PROGRESSIVE INDUCTOR MODELING
VIA A FINITE ELEMENT SUBPROBLEM METHOD

Patrick Dular1,2, Laurent Krähenbühl3 and Christophe Geuzaine1

1 University of Liège, Dept. of Electrical Engineering and Computer Science, ACE, B-4000 Liège, Belgium
2 F.R.S.-FNRS, Fonds de la Recherche Scientifique, Belgium
3 Université de Lyon, Ampère (CNRS UMR5005), École Centrale de Lyon, F-69134 Écully Cedex, France

Abstract - The modeling of inductors is split into a sequence of progressive finite element subproblems. The source fields generated by the coil conductors alone, with a wire representation, are calculated at first via either the Biot-Savart law or finite elements. The associated reaction fields for each added or modified region, mainly the magnetic cores, and in return for the source conductor regions themselves when massive, are then calculated with finite element models. Changes of magnetic regions go from perfect magnetic properties up to volume linear and nonlinear properties. The resulting subproblem method allows efficient solving of parameterized analyses thanks to a proper mesh for each subproblem and the reuse of previous solutions to be locally corrected.

I. INTRODUCTION

Instead of solving a complete inductor problem, including coil conductors and magnetic cores, it is here proposed to perform successive finite element (FE) calculations via a subproblem (SP) method (SPM) [1]-[3], mainly by separating the material regions, and giving them models of different accuracy levels. The aim is to lighten the computational efforts in the preliminary stages of a design, e.g. in parameterized analyses, before refining the models. The idea of source and reaction fields is considered but, at the difference with the common method that adds these fields in the whole domain to define the total field, the source fields are here to be defined, via projections, only in the FE mesh of the added regions as local volume sources (VSs) [1]-[3]. They can even be initially reduced to the boundary of the added region, as local surface sources (SSs), which is an important aspect developed here, in particular for efficient calculations with source fields calculated via the Biot-Savart law.

Progressive SPs can tackle the added magnetic core regions at different levels of precision, considering them as perfect magnetic regions, with the source fields acting as SSs, up to linear and nonlinear volume regions. The accuracy of the coil conductor models can increase as well, starting with the wire (filament; with negligible section) representation of the conductors with Biot-Savart models up to their volume FE models, both in magnetostatics and magnetodynamics. Sequences of such SP solutions and/or corrections are here to be defined, via the material relations

\[ h_p = \mu_p^{-1} b_p + h_{s,p}, \quad j_p = \sigma_p e_p + j_{k,p}, \]

\[ (1a-b) \]

where \( \mu_p \) is the magnetic permeability (possibly function of \( b_p \) in a nonlinear material), \( \sigma_p \) is the electric conductivity, and \( h_{s,p} \) and \( j_{k,p} \) are VSs [2], [3]. They can be remnant fields in magnets or fixed current densities in conductors. They can also be defined by

\[ h_{s,p} = (\mu_p^{-1} - \mu_q^{-1}) b_q, \quad j_{k,p} = (\sigma_p - \sigma_q) e_q, \]

\[ (2a-b) \]

for changes from \( \mu_q \) and \( \sigma_q \) for previous SP \( q \) to \( \mu_p \) and \( \sigma_p \) for SP \( p \) in some regions [2], [3]. Also, BCs are defined for SSs, possibly expressed from previous solutions, i.e.

\[ n \times h_{p} \Gamma_{p} = j_{f,p}, \quad n \cdot b_{p} \Gamma_{p} = f_{j,p}, \quad n \times e_{p} \Gamma_{p} = h_{k,p}, \]

\[ (3a-b) \]

with \( \Gamma_{p} \) the unit normal exterior to \( \Omega_{p} \). Some paired portions of \( \Gamma_{p} \) can define double layers, with the thin region in between exterior to \( \Omega_{p} \); in particular, these will be associated with the boundary of the volume conductors or cores. They are denoted \( \Gamma'_{p} \) and \( \Gamma''_{p} \) and are geometrically defined as a single surface \( \Gamma_{p} \) with interface conditions (ICs), fixing the discontinuities \(| \gamma_{p}' \cdot v_{p}^p - \gamma_{p}'' \cdot v_{p}^p |\) by

\[ n \times h_{p} \Gamma_{p} = [j_{f,p}]_{\Gamma_{p}}, \quad n \cdot b_{p} \Gamma_{p} = [f_{j,p}]_{\Gamma_{p}}, \quad n \times e_{p} \Gamma_{p} = [h_{k,p}]_{\Gamma_{p}}. \]

\[ (4a-b-c) \]

With the magnetic vector potential \( a_p \) and electric scalar potential \( \psi_p \) defined via \( b_p = \text{curl} a_p \) and \( e_p = -\partial t a_p - \text{grad} \psi_p = \partial n a_p - \text{up}, \) and the resulting BC and IC

\[ n \times a_{p} \Gamma_{p} = a_{f,p}, \quad n \times a_{p} \Gamma_{p} = [a_{f,p}]_{\Gamma_{p}}. \]

\[ (5a-b) \]

the \( a_p \) weak formulation of the magnetodynamic problem is obtained from the weak form of the Ampère equation, i.e. [2]

\[ (\mu_p^{-1} \text{curl} a_p, \text{curl} a')_\Omega + (h_{s,p}, \text{curl} a')_\Omega = -(j_{s,p}, a')_\Omega \]

\[ + \langle \sigma_p \partial t a_p, a' \rangle_{\Omega_p} + \langle \sigma_p u_p, a' \rangle_{\Omega_p} + \langle n \times h_{p} a', a' \rangle_{\Gamma_{p}'} = 0, \quad a' \in F_{\Gamma_{p}}(\Omega_{p}), \]

\[ (6) \]

where \( F_{\Gamma_{p}}(\Omega_{p}) \) is a curl-conform function space defined on \( \Omega_{p} \), gauged in \( \Omega_{p} \subset C \), and containing the basis functions for \( a_p \) and for the test function \( a' \) (at the discrete level, this space is defined by edge FEIs; the gauge is based on the tree-coo-technique); \( \cdot, \cdot \rangle_{\Omega} \) and \( < \cdot, \cdot >_{\Gamma} \) denote a volume integral in \( \Omega \) and a surface integral on \( \Gamma \), respectively, of the product of their field arguments.

With the SPM, a complete problem is split into a series of SPs that define a sequence of changes, with the complete solution given by the sum of the SP solutions [1]-[3]. Each SP is defined in its particular domain and mesh, usually overlapping those of the other SPs. At the discrete level, this allows distinct meshes with suitable refinements and possible domain overlapping between separate SPs.
III. PROGRESSIVE INDUCTOR PROBLEMS

A. Conductor and core models

Coil conductor alone (COIL-BS) — A volume conductor, possibly stranded, can be first simplified to a wire geometry \( \Omega_{s,p} \) alone, from which the generated field is defined via the Biot-Savart formula, being a direct solution (no need of FE calculation) of the related SP \( p = \text{coil-BS} \), with fixed source \( f_{s,p} \) in \( \Omega_{s,p} \). Fields \( b_p \) and \( a_p \), with \( e_p = -\partial_t a_p \), are then defined via line integrals along the wire. They are to be calculated afterward only in some particular regions, as VSS or SSs when adding other regions or for a change to a volume conductor.

Perfect magnetic core (CORE-PMC) — An SP \( p = \text{core-pmc} \) is defined in a new domain \( \Omega_p \) by considering some added core regions \( \Omega_{p,i} \) (i is the region index) as being perfect, i.e., of infinite \( \mu_p \). The interior of \( \Omega_{p,i} \) with zero total \( h \) inside, is extracted from the studied domain \( \Omega_p \) and treated via BC (3a) to fix a zero trace of total \( h \) on their boundaries \( \Gamma_{p,i} = \partial \Omega_{p,i} \), thus coupling both the unknown field and the field from previous SP(s) \( q \), acting as a SS [3], i.e.,

\[
\mathbf{n} \times \mathbf{h}_p \big|_{\Gamma_{p,i}} = - \mathbf{n} \times \mathbf{h}_q \big|_{\Gamma_{p,i}}. \tag{7}
\]

The solution can serve as a reference solution for any finite permeability further considered.

Volume core (CORE-VOL) — A volume magnetic core \( \Omega_{p,i} \) is considered in an SP \( p = \text{core-vol} \). For a newly added core, VSSs (2a-b) using previous solution(s) \( q \) are used. If the core has been initially added as a perfect magnetic core, considering also a zero solution \( b_q \) in \( \Omega_{p,i} \), the VSSs are zero; \( b_q \) is considered to be carried in the double layer of \( \Gamma_{p,i} \). A trace discontinuity of \( a_q \) (\( b_q \)) thus occurs, of which the opposite value defines an SS for SP \( p \) in (5b) ((4b)), strongly expressed in function space \( F_p, l(\Omega) \), i.e., also with zero SS in (4a),

\[
[n \times a_p]_{\Gamma_{p,i}} = - [n \times a_q]_{\Gamma_{p,i}}, \quad [n \times h_p]_{\Gamma_{p,i}} = 0. \tag{8a-b}
\]

Changes to nonlinear magnetic properties can be done with VS (2a), with the new \( \mu_p^{-1} \) function of the total field \( (b_q + b_p) \), i.e., \( \mu_p^{-1} = \mu_p^{-1}(b_q + b_p) \). Nonlinear iterations are needed for the related SP up to the convergence of \( a_p \). Other changes of the core model could concern changes to equivalent properties of the core laminations in a homogenized model or could go up to the fine description of the laminations.

Volume coil (COIL-VOL) — A wire conductor \( \Omega_{s,q} \) from a previous SP \( q \) can be corrected to its actual volume geometry \( \Omega_{s,p} \) \( \subset \Omega_{s,q} \) by a new SP \( p = \text{coil-vol} \), carrying an unchanged total current and defined, with its surrounding, with a FE mesh. To overcome the singularity proper to the Biot-Savart field along wire \( \Omega_{s,q} \), the key is to get rid of the singular solution \( q \) inside \( \Omega_{s,p} \), keeping the solution unchanged outside, simultaneously to adding the volume conductor \( \Omega_{s,p} \). As it will be explained, this is done via ICS with SSs through \( \Gamma_{s,p} = \partial \Omega_{s,p} \), i.e.,

\[
[n \times h_p]_{\Gamma_{s,p}} = - [n \times h_q]_{\Gamma_{s,p}}, \quad [n \times b_p]_{\Gamma_{s,p}} = - [n \times b_q]_{\Gamma_{s,p}}, \tag{9a-b}
\]

assembled together with \( f_{s,p} \) fixed as a VS in \( \Omega_{s,p} \), i.e., the current fixed in \( \Omega_{s,p} \) (dynamic). Solution \( q \) includes the possible added core solution. The Biot-Savart calculation is required only on \( \Gamma_{s,p} \) to evaluate the SSs. SS (9a) is weakly expressed in the weak formulation (6) whereas (9b) strongly fixes a discontinuity of \( n \times a_p \) through \( \Gamma_{s,p} \) in \( F_p, l(\Omega_p) \).

B. SP Corrections and their sources

Any SP \( p \) is defined as a correction of a previous (or several) SP(s) \( q \), without involving the already considered sources. It requires VSSs and/or SSs in some regions \( \Gamma_{p,i} \) or \( \Gamma_{q,i} \) evaluated from previous SP(s) \( q \). These sources, coming from previous meshes or Biot-Savart evaluations of SPs \( q \), have to be properly discretized in the mesh of SP \( p \) to assure the conformity of the sequenced FE weak formulations. They are obtained by means of Galerkin projections of the primary field \( a_q \) between the meshes [3]. For a VS, an alternative to the projection on a volume \( \Omega_{p,i} \) consists in evaluating and projecting the source on its surface \( \Gamma_{p,i} \), that then defines the BC of a physical local FE problem. A Biot-Savart VS gains at being processed in this way. A change with a significant effect on the previously solved SPs has to be further considered as a source for these, which thus requires iterative corrections. Various correction schemes will be studied (e.g., Fig. 1, using SSs).

C. Inductance and resistance calculation

The self inductance of a wire conductor, and the possible mutual inductances with other wire conductors, can be calculated via double integral Neumann formulas. The resistance can be approximated as well. After a volume correction SP \( p \), the corrected inductance can be shown to be advantageously obtained with the solution \( a_p \) only in \( \Omega_{p,q} \), i.e., via \( \{j_{p,q}, a_{p,q}\}_{\Omega_p} \), defining the total magnetic flux, thus as the new global value without any reference to the wire inductance approximation. An added magnetic core in an SP gives an inductance change that can be calculated by a volume integral limited to the added region, which is another advantage of the SPM.

IV. CONCLUSION

The developed FE-SPM allows to split inductor modeling into SPs of lower complexity regarding meshing operations and computational aspects. Source field FE or Biot-Savart calculations are followed by approximate reaction field solutions related to approximate BCs, up to their accurate volume distributions in both coil conductor and magnetic core. Significant corrections are progressively obtained, for the nonlinear magnetic properties and the skin and proximity effects, and the related inductances and resistances.

REFERENCES