ABSTRACT: New design formulae for the stability of structural members subjected to combined bending moments and axial forces (so-called beam-columns) have been recently developed and included in the EN version of the European Norm Eurocode 3 Part 1-1 (EC3). It has however to be recognized that these formulae have been validated mostly through comparisons with results of experimental tests and numerical simulations carried out on columns with open sections. The question may therefore be raised to know whether the new formulae are adapted to tubular columns with a similar degree of accuracy and safety than for open section columns. The aim of the paper is to indicate how present EC3 formulae fit with the available results on tubular columns, and then to suggest improvements of the formulae when applied to the latter.

1 INTRODUCTION

Since many decades, researchers are contributing regularly to the development of efficient design formulae for the design and the verification of steel structural columns subjected to combined bending moments and axial forces. Recently, further to several years of intensive research and discussions within the Technical Committee 10 “Stability” of the European Convention for Constructional Steelwork (ECCS), new formulae have been developed and included in the EN version of the European Norm Eurocode 3 Part 1-1 (EN 1993-1-1, 2005).

These formulae, the background and the mode of application of which are described in a recent ECCS publication (Boissonnade et al. 2006), have been validated by means of extensive comparisons with more than 20,000 numerical simulations and 250 to 300 experimental tests. Their accurate and their much less conservative character than the ones of the previous version ENV 1993-1-1 (1992) have also been widely demonstrated.

In the design guide n°2 (DG2) of CIDECT (Comité International pour le Développement et l’Etude de la Construction Tubulaire), an explicit reference is made to Eurocode 3 ENV for the verification of the resistance and the stability of beam-columns with hollow sections. In the framework of the forthcoming revision of DG2, it should be logically decided to refer to the newly developed EN formulae.

It has however to be recognized that the new EN formulae have been validated mostly through comparisons with results of experimental tests and numerical simulations carried out on columns with open sections, and only with a very limited number of experimental tests on tubular columns. The question may therefore be raised to know whether the new formulae are adapted to the tubular columns. The reply to this question is brought through a recently finalised CIDECT-funded project devoted to the application of the new EC3 formulae to columns with tubular sections.

2 METHODOLOGY

In fact, two aspects have to be considered in beam-column interaction formulae: the resistance of the cross-sections and the stability of the member (the member effects). The first aspect strictly depends on the resistance class of the cross-sections while the second one relate to the member slenderness. EC3 suggests plastic resistance values for class 1 or 2 cross-sections and elastic ones for class 3 or 4 cross-sections.

In the new EC3 design formulae, the way of taking into account member effects is rather accurate and for sure is similar whatever the cross-section class and shape. As a result, it can be stated that the degree of accuracy and safety of the new EC3 beam-column formulae widely depends on the “in formulae embedded” cross-sectional resistance criteria.

The present paper mainly focuses on how the cross-sectional resistance aspects are covered in the...
case of columns with class 1 and 2 rectangular hollow sections (RHS).

Besides that, Eurocode 3 presents two sets of beam-column formulae - Annex A (Method 1) and Annex B (Method 2) - and it is up to each country to decide whether the first one, the second one or both of them are to be recommended. The key words on which Method 1 has been developed are “generality”, “transparency”, “consistency” and “accuracy”. The corresponding interaction formulae have so been derived as far as possible on theoretical bases so as to be able to cover any loading, i.e. axial compression only, axial compression and monoaxial bending or biaxial bending, with or without lateral torsional buckling ... In addition, they have been developed in such a way that each coefficient in the formulae represents a single physical effect.

But on the other hand, as already said, almost all these coefficients have been derived for members with I or H cross-sections. Therefore the objectives of the present paper are twofold:

• indicate how present EC3 formulae fit for RHS columns;
• suggest improvements of the formulae when applied to RHS members;

\[
\frac{N_{Ed}}{N_{pl,Rd}} + \mu_y \left[ \frac{C_{my} M_{y,Ed}}{N_{Ed} / N_{er,y}} C_{yy} M_{pl,y,Rd} \right] + \alpha^* \left[ \frac{C_{mc} M_{z,Ed}}{N_{Ed} / N_{er,z}} C_{yz} M_{pl,z,Rd} \right] \leq 1
\]

\[
\frac{N_{Ed}}{N_{pl,Rd}} + \mu_z \left[ \frac{C_{my} M_{y,Ed}}{N_{Ed} / N_{er,y}} C_{yy} M_{pl,y,Rd} \right] + \beta^* \left[ \frac{C_{mc} M_{z,Ed}}{N_{Ed} / N_{er,z}} C_{yz} M_{pl,z,Rd} \right] \leq 1
\]

where \(N_{Ed}, M_{y,Ed}\) and \(M_{z,Ed}\) are the design values of the compression force and the maximum moments along the member about the y–y and z–z axes, respectively; \(N_{pl,Rd}, M_{pl,y,Rd}, M_{pl,z,Rd}\) are the design plastic resistances to the normal force and bending moments; \(\chi_y\) and \(\chi_z\) are the reduction factors due to flexural buckling under pure compression; \(C_{my}\) and \(C_{mc}\) are the equivalent uniform moment factors; \(N_{er,y}\) and \(N_{er,z}\) are the Euler elastic critical loads; \(\mu_y\) and \(\mu_z\) are defined as follows

\[
\mu_y = \frac{1 - N_{Ed} / N_{er,y}}{1 - \chi_y N_{Ed} / N_{er,y}}
\]

\[
\mu_z = \frac{1 - N_{Ed} / N_{er,z}}{1 - \chi_z N_{Ed} / N_{er,z}}
\]

\(\alpha^*\) and \(\beta^*\) are plasticity factors accounting for the cross-sectional biaxial bending resistance interaction (for Class 1 and 2 cross-sections):

\[
\alpha^* = 0.6 \sqrt{\frac{w_y}{w_z}}
\]

\[
\beta^* = 0.6 \sqrt{\frac{w_z}{w_y}}
\]

3 APPLICATION OF THE NEW EUROCODE 3 METHOD 1 FORMULAE TO RHS COLUMNS

3.1 Method 1 stability formulae

EC3 distinguishes “members susceptible to torsional deformations” from “members not susceptible to torsional deformations” ones. In Method 1, no torsional deformations occur if:

\[I_y \geq I_t\]

where \(I_y\) and \(I_t\) represent respectively the torsional and flexural (strong axis) rigidities of the cross-section.

In the opposite case \((I_y < I_t)\), lateral torsional buckling (LTB) effects have to be considered, except if efficient restraints prevent them from developing. An expression is suggested in the EN 1993-1-1 Annex A to check the efficiency of the restraints.

Usually, for RHS cross-sections, the above criterion is satisfied; so all the LTB effects can be neglected. Accordingly, a RHS column submitted to axial forces and bending moments has to be verified by Eqs. (2) and (3) as follows:

\[C_{yy} = 1 + \left( w_z - 1 \right) \left[ 2 - \frac{1.6}{w_z} \left( \frac{\lambda_{max}}{C_{my} M_{y,Ed}} \right)^2 \right] \frac{N_{Ed}}{N_{pl,Rd}} \geq \frac{W_{pl,y}}{W_{pl,y}} \]

\[C_{yz} = 1 + \left( w_z - 1 \right) \left[ 2 - \frac{14 C_{mc} M_{z,Ed}}{w_z} \left( \frac{N_{Ed}}{N_{pl,Rd}} \right) \right] \geq \alpha^* \frac{W_{pl,z}}{W_{pl,z}} \]

\[C_{yy} = 1 + \left( w_z - 1 \right) \left[ 2 - \frac{1.6}{w_z} \left( \frac{\lambda_{max}}{C_{mc} M_{z,Ed}} \right)^2 \right] \frac{N_{Ed}}{N_{pl,Rd}} \geq \frac{W_{pl,z}}{W_{pl,z}} \]

\[C_{yz} = 1 + \left( w_z - 1 \right) \left[ 2 - \frac{14 C_{my} M_{y,Ed}}{w_z} \left( \frac{N_{Ed}}{N_{pl,Rd}} \right) \right] \geq \beta^* \frac{W_{pl,y}}{W_{pl,y}} \]

\[C_{yy} = 1 + \left( w_z - 1 \right) \left[ 2 - \frac{1.6}{w_z} \left( \frac{\lambda_{max}}{C_{mc} M_{z,Ed}} \right)^2 \right] \frac{N_{Ed}}{N_{pl,Rd}} \geq \frac{W_{pl,z}}{W_{pl,z}} \]

All these plasticity factors tends to 1.0 for Class 3 or 4 cross-sections.

It is also important to notice that besides this stability check, the resistance check of the member end sections has always to be verified.
3.2 Application to RHS stub columns under monoaxial compression and bending moment

a) Format of the $C_{ij}$ factors
Under axial compression and monoaxial bending, the stability formulae reduce to simpler expressions. For the particular case of strong axis bending they write:

$$\frac{N_{Ed}}{\chi_y N_{pl,Rd}} + \frac{1}{1 - \chi_y} \frac{C_m M_{y,Ed}}{C_{yy} M_{pl,y,Rd}} \leq 1$$  \hspace{1cm} (12)

or

$$\frac{n}{\chi_y} + \frac{1}{1 - \chi_y} \frac{C_m m_y}{C_{yy}} \leq 1$$  \hspace{1cm} (13)

where $n = N_{Ed}/N_{pl,Rd}$ and $m_y = M_{Ed}/M_{pl,y,Rd}$.

As said before, plasticity effects ($M-N$ cross-sectional resistance interaction in Class 1 or 2 cross-sections) are covered by the factors $C_{ij}$ ($C_{yy}$ in this case). These coefficients fully play their role in stub columns where no member instability occurs and failure is associated to the plastic resistance of the most loaded cross-section along the column. In more slender columns, the amount of plasticity which develops at column failure in the most loaded cross-section along the column will obviously depend on the slenderness of the column (for instance, no plasticity will occur if the column slenderness is high as, in such a case the column will fail by pure elastic instability. These $C_{ij}$ factors are so composed of two parts:

• a “cross-section” part relative to cross-section resistance; it is expressed as a function of the geometry of section, i.e. parameters $h$, $b$, $t$, $r_m$ (Figure 1);

• a “member” part relative to member slenderness $\overline{\lambda}_{max}$ (maximum value of $\overline{\lambda}_y$ and $\overline{\lambda}_z$, the relative slenderness for column buckling about y – y and z – z axes, respectively).

Accordingly, factors $C_{ij}$ can be expressed under the some of these two parts: $C_{ij} = C_{ij}(n, h, b, t, r_m) + C_{ij}(n, \overline{\lambda}_{max})$.

Figure 1: Geometry of a Rectangular hollow section (RHS)

In case of stub members, Eq. (12) should be reduced to an expression equivalent to the resistance of cross-section; when the member slenderness tends to zero, then $C_{n,y} \rightarrow 1$, $\chi_y \rightarrow 1$, and $1/(1 - \chi_y N_{Ed}/N_{cr,y}) \rightarrow 1$, and then Eq. (12) becomes

$$\frac{N_{Ed}}{N_{pl,Rd}} + \frac{M_{y,Ed}}{M_{pl,y,Rd}} \leq 1$$  \hspace{1cm} (14)

or

$$m_y \leq (1 - n)C_{yy}$$  \hspace{1cm} (15)

with $C_{yy} = 1 + 2(w_y - 1)n \geq \frac{W_{pl,y}}{W_{pl,y}}$  \hspace{1cm} (16)

where $C_{ij}(n, \overline{\lambda}_{max}) = 0$.

In Eq. (16), the factor 2 is the theoretical value that is derived from the exact $M-N$ interaction for full rectangular cross-sections (Boissonnade et al.). Let’s verify the validation of this factor through following comparative graphs performed for RHS 160x80x6.3 (Figure 2) and RHS 75x75x3 (Figure 3):

• “EC3_SM” curve represents stub member $M_y$-N resistance following EC3;

• “EC3_CS” curve represents cross-section $M_y$-N resistance following EC3, section §6.2.9.1;

• “Num” curve represents $M_y$-N resistance carried out by numerical simulations.

It is to notice that numerical simulations have been performed, by a home made FEM Finelg, with exactly equivalent conditions on material and restraint as in the formulae. Steel material was assumed elastic perfectly-plastic and defined by the elastic modulus $E = 0.2 \cdot 10^6$ N/mm², the Poisson coefficient $\nu = 0.3$, and a value of elastic limit $f_y$.

The comparative study allows to following statements:

• The proposed formulae of EC3 for cross-section resistance do not very fit with RHS. The given figures show a quite good approximation for $M_y$-N resistance at high compression level ($N_{Ed}/N_{pl,Rd} > 0.7$), but obviously no more precise at lower compression level, especially for $M_y$-N resistance of minor axis (Figure 2b).

• The proposed formulae of EC3 applied to RHS stub members get a quite good approximation at high compression level ($N_{Ed}/N_{pl,Rd} > 0.6$), an considerable conservative estimation at lower compression level, also especially for $M_y$-N resistance of minor axis (Figure 2b).

Then, the factor 2 of Eq. (16) should be improved such as to exploit more the plastic capacity of the sections and then to reconcile both resistance criteria relative to cross-section and stub member.
b) Djalaly formula for Mi-N cross-section resistance

Djalaly H. et al. proposed following formulae to calculate $M_y$-$N$ cross-section resistance (or $M_z$-$N$ by exchanging $b$ and $h$) of a RHS:

$$m_y = 1 - f_i n^2, \text{ when } 0 \leq n \leq \frac{\gamma}{1+\gamma}$$  \hspace{1cm} (17)

$$m_y = (1-n) f_i, \text{ when } \frac{\gamma}{1+\gamma} < n \leq 1$$  \hspace{1cm} (18)

, with $f_i = \frac{(1+\gamma)^2}{\sqrt{2+\gamma+2\omega}}$  \hspace{1cm} (19)

$$f_i = \frac{(1+\omega)(1+\gamma)}{1+\omega+\gamma/2}$$  \hspace{1cm} (20)

$$\omega = \frac{t}{h-2t}$$  \hspace{1cm} (21)

$$\gamma = \frac{h-2t}{b}$$  \hspace{1cm} (22)

Again, the proposed formulae were verified with results of numerical simulations. Figure 4 and Figure 5 report results carried out for RHS 160x80x6.3 and RHS 75x75x3, respectively:

- “Djalaly” curve represents $M_y$-$N$ cross-section resistance following Djalaly formulae;
- and “Num” curve represents $M_y$-$N$ resistance carried out by numerical simulations.

It is obviously seen that the proposed formulae gives a very good approximation to the numerical ones. They should be used to improve the “cross-section resistance” part in the interaction formulae of EC3.
c) Proposal of University of Liege (ULg) for the factor “2”

It is also obviously seen that a $M_f N$ cross-section resistance formula has to have a similar form than the one of Eq. (15) in order to be able to be implanted into the interaction formulae of EC3. Accordingly a calibrated formula (ULg’s formula) has been developed, based on Djalaly formulae as follows:

$$m_y \leq (1-n)(1+g_1 n+g_2 n^2)$$

With, $g_1 = \frac{2(1+\gamma)^2(1+\omega)}{(1+2\gamma)(2+\gamma+2\omega)}$ (24)

and $g_2 = \frac{(1+\gamma)^2(-1+2\omega)}{\gamma(1+2\gamma)(2+\gamma+2\omega)}$ (25)

The factor 2 in the coefficient $C_{yy}$ (and $C_{yz}$) should be replaced by the following factor $a_y$:

$$a_y = \frac{g_1 + g_2 n}{w_y - 1}$$ (26)

, and then

$$C_{yy} = 1 + a_y \left( w_y - 1 \right) \frac{N_{Ed}}{N_{pl,Rd}} \geq \frac{W_{pl,y}}{W_{pl,y}}$$ (27)

In Figure 4 and Figure 5, a quite good approximation of ULg’s curves to the numerical ones validates the proposed formulae. Furthermore, with the same purpose, the factors 2 in coefficients $C_{yy}$ and $C_{xz}$ should be also replaced by a factor $a_z$ given by Eq. (30):

$$a_z = \frac{g_1 + g_2 n}{w_z - 1}$$ (28)

, with $g_1^*$ and $g_2^*$ calculated following Eqs (24) and (25), but by exchanging b and h.

3.3 Application to RHS stub columns under biaxial bending

a) Verification of $\alpha^*$ and $\beta^*$

When the axial force $N_{Ed}$ is negligible and the member length is very small, and then the influence of instability vanishes, the problem reduces to cross-section resistance affected by the interaction of biaxial bending. Eqs. (2) and (3) are reduced to stub member formulae under biaxial bending:

$$m_y + \alpha^* m_z \leq 1$$ (29)

, and $\beta^* m_y + m_z \leq 1$ (30)

, while EC3 proposes another formula specifically to cross-section resistance as follows:

$$m_y^e + m_z^e \leq 1$$ (31)

, $m_y$ and $m_z$ are respectively $M_{y,Ed}/M_{pl,y,Rd}$ and $M_{z,Ed}/M_{pl,z,Rd}$, and $\alpha$ and $\beta$, the factors depending of cross-sectional shape to take into account plastic effects ($\alpha \beta \geq 1$). According to the point §6.2.9.1(6) of EC3,

$$\begin{align*}
\alpha &= 1.66 \frac{1}{1-1.13n^2} \quad \text{but} \quad \alpha, \beta \leq 6
\end{align*}$$ (32)

It can be seen that $\alpha^*$ and $\beta^*$ should be derived from the linearization of Eq. (31) (Figure 6) to be able to keep the linear form of the interaction formulae (29) and (30).

![Figure 6: Biaxial bending interaction](image)

The proposed $\alpha^*$ and $\beta^*$ in Eqs. (6) & (7) are obviously suitable for I or H profile members, but again maybe not for RHS members. The verification was performed through following comparative graphs on RHS 160x80x6.3 (Figure 7a) and RHS 75x75x3 (Figure 7a):

- “EC3_SM” curve represents stub member $M_y - M_z$ resistance following Eqs. (29) & (31);
- “EC3_CS” curve represents cross-section $M_y - M_z$ resistance following Eq. (31);
- “Num” curve represents $M_y - M_z$ resistance carried out by numerical simulations.

That allows to following statements:

- A good agreement between “EC3_SM” and “Num” curves means that the proposed formulae of EC3 for cross-section resistance are fit for RHS.
- A considerable lack between “EC3-CS” and “Num” curves means that the factors $\alpha^*$ and $\beta^*$...
should be improved to exploit more the plastic capacity of RHS.

Figure 7: $M_y-M_z$ cross-section resistance following EC3 and numerical methods

(b) SHS 75x75x3

Figure 9 again demonstrates that the new derived $\alpha^*$ and $\beta^*$ fit well with RHS 160x80x6.3 and RHS 75x75x3, through “ULg_SM” curves.

Figure 8: Biaxial bending interaction under $m_y$-$m_z$ form

Figure 9: $M_y-M_z$ cross-section resistance following ULg proposal and numerical method

3.4 Application to RHS long columns

The proposed improvements on parameters $C_{ij}$ and on $\alpha^*$ and $\beta^*$ have been verified again with long members. Then, comparative study was performed on two profiles:
- hot forming RHS 160x80x6.3, radius $r_m$ equal to 9.4 mm, imperfection factor $\alpha$ (under compression) equal to 0.21,
- and cold forming SHS 75x75x3, $r_m = 4.5$ mm, $\alpha = 0.49$.

For simplicity’s sake, bending moments were assumed uniform, i.e. $\psi_y = 1$ or $\psi_z = 1$; and free end rotations in two directions y-y and z-z.

a) Members under 2D axial compression and bending moment ($M_{y,Ed}$-$N_{Ed}$ or $M_{z,Ed}$-$N_{Ed}$)

Figure 10, Figure 11 and Figure 12 presents different interaction curves carried out by different methods:
- “EC3”, EC3 following Eqs. (2) and (3),
- “ULg”, ULG proposal with improvements on $C_{ij}$ and on $\alpha^*$ and $\beta^*$,
- “Num”, numerical simulations,
and for different member slenderness ($\bar{\lambda}_y$ or $\bar{\lambda}_z$) ranging from 0.0 to 2.0.

The comparative study allows to following statements:

- When the column is highly slender ($\bar{\lambda}_y$ or $\bar{\lambda}_z > 2.0$), both EC3 and ULg curves joint together and present elastic behaviour,
- Compared with numerical simulations, ULg curves overestimates a bit of plastic effects of slender members, whereas EC3 curves underestimate the latter. It means the member part within $C_{ij}$ should be also improved to fit better with RHS.

![Figure 10: Hot forming RHS 160x80x6.3 ($r_m = 9.4$ mm) under only $M_{y,Ed}$ and $N_{Ed}$, $\psi_y = 1$](image1)

![Figure 11: Hot forming RHS 160x80x6.3 ($r_m = 9.4$ mm) under only $M_{x,Ed}$ and $N_{Ed}$, $\psi_x = 1$](image2)

![Figure 12: Cold forming RHS 75x75x3 ($r_m = 4.5$ mm) under only $M_{y,Ed}$ and $N_{Ed}$, $\psi_y = 1$](image3)

(b) Members under axial compression and bending moments $N_{Ed}$, $M_{y,Ed}$ and $M_{z,Ed}$

Again, Figure 12 and Figure 13 present different interaction curves carried out by EC3, ULg formulae and numerical simulations, with different level of axial force $n (N_{Ed}/N_{pl,Rd})$ from 0.0 to 1.0, and with two member lengths $L$ equal to zero and 3890.05 mm.

Similar comments to the last point should be obtained with the given results:

- When the column is highly slender, both EC3 and ULg curves joint together and present elastic behaviour,
- Compared with numerical simulations, ULg curves should overestimate a bit of plastic effects of slender members, whereas EC3 curves underestimate the latter.

There are certainly works on improvement of parameters $C_{ij}$, especially its member part such as to fit better to RHS. Furthermore, more numerical simulation and collection of experimental data should be forthcoming to be able to validate the proposal.

Whatever, it can be seen that the proposed modification on $C_{ij}$ and on $\alpha^*$ and $\beta^*$ can increase effectively resistance of RHS members that is still much conservative following the actual EC3.

![Figure 13: Hot forming RHS 160x80x6.3 ($r_m = 9.4$ mm) under only $M_{y,Ed}$ and $N_{Ed}$, $\psi_y = 1$](image4)

(a) Column length $L$ equal to zero

![Figure 13: Hot forming RHS 160x80x6.3 ($r_m = 9.4$ mm) under only $M_{y,Ed}$ and $N_{Ed}$, $\psi_y = 1$](image5)

(b) Column length $L$ equal to 3890.05 mm
Figure 13: Hot forming RHS 160x80x6.3 (r_m = 9.4 mm), ψ_y=1, E=0.2 \times 10^6 \text{N/mm}^2, f_y=410 \text{N/mm}^2

4 CONCLUSIONS AND PERSPECTIVES

The paper presents how to apply the new Eurocode 3 beam-column column formulae to tubular construction. Investigations have been limited only within class 1 or 2 sections, and especially with RHS members.

Parameters on \( C_{ij} \) and on \( \alpha^* \) and \( \beta^* \) were deeply highlighted to be able to improve the interaction formulae, being closer to the reality, and then more economic. Accordingly, improvements on \( C_{ij} \) and on \( \alpha^* \) and \( \beta^* \) have been proposed. These improvements allow increasing effectively resistance of RHS members that is still much conservative following the actual EC3. But, compared to numerical simulations, the proposal seems to overestimate a bit of plastic effects of slender members. It may be due to the member part of parameters \( C_{ij} \) that is not really fit to RHS members. That is may be the work in the next step.

Furthermore, more numerical simulation and collection of experimental data should be forthcoming to be able to validate the proposal.

PREFERENCES


