of human capital with cdf* $F : \mathbb{R}^+ \to [0,1]$ *and a tax system* ($\tau \in (0,1)$ *and* $\lambda \in [0,1)$)*, a static equilibrium is a collection of wages* (wh, $w^T h$)*, profits* ($\pi(z)$)*, cutoffs* (h_W, h_M)*, demand for human capital* (H)*, and final good production* ($Y = \int_{h_M}^{\infty} y(z)\ dF(z)$) *such that:*

1. *Given wages, firms maximize profit.*
2. *Given wages and a tax system, agents maximize utility by following a cutoff strategy in which agents with human capital in* $[0, h_W)$ *become workers, agents with human capital in* $[h_W, h_M)$ *are teachers and agents with human capital above* h_M *work as managers.*
3. *Labor market clears:* $\int_{h_M}^{\infty} H(z)\ dF(z) = \int_0^{h_W} h\ dF(h)$
4. *Government budget is balanced:* $\int_{h_W}^{h_M} w^T h\, dF(h) = \int_0^{h_W} wh \left[1 - (1-\tau)\left(\frac{wh}{Y}\right)^{-\lambda}\right] dF(h) + \int_{h_W}^{h_M} w^T h \left[1 - (1-\tau)\left(\frac{w^T h}{Y}\right)^{-\lambda}\right] dF(h) + \int_{h_M}^{\infty} \pi(z) \left[1 - (1-\tau)\left(\frac{\pi(z)}{Y}\right)^{-\lambda}\right] dF(z)$

C Additional Figures

C.1 Comparative Statics for Tax Progressivity

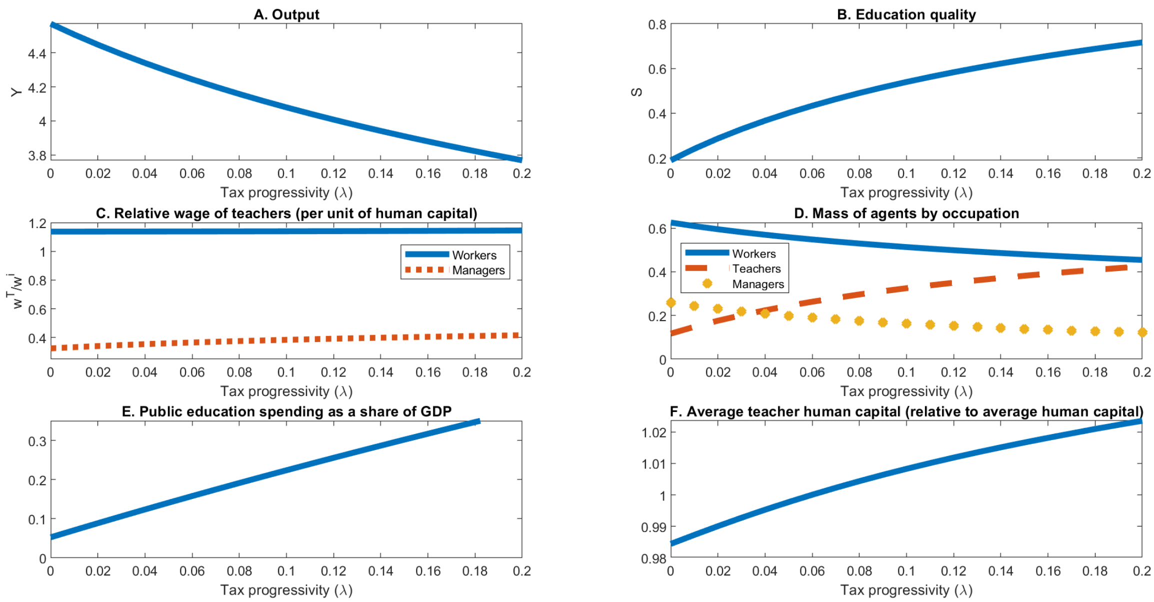

Figure C.1: Comparative statics for tax progressivity (λ).
Notes: In this figure, we use the following parameter values: $\gamma = 0.2$, $\tau = 0.1$ $\alpha = 0.05$, $\varphi = 1.75$ and F is a log-normal distribution with parameter values equal to zero and 1 respectively.

C.2 Human Capital Distribution, Public Education and Growth

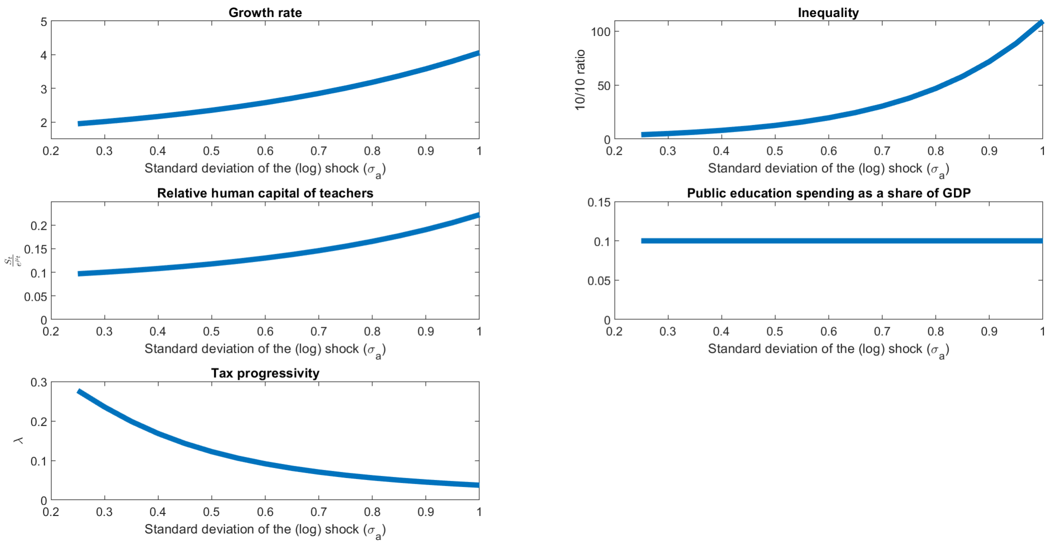

Figure C.2: Comparative statics in a balanced growth path with progressive tax system and constant public education expenditures to GDP (10%): standard deviation of the (logarithm of the) shock (σ_a).
Notes: In this figure, we use the following parameter values: $\alpha = 0.3$, $\beta_1 = 0.5$, $\gamma = 0.1$, $\varphi = 1$, $\mu_a = 2$ and $\tau = 0.05$. The figure shows growth, inequality, teachers' human capital (relative to the average human capital in the economy) and public education spending for different values of the standard deviation of the (log) shock to human capital (σ_a) for a fixed ratio of public education expenditures to GDP. Tax progressivity (λ) is adjusted to keep that ratio constant across different values of σ_a and with a fixed tax level (τ).

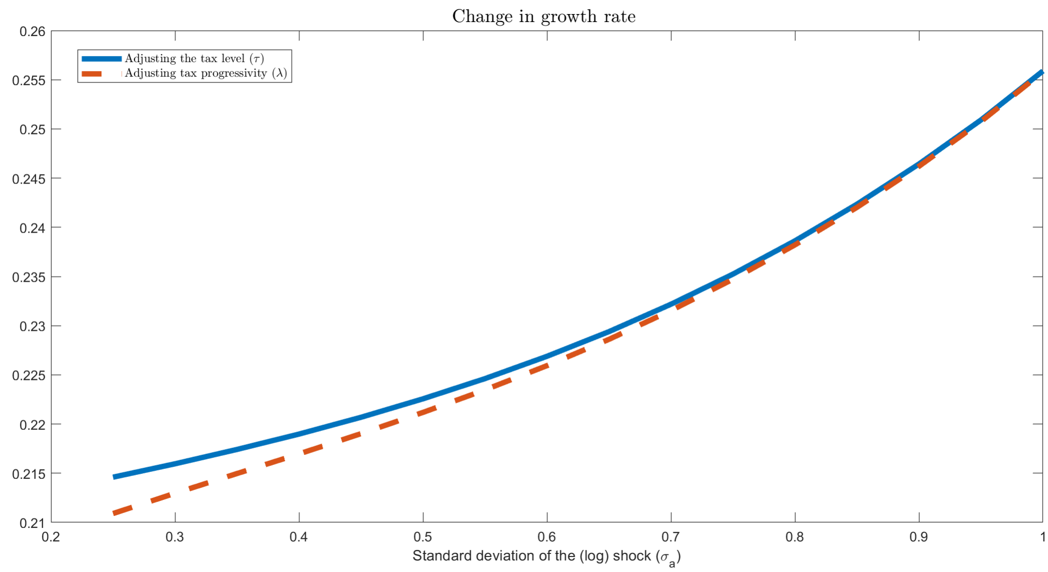

Figure C.3: Growth effect of a one percentage point increase in public education expenditures for different values of the standard deviation of the (logarithm of the) shock (σ_a).
Notes: In this figure, we use the following parameter values: $\alpha = 0.3$, $\beta_1 = 0.5$, $\gamma = 0.1$, $\varphi = 1$, $\mu_a = 2$, $\tau = 0.05$ and $\lambda = 0.05$. The figure shows the change in economic growth from a one percentage point increase in public education spending to GDP for different values of the standard deviation of the (log) shock (σ_a). The solid line refers to an increase in public education through an increase in the tax level (τ) and the dashed line through an increase in tax progressivity (λ).

C.3 Examples of Positive and Negative Relationships Between Public Education and Income Inequality

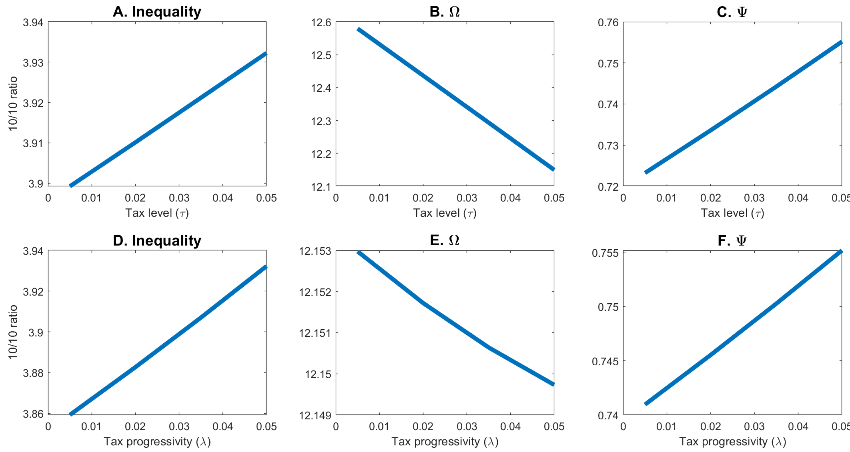

Figure C.4: Comparative statics in a balanced growth path with positive relationship between public education and income inequality : tax level (τ) and tax progressivity (λ).
Notes: In this figure, we use the following parameter values: $\alpha = 0.7$, $\beta_1 = 0.25$, $\gamma = 0.5$, $\varphi = 0.4$, $\mu_a = 3$, $\sigma_a = 0.25$, $\lambda = 0.05$ (Panels A to C) and $\tau = 0.05$ (Panels D to F). This figures shows examples of a positive relationship between the tax level (τ) as well as tax progressivity (λ) and income inequality. $\Omega = \frac{\int_0^{h^W} h \ dF(h)}{\int_0^{F^{-1}(0.1)} h \ dF(h)}$ and $\Psi = \frac{\int_{F^{-1}(0.9)}^{\infty} \pi(z) \ dF(z)}{\int_{h_M}^{\infty} \pi(z) \ dF(z)}$. They respectively relate to labor supply at the bottom of the human capital distribution and to profit concentration at the top. Parameter values have been chosen so that there is no teacher in either the top or bottom 10% of the distribution at any reported tax rate.

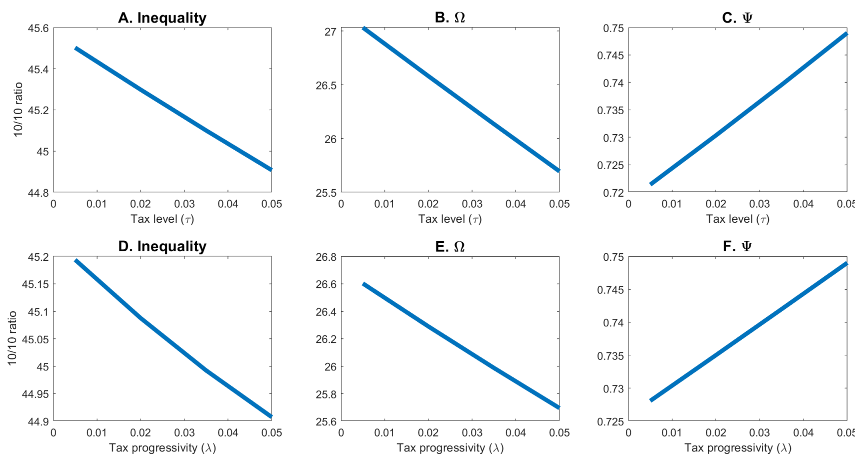

Figure C.5: Comparative statics in a balanced growth path with negative relationship between public education and income inequality : τ and λ.
Notes: In this figure, we use the following parameter values: $\alpha = 0.3$, $\beta_1 = 0.5$, $\gamma = 0.3$, $\varphi = 1$, $\mu_a = 3$, $\sigma_a = 0.75$, $\lambda = 0.05$ (Panels A to C) and $\tau = 0.05$ (Panels D to F). This figures shows examples of a negative relationship between the tax level (τ) as well as tax progressivity (λ) and income inequality. $\Omega = \frac{\int_0^{h^W} h \ dF(h)}{\int_0^{F^{-1}(0.1)} h \ dF(h)}$ and $\Psi = \frac{\int_{F^{-1}(0.9)}^{\infty} \pi(z) \ dF(z)}{\int_{h_M}^{\infty} \pi(z) \ dF(z)}$. They respectively relate to labor supply at the bottom of the human capital distribution and to profit concentration at the top. Parameter values have been chosen so that there is no teacher in either the top or bottom 10% of the distribution at any reported tax rate. Parameter values have been chosen so that there is no teacher in either the top or bottom 10% of the distribution at any reported tax rate.

C.4 Relationship Between Public Education and Income Inequality for Alternative Measures of Inequality

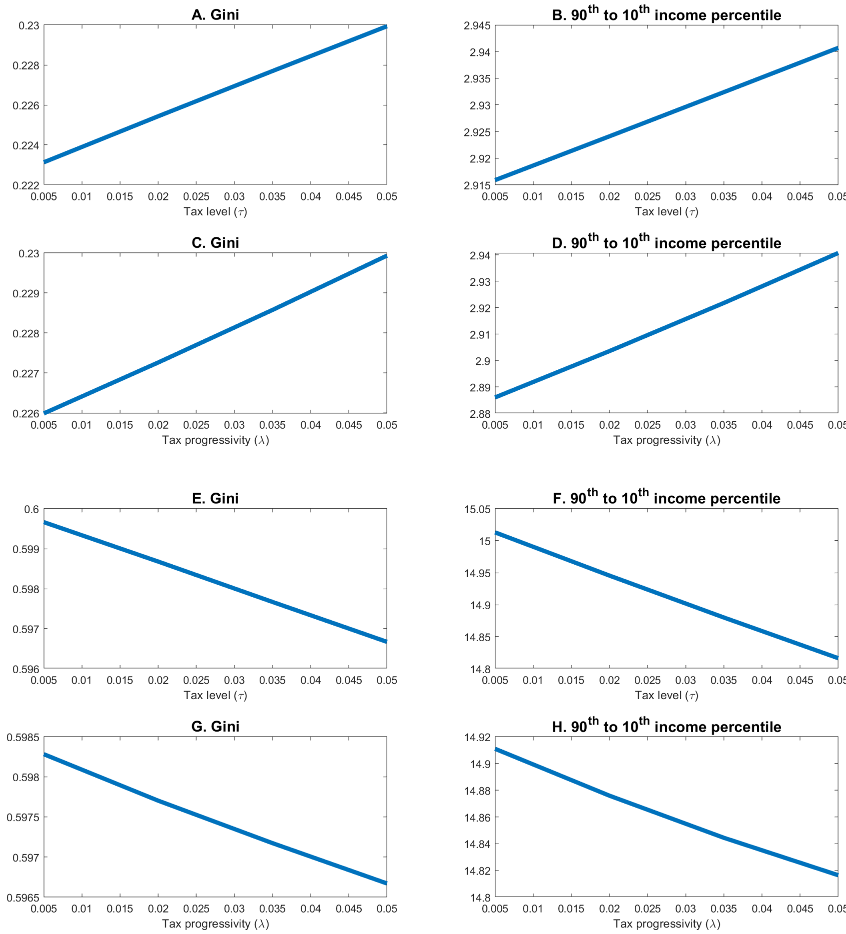

Figure C.6: Alternative measures of inequality.
Notes: In this figure, we use the following parameter values: $\alpha = 0.7$, $\beta_1 = 0.25$, $\gamma = 0.5$, $\varphi = 0.4$, $\mu_a = 3$ and $\sigma_a = 0.25$ for Panels A to D; and $\alpha = 0.3$, $\beta_1 = 0.5$, $\gamma = 0.3$, $\varphi = 1$, $\mu_a = 3$, $\sigma_a = 0.75$ for Panels E to H. $\lambda = 0.05$ (Panels A to B and E to F) and $\tau = 0.05$ (Panels D to F and G to H). This figure shows that both positive and negative relationship between income inequality and the tax level (τ) as well as tax progressivity (λ) can be obtained for alternative measures of income inequality.

C.5 Cross-State Heterogeneity

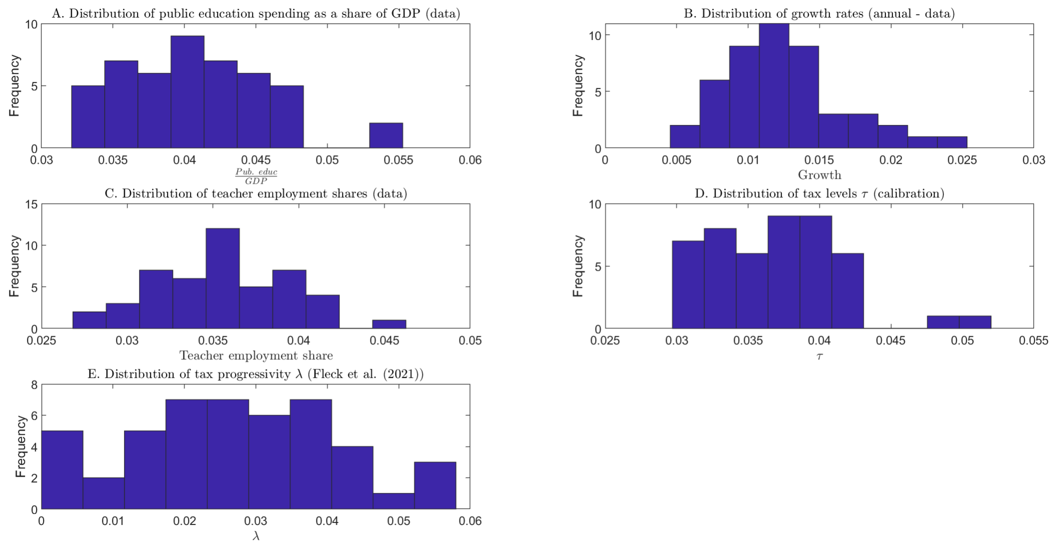

Figure C.7: Distribution of moments (data) and parameters.

D Data Appendix

This section describes the data used in the empirical analysis in Section 4 and in the quantitative analysis in Section 5. Data is collected at the US state level for years 1960, 1970, 1980, 1990, 2000 and 2010.[26]

Census data: We use US Census data to estimate employment shares and relative wages by occupation and state.[27] We keep individuals aged between 18 and 64 year old, following Autor and Dorn (2013). We drop agents for which the occupation is unknown as well as those for which the number of working hours (per week) and weeks (per year) is not reported. Following Ales, Kurnaz, and Sleet (2015), underemployed individuals (i.e., people with less than 250 hours worked) and agents earning less than 100 US dollars per year are dropped as well. Regarding occupations, we use the occ1990 occupation system. We first drop military occupations, unemployed and unknown occupations (occ1990 greater than 904). We then use occ1990 classification to identify (non-postsecondary) teachers (occ1990 between 155 and 159) and managers (occ1990 below 155 and between 160 and 200). All other remaining occupations are classified as workers. We then compute average (log) hourly wages and employment shares for each of our three occupation groups. We also compute the average level of education and age by occupation and the share of workers with at least a college/high school degree.

Public education data: We obtain data on public education spending on elementary and secondary schools from the US Census Bureau Statistical Abstracts of the United States for years 1960 to 2000.[28] For the year 2010, we use the National Center for

[26] We focus on contiguous United States excluding the District of Columbia.

[27] Steven Ruggles, Sarah Flood, Ronald Goeken, Megan Schouweiler and Matthew Sobek. IPUMS USA: Version 12.0 [dataset]. Minneapolis, MN: IPUMS, 2022. Data is available at https://usa.ipums.org/usa/.

[28] Available at https://www.census.gov/library/publications/time-series/statistical_abstracts.html.

Education Statistics Digest of Education Statistics.[29] We also collect data on enrollment, number of teachers and average teacher salary.

Personal income data: State-level GDP data is not available before 1997. As a result, we use data on personal income instead which is available from the Bureau of Economic Analysis.[30] We compute real personal income per capita using Bureau of Economic Analysis data on population and Consumer price Index (from the US Bureau of Labor Statistics) to obtain measures of state level growth rates in real income per capita.[31] We measure state-level public education investment as the ratio of public education spending to personal income.

Intergenerational income mobility data: Chetty, Hendren, Kline, and Saez (2014) provide state-level estimates of the rank-rank slope for intergenerational mobility. The rank-rank slope measures the correlation between the ranks of the parent and of the child in the income distribution.[32]

[29] Data can be found at https://nces.ed.gov/programs/digest/.
[30] Data available at https://apps.bea.gov/iTable/iTable.cfm?reqid=70step=1acrdn=2.
[31] CPI data available at https://www.bls.gov/cpi/data.htm.
[32] Data available at https://opportunityinsights.org/data.

E Calibration: Parameters and Moment Matching

	AL	AZ	AR	CA	CO	CT	DE	FL	GA	ID	IL	IN	IA	KS	KY	LA
φ	1.308	0.923	0.636	0.861	1.821	1.616	0.706	1.279	1.324	0.901	0.710	0.382	0.380	1.530	0.860	3.990
α	0.675	0.633	0.709	0.597	0.583	0.563	0.613	0.645	0.632	0.675	0.627	0.702	0.695	0.642	0.697	0.682
β_1	0.406	0.306	0.386	0.248	0.296	0.376	0.402	0.322	0.388	0.263	0.392	0.383	0.323	0.339	0.377	0.419
μ_a	2.106	2.411	2.112	2.626	2.393	2.198	2.052	2.435	1.993	2.467	2.139	2.164	2.388	2.212	2.176	1.995
σ_a	0.110	0.195	0.196	0.224	0.133	0.124	0.197	0.121	0.150	0.184	0.219	0.224	0.230	0.116	0.139	0.039
γ	0.171	0.008	0.025	0.057	0.000	0.125	0.086	0.135	0.046	0.013	0.000	0.001	0.009	0.083	0.095	0.199
τ	0.037	0.032	0.040	0.031	0.031	0.035	0.033	0.032	0.037	0.035	0.033	0.037	0.039	0.038	0.038	0.039
λ	0.001	0.013	0.037	0.036	0.021	0.016	0.058	0	0.041	0.037	0.018	0.018	0.026	0.034	0.016	0.006

	ME	MD	MA	MI	MN	MS	MO	MT	NE	NV	NJ	NM	NY	NC	ND	OH
φ	0.633	0.711	1.032	0.405	0.923	0.660	0.834	0.651	0.972	0.571	0.545	0.738	0.508	1.025	0.330	0.651
α	0.670	0.544	0.559	0.656	0.619	0.715	0.668	0.687	0.669	0.717	0.581	0.638	0.603	0.659	0.697	0.661
β_1	0.314	0.430	0.341	0.377	0.331	0.432	0.383	0.261	0.333	0.268	0.359	0.310	0.353	0.408	0.272	0.416
μ_a	2.271	2.085	2.381	2.120	2.290	2.024	2.181	2.576	2.343	2.619	2.164	2.296	2.211	2.044	2.958	2.052
σ_a	0.197	0.229	0.181	0.189	0.170	0.188	0.162	0.145	0.149	0.160	0.278	0.211	0.209	0.173	0.183	0.149
γ	0.002	0.040	0.032	0.164	0.048	0.004	0.049	0.089	0.046	0.079	0.020	0.044	0.150	0.003	0.131	0.161
τ	0.042	0.033	0.031	0.040	0.037	0.039	0.034	0.042	0.035	0.030	0.039	0.043	0.041	0.033	0.035	0.038
λ	0.044	0.026	0.034	0.038	0.039	0.027	0.025	0.032	0.032	0.024	0.042	0.036	0.047	0.031	0.003	0.02

	OK	OR	PA	RI	SC	SD	TN	TX	UT	VT	VA	WA	WV	WI	WY
φ	1.437	0.635	0.456	0.378	0.567	0.690	0.693	1.050	1.328	0.561	1.144	1.253	0.411	0.444	0.430
α	0.678	0.650	0.645	0.629	0.687	0.692	0.677	0.629	0.639	0.645	0.552	0.622	0.730	0.679	0.724
β_1	0.355	0.293	0.388	0.346	0.398	0.298	0.384	0.332	0.250	0.298	0.396	0.294	0.358	0.368	0.275
μ_a	2.236	2.318	2.250	2.421	2.043	2.690	2.239	2.216	2.477	2.389	2.113	2.416	2.232	2.200	2.552
σ_a	0.107	0.210	0.215	0.155	0.220	0.147	0.206	0.204	0.152	0.213	0.210	0.151	0.134	0.210	0.135
γ	0.095	0.003	0.109	0.279	0.005	0.113	0.000	0.008	0.001	0.000	0.013	0.000	0.188	0.052	0.119
τ	0.038	0.040	0.036	0.039	0.039	0.034	0.031	0.042	0.036	0.042	0.033	0.033	0.049	0.039	0.052
λ	0.016	0.044	0.021	0.018	0.026	0.007	0.02	0.002	0.04	0.053	0.013	0.029	0.032	0.058	0.004

Table E.1: Calibration: parameter values by US state

	DATA							MODEL						
	Growth (annual)	rank-rank correlation	Teacher share	Manager share	Rel. logwage teacher-worker	Rel. logwage manager-teacher	$\frac{pub.\ educ}{GDP}$	Growth (annual)	rank-rank correlation	Teacher share	Manager share	Rel. logwage teacher-worker	Rel. logwage manager-teacher	$\frac{pub.\ educ}{GDP}$
AL	0.013	0.396	0.033	0.201	0.305	0.278	0.039	0.013	0.391	0.032	0.201	0.305	0.278	0.039
AZ	0.008	0.293	0.031	0.239	0.236	0.312	0.034	0.008	0.294	0.031	0.239	0.236	0.312	0.034
AR	0.014	0.371	0.038	0.182	0.281	0.270	0.045	0.014	0.371	0.038	0.182	0.283	0.268	0.045
CA	0.009	0.239	0.031	0.255	0.308	0.308	0.035	0.009	0.238	0.031	0.255	0.308	0.308	0.035
CO	0.010	0.283	0.035	0.277	0.147	0.381	0.034	0.010	0.283	0.035	0.277	0.147	0.381	0.034
CT	0.017	0.364	0.035	0.285	0.271	0.316	0.038	0.017	0.360	0.035	0.285	0.271	0.316	0.038
DE	0.007	0.395	0.034	0.254	0.320	0.246	0.039	0.007	0.386	0.034	0.254	0.320	0.246	0.039
FL	0.008	0.313	0.029	0.230	0.276	0.274	0.033	0.008	0.309	0.029	0.230	0.276	0.274	0.033
GA	0.008	0.367	0.039	0.229	0.232	0.350	0.043	0.008	0.374	0.039	0.229	0.232	0.350	0.043
ID	0.009	0.252	0.036	0.206	0.236	0.309	0.040	0.009	0.251	0.035	0.206	0.236	0.309	0.039
IL	0.009	0.382	0.032	0.246	0.262	0.281	0.036	0.009	0.377	0.032	0.246	0.262	0.281	0.036
IN	0.009	0.370	0.033	0.203	0.286	0.188	0.039	0.009	0.368	0.033	0.203	0.286	0.188	0.039
IA	0.013	0.309	0.035	0.207	0.292	0.185	0.043	0.013	0.310	0.035	0.207	0.292	0.185	0.042
KS	0.013	0.321	0.039	0.231	0.224	0.313	0.043	0.013	0.326	0.039	0.231	0.224	0.313	0.043
KY	0.013	0.365	0.035	0.197	0.268	0.245	0.041	0.013	0.363	0.035	0.197	0.268	0.245	0.041
LA	0.020	0.396	0.036	0.197	0.252	0.304	0.041	0.020	0.403	0.036	0.197	0.252	0.304	0.041
ME	0.013	0.295	0.042	0.222	0.239	0.243	0.048	0.013	0.300	0.042	0.222	0.239	0.243	0.048
MD	0.013	0.414	0.033	0.311	0.295	0.266	0.036	0.013	0.414	0.033	0.311	0.295	0.266	0.036
MA	0.016	0.330	0.033	0.303	0.229	0.295	0.035	0.016	0.328	0.033	0.303	0.229	0.296	0.035
MI	0.006	0.355	0.035	0.228	0.382	0.151	0.045	0.006	0.361	0.035	0.228	0.382	0.151	0.045
MN	0.013	0.323	0.037	0.255	0.244	0.272	0.041	0.013	0.318	0.037	0.255	0.244	0.272	0.040
MS	0.017	0.422	0.037	0.180	0.258	0.275	0.043	0.017	0.417	0.037	0.180	0.258	0.275	0.043
MO	0.011	0.372	0.033	0.219	0.249	0.262	0.037	0.011	0.369	0.032	0.219	0.249	0.262	0.037
MT	0.017	0.245	0.039	0.216	0.259	0.188	0.046	0.017	0.250	0.039	0.216	0.259	0.188	0.046
NE	0.015	0.320	0.035	0.219	0.227	0.275	0.039	0.015	0.318	0.035	0.219	0.227	0.275	0.039
NV	0.005	0.257	0.027	0.192	0.272	0.194	0.032	0.005	0.257	0.027	0.192	0.272	0.194	0.032
NJ	0.011	0.352	0.039	0.274	0.335	0.255	0.045	0.011	0.344	0.039	0.274	0.335	0.255	0.045
NM	0.014	0.301	0.042	0.230	0.293	0.278	0.048	0.014	0.297	0.041	0.230	0.293	0.278	0.048
NY	0.010	0.344	0.038	0.261	0.384	0.185	0.047	0.010	0.338	0.038	0.261	0.385	0.185	0.047
NC	0.010	0.391	0.034	0.216	0.221	0.334	0.037	0.010	0.393	0.034	0.216	0.221	0.334	0.037
ND	0.025	0.261	0.029	0.210	0.336	0.126	0.037	0.025	0.261	0.029	0.210	0.336	0.126	0.037
OH	0.008	0.401	0.033	0.224	0.337	0.190	0.041	0.008	0.401	0.033	0.224	0.337	0.190	0.041
OK	0.016	0.335	0.036	0.207	0.229	0.295	0.041	0.016	0.340	0.036	0.207	0.229	0.295	0.041
OR	0.009	0.282	0.040	0.236	0.248	0.247	0.045	0.009	0.280	0.039	0.236	0.248	0.247	0.045
PA	0.013	0.370	0.032	0.234	0.360	0.187	0.039	0.013	0.373	0.032	0.234	0.360	0.187	0.039
RI	0.012	0.335	0.032	0.248	0.434	0.108	0.042	0.012	0.332	0.032	0.248	0.434	0.108	0.042
SC	0.010	0.379	0.036	0.200	0.288	0.258	0.043	0.010	0.381	0.036	0.200	0.288	0.258	0.043
SD	0.021	0.293	0.029	0.207	0.285	0.203	0.035	0.021	0.285	0.029	0.207	0.285	0.202	0.035
TN	0.013	0.367	0.030	0.208	0.261	0.282	0.034	0.013	0.370	0.030	0.208	0.261	0.282	0.034
TX	0.015	0.317	0.040	0.225	0.253	0.367	0.044	0.015	0.319	0.040	0.225	0.253	0.367	0.044
UT	0.013	0.238	0.039	0.233	0.177	0.348	0.041	0.013	0.240	0.039	0.234	0.177	0.348	0.041
VT	0.016	0.288	0.042	0.244	0.246	0.223	0.048	0.016	0.285	0.042	0.244	0.246	0.223	0.048
VA	0.014	0.383	0.035	0.284	0.252	0.375	0.036	0.014	0.380	0.035	0.284	0.252	0.375	0.036
WA	0.011	0.276	0.035	0.255	0.172	0.323	0.036	0.011	0.282	0.035	0.255	0.172	0.323	0.036
WV	0.015	0.337	0.041	0.179	0.347	0.129	0.054	0.015	0.344	0.041	0.179	0.347	0.129	0.054
WI	0.012	0.345	0.037	0.215	0.310	0.192	0.045	0.012	0.354	0.037	0.215	0.310	0.192	0.045
WY	0.022	0.255	0.046	0.227	0.248	0.163	0.055	0.022	0.263	0.044	0.194	0.275	0.131	0.055

Table E.2: Calibration: data vs. model by state

F Second-Order Partial Derivative of the Gini Coefficient with Respect to Public Education and Parameters

	φ	α	β_1	μ_a	σ_a	γ
AL	-0.054	0.028	-0.025	0.000	-0.512	0.329
AZ	-0.108	-0.074	-0.014	0.000	-0.291	0.339
AR	-0.110	-0.078	-0.017	0.000	-0.188	0.411
CA	-0.123	-0.081	-0.014	0.000	-0.243	0.291
CO	-0.077	-0.047	-0.022	0.000	-0.753	0.301
CT	-0.068	0.007	-0.035	0.000	-0.628	0.235
DE	-0.125	-0.084	-0.023	0.000	-0.201	0.284
FL	-0.061	0.017	-0.021	0.000	-0.473	0.302
GA	-0.101	-0.008	-0.024	0.000	-0.533	0.379
ID	-0.100	-0.053	-0.015	0.000	-0.292	0.363
IL	-0.124	-0.106	-0.016	0.000	-0.186	0.281
IN	-0.161	-0.122	-0.022	0.000	-0.060	0.357
IA	-0.137	-0.131	-0.003	0.000	-0.042	0.310
KS	-0.062	0.026	-0.026	0.000	-0.601	0.357
KY	-0.074	-0.018	-0.019	0.000	-0.308	0.355
LA	-0.015	0.102	-0.031	0.000	-1.744	0.356
ME	-0.044	-0.036	0.008	0.000	-0.230	0.372
MD	-0.143	-0.126	-0.020	0.000	-0.156	0.194
MA	-0.112	-0.082	-0.027	0.000	-0.342	0.221
MI	-0.109	-0.073	-0.006	0.000	-0.068	0.256
MN	-0.096	-0.056	-0.015	0.000	-0.306	0.305
MS	-0.108	-0.078	-0.020	0.000	-0.201	0.419
MO	-0.096	-0.044	-0.025	0.000	-0.276	0.334
MT	-0.079	-0.037	-0.008	0.000	-0.195	0.318
NE	-0.114	-0.085	-0.025	0.000	-0.383	0.327
NV	-0.085	-0.047	-0.008	0.000	-0.171	0.372
NJ	-0.174	-0.164	-0.013	0.000	-0.077	0.217
NM	-0.118	-0.072	-0.014	0.000	-0.200	0.325
NY	-0.123	-0.089	-0.007	0.000	-0.084	0.222
NC	-0.097	-0.034	-0.026	0.000	-0.355	0.373
ND	-0.099	-0.069	-0.002	0.000	-0.044	0.287
OH	-0.083	-0.035	-0.015	0.000	-0.202	0.270
OK	-0.054	0.021	-0.025	0.000	-0.573	0.367
OR	-0.133	-0.080	-0.013	0.000	-0.161	0.308
PA	-0.141	-0.109	-0.010	0.000	-0.085	0.263
RI	-0.091	-0.044	-0.022	0.000	-0.055	0.207
SC	-0.152	-0.132	-0.047	0.000	-0.166	0.366
SD	-0.077	-0.030	-0.011	0.000	-0.220	0.328
TN	-0.118	-0.087	-0.018	0.000	-0.195	0.338
TX	-0.108	-0.041	-0.024	0.000	-0.326	0.347
UT	-0.073	0.054	0.026	0.000	-0.437	0.340
VT	-0.135	-0.118	-0.011	0.000	-0.122	0.295
VA	-0.122	-0.073	-0.038	0.000	-0.374	0.299
WA	-0.072	0.018	-0.003	0.000	-0.492	0.342
WV	-0.070	-0.017	-0.009	0.000	-0.132	0.328
WI	-0.133	-0.120	-0.012	0.000	-0.093	0.300
WY	-0.071	-0.034	-0.005	0.000	-0.108	0.317

Table F.1: Second-order partial derivative of the Gini coefficient with respect to the tax level (τ) and parameters by state

	φ	α	β_1	μ_a	σ_a	γ
AL	-0.015	-0.059	-0.008	0.000	-0.117	0.165
AZ	-0.014	-0.010	-0.002	0.000	-0.051	0.212
AR	-0.013	-0.021	-0.003	0.000	-0.034	0.213
CA	-0.020	-0.030	-0.004	0.000	-0.062	0.215
CO	-0.003	-0.018	0.014	0.000	-0.154	0.212
CT	-0.015	-0.029	-0.010	0.000	-0.171	0.188
DE	-0.023	-0.041	-0.007	0.000	-0.048	0.226
FL	-0.011	-0.044	-0.006	0.000	-0.106	0.172
GA	-0.042	-0.035	-0.020	0.000	-0.163	0.207
ID	-0.013	-0.016	-0.002	0.000	-0.059	0.211
IL	-0.011	-0.013	-0.001	0.000	-0.038	0.215
IN	-0.054	-0.051	-0.017	0.000	-0.030	0.199
IA	-0.010	-0.014	0.001	0.000	-0.004	0.227
KS	-0.010	-0.031	-0.005	0.000	-0.129	0.198
KY	-0.011	-0.040	-0.003	0.000	-0.056	0.187
LA	0.000	-0.057	-0.005	0.000	-0.412	0.159
ME	-0.038	-0.093	-0.023	0.000	-0.058	0.143
MD	-0.015	-0.021	-0.002	0.000	-0.033	0.229
MA	-0.015	-0.020	-0.007	0.000	-0.070	0.219
MI	-0.018	-0.061	-0.001	0.000	-0.016	0.208
MN	-0.021	-0.026	-0.002	0.000	-0.063	0.217
MS	-0.011	-0.011	-0.003	0.000	-0.034	0.210
MO	-0.013	-0.029	-0.009	0.000	-0.048	0.205
MT	-0.010	-0.037	-0.001	0.000	-0.032	0.208
NE	-0.008	-0.005	0.016	0.000	-0.082	0.227
NV	-0.009	-0.040	-0.001	0.000	-0.025	0.184
NJ	-0.020	-0.024	-0.002	0.000	-0.019	0.247
NM	-0.017	-0.026	-0.003	0.000	-0.045	0.224
NY	-0.020	-0.050	-0.002	0.000	-0.024	0.224
NC	-0.014	-0.007	-0.008	0.000	-0.074	0.211
ND	-0.013	-0.055	-0.001	0.000	-0.009	0.189
OH	-0.014	-0.056	-0.003	0.000	-0.042	0.191
OK	-0.010	-0.036	-0.005	0.000	-0.114	0.184
OR	-0.022	-0.012	-0.005	0.000	-0.030	0.241
PA	-0.032	-0.052	-0.002	0.000	-0.032	0.215
RI	-0.015	-0.078	-0.023	0.000	-0.020	0.188
SC	-0.042	-0.060	-0.035	0.000	-0.047	0.183
SD	-0.011	-0.047	-0.002	0.000	-0.038	0.183
TN	-0.013	-0.011	-0.003	0.000	-0.036	0.210
TX	-0.020	-0.016	-0.006	0.000	-0.090	0.205
UT	0.038	0.000	0.031	0.000	-0.042	0.232
VT	-0.011	-0.010	-0.005	0.000	-0.016	0.246
VA	-0.027	-0.025	-0.009	0.000	-0.109	0.209
WA	0.006	0.067	0.028	0.000	-0.073	0.161
WV	-0.010	-0.079	-0.001	0.000	-0.022	0.183
WI	-0.013	-0.028	0.006	0.000	-0.023	0.218
WY	-0.010	-0.049	0.000	0.000	-0.018	0.193

Table F.2: Second-order partial derivative of the Gini coefficient with respect to tax progressivity (λ) and parameters by state