Computation of Source for Non-Meshed Coils in a Reduced Domain with A–V Formulation

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The discretized source magnetic vector potential $\mathbf{A_j}$, interpolated from source field $\mathbf{H_s}$ with $\mathbf{H}(\mathbf{curl}, \Omega)$ semi-norm, is studied for non-meshed coils with magnetic vector potential $\mathbf{A}$ and electric scalar potential V formulation. As a novelty, source potential $\mathbf{A_j}$ is computed in a reduced domain Ω_{red} instead of the complete domain Ω. For domains with fixed and moving parts, potential $\mathbf{A_j}$ can be computed on each part, for each of the related current sources, with no need to ensure its continuity between these parts.

***Index Terms*—Current Source, Finite Element, Induction Machine, Magnetic Vector Potential, Movement.**

I. Introduction

TOTAL or reduced magnetic scalar potential formulations are widely used. However, these formulations require artificial cuts [1], contrary to the $\mathbf{A}-V$ formulation [2]. Coils are generally meshed with the $\mathbf{A}-V$ formulation. Using non-meshed coils, as planned in this paper, presents some advantages. A useful source for non-meshed coils is the discretized vector potential $\mathbf{A_j}$ interpolated from $\mathbf{H_s}$ with $\mathbf{H}(\mathbf{curl}, \Omega)$ semi-norm [3]. In moving systems, both fixed and moving regions (e.g., stator and rotor) occur, at the interface of which the continuity of fields and potentials are commonly to be ensured. This paper aims at developing a method that allows to compute source $\mathbf{A_j}$ in some reduced parts of the domain without having to ensure its continuity at the interfaces with other parts.

II. Magnetodynamic $\mathbf{A}-V$ formulation

From magnetic flux density $\mathbf{B}$ and electric field $\mathbf{E}$, magnetic vector potential $\mathbf{A}$ and electric scalar potential V are defined in magnetodynamics such that [2]:

$$\mathbf{B} = \mathbf{curl}\ \mathbf{A}, \quad (1)$$
$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \mathbf{grad}\ V. \quad (2)$$

The considered complete domain Ω, of boundary Γ with normal $\mathbf{n}$, includes some coils in a subdomain $\Omega_c \subset \Omega$. Applying the Galerkin method to Maxwell's equations with potentials $\mathbf{A}$ (1) and V (2), magnetodynamics $\mathbf{A}-V$ formulation on domain Ω is such that:

$$\int_\Omega \left[\nu \mathbf{curl}\mathbf{W_k} \cdot \mathbf{curl}\mathbf{A} + \sigma \mathbf{W_k} \cdot \left(\frac{\partial \mathbf{A}}{\partial t} + \mathbf{grad}V\right)\right] d\Omega \quad (3a)$$
$$= \int_\Omega \mathbf{curl}\ \mathbf{W_k} \cdot \mathbf{H_s}\ d\Omega \quad (3b)$$
$$+ \int_\Gamma \mathbf{W_k} \cdot (\mathbf{n} \times \mathbf{H_s})\, d\Gamma,\ \forall\ k = 1 \cdots n_e, \quad (3c)$$
$$\int_\Omega \sigma\ \mathbf{grad}\ N_l \cdot \left(\frac{\partial \mathbf{A}}{\partial t} + \mathbf{grad}V\right) d\Omega = 0, \forall\, l = 1 \cdots n_n, \quad (4)$$

with ν the reluctivity, σ the conductivity, $\mathbf{H_s}$ the source magnetic field, n_n and n_e the numbers of nodes and edges in the mesh of Ω, and N_l and $\mathbf{W_k}$ the nodal and edge shape functions. Potential $\mathbf{A}$ is discretized with edge finite elements, whereas potential V is discretized with nodal finite elements.

Terms (3b)–(3c) are deduced from $\int_\Omega \mathbf{W_k} \cdot \mathbf{J_s}\ d\Omega$, with $\mathbf{J_s}$ the source current density defined such that $\mathbf{J_s} = \mathbf{curl}\ \mathbf{H_s}$. The trace $\mathbf{n} \times \mathbf{H_s}$ in term (3c) is zero when Γ is at infinity. But when it is integrated only in a reduced domain, it does not cancel out. In an electrical machine, the fixed and moving parts of Ω are the stator Ω_s and the rotor Ω_r. An example of a reduced domain with coils can be Ω_s.

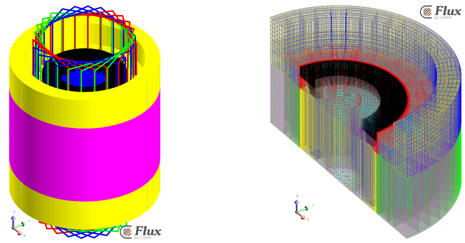
III. $\mathbf{A}-V$ formulation with computation of source $\mathbf{H_s}$ on a reduced domain

Let Ω_{red} be a reduced domain, of boundary Γ_{red}, such that $\Omega_c \subset \Omega_{red} \subset \Omega$ and with no contact between Ω_{red} and Ω_c. If source field $\mathbf{H_s}$ is computed only in Ω_{red}, the right-hand side terms (3b)–(3c) of $\mathbf{A}-V$ formulation are not integrated on Ω and Γ anymore, but on Ω_{red} and Γ_{red}. Trace $\mathbf{n} \times \mathbf{H_s}$ in (3c) is not zero anymore because Γ_{red} is not at infinity. This term (3c) can be simplified as follows.

Edge functions $\mathbf{W_k}$ are split as $\mathbf{W_k} = \mathbf{W}_{\omega_\mathbf{k}} + \mathbf{W}_{\gamma_\mathbf{k}}$, with $\mathbf{W}_{\omega_\mathbf{k}}$ for edges in Ω_{red} but not on Γ_{red}, and $\mathbf{W}_{\gamma_\mathbf{k}}$ for edges on Γ_{red}. By replacing $\mathbf{W_k}$ by $\mathbf{W}_{\omega_\mathbf{k}}$, the surface integral term (3c) is zero because $\mathbf{W}_{\omega_\mathbf{k}}$ is zero on Γ_{red}, the integration domain of (3c). By replacing $\mathbf{W_k}$ by $\mathbf{W}_{\gamma_\mathbf{k}}$, the right hand-side terms (3b)–(3c) are equals together to zero, because domain Ω_c is included in Ω_{red} with no contact with Γ_{red}, so $\int_{\Omega_{red}} \mathbf{W}_{\gamma_\mathbf{k}} \cdot \mathbf{J_s}\ d\Omega = 0$.

As a result, the first equation of $\mathbf{A}-V$ formulation (3) with computation of source $\mathbf{H_s}$ in reduced domain Ω_{red} is:

$$\int_\Omega \left[\nu \mathbf{curl}\mathbf{W_k} \cdot \mathbf{curl}\mathbf{A} + \sigma \mathbf{W_k} \cdot \left(\frac{\partial \mathbf{A}}{\partial t} + \mathbf{grad}V\right)\right] d\Omega \quad (5a)$$
$$= \int_{\Omega_{red}} \mathbf{curl}\ \mathbf{W}_{\omega_\mathbf{k}} \cdot \mathbf{H_s}\ d\Omega,\ \forall\ k = 1 \cdots n_e. \quad (5b)$$



(a) Complete geometry

(b) Mesh with symmetry and periodicity

Fig. 1. Geometry and mesh of the induction machine

IV. $\mathbf{A}-V$ Formulation with Source $\mathbf{A_j}$ Interpolated from $\mathbf{H_s}$ with $\mathbf{H}(\mathbf{curl},\Omega_{red})$ Semi-Norm

Sources for non-meshed coils can be computed from source $\mathbf{H_s}$ or source magnetic vector potential $\mathbf{A_s}$ by using analytical forms deduced from Biot-Savart law's [4]. With ν_0 the vacuum reluctivity, $\mathbf{A_s}$ is defined from $\mathbf{H_s}$ via:

$$\mathbf{H_s} = \nu_0 \ \mathbf{curl} \ \mathbf{A_s}. \quad (6)$$

Source vector potential $\mathbf{A_j}$ is discretized on edge finite elements of Ω_{red}. Function space $\mathbf{L^2}(\Omega)$, space of edge finite elements $\mathbf{H}(\mathbf{curl},\Omega_{red})$ and its semi-norm are defined as [5]:

$$\mathbf{L^2}(\Omega) = \left\{ \mathbf{u} : \Omega \to \mathbb{R}^m, \int_\Omega |\mathbf{u}(\mathbf{x})|^2_{\mathbb{R}} \, d\Omega < \infty \right\} \quad (7)$$

$$\mathbf{H}(\mathbf{curl},\Omega) = \left\{ \mathbf{u} \in \mathbf{L^2}(\Omega), \mathbf{curl}\ \mathbf{u} \in \mathbf{L^2}(\Omega) \right\}, \quad (8)$$

$$|\mathbf{u}|_{\mathbf{H}(\mathbf{curl},\Omega)} = \sqrt{\int_\Omega \mathbf{curl}\ \mathbf{u} \cdot \mathbf{curl}\ \mathbf{u} \ d\Omega}. \quad (9)$$

Source $\mathbf{A_j}$ is computed by interpolation with $\mathbf{H}(\mathbf{curl},\Omega_{red})$ semi-norm (9) and equation (6) via [3]:

$$\min_{\mathbf{A_j} \in \mathbf{H}(\mathbf{curl},\Omega_{red})} \frac{1}{2} \left| \mathbf{curl}\ \mathbf{A_j} - \mathbf{curl}\ \mathbf{A_s} \right|_{\mathbf{H}(\mathbf{curl},\Omega_{red})} \Leftrightarrow \quad (10)$$

$$\int_{\Omega_{red}} \mathbf{curl}\mathbf{W_k} \cdot \mathbf{curl}\mathbf{A_j} \ d\Omega = \int_{\Omega_{red}} \mathbf{curl}\mathbf{W_k} \cdot \frac{\mathbf{H_s}}{\nu_0} \ d\Omega. \quad (11)$$

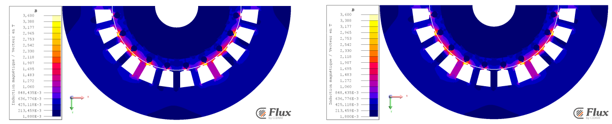
The first equation (5) of $\mathbf{A}-V$ formulation with source $\mathbf{A_j}$ computed in Ω_{red} instead of $\mathbf{H_s}$ (6) then becomes:

$$\int_\Omega \left[\nu \mathbf{curl}\mathbf{W_k} \cdot \mathbf{curl}\mathbf{A} + \sigma \mathbf{W_k} \cdot \left(\frac{\partial \mathbf{A}}{\partial t} + \mathbf{grad} V \right) \right] d\Omega \quad (12a)$$

$$= \int_{\Omega_{red}} \nu_0 \ \mathbf{curl}\ \mathbf{W}_{\omega_\mathbf{k}} \cdot \mathbf{curl}\ \mathbf{A_j} \ d\Omega, \ \forall \ k = 1 \cdots n_e \quad (12b)$$

V. Application to an Induction Machine

The induction machine shown in Fig. 1 is described in [3]. Rotor domain Ω_r is composed of 2 regions (in cyan and black in Fig. 1). Stator domain Ω_s is composed of 3 regions (in magenta, yellow and turquoise in Fig. 1). The non-meshed coils belong to Ω_s, with reduced domain Ω_{red} thus chosen as Ω_s. The induction machine is studied in the frequency domain from the $\mathbf{A}-V$ formulation with source $\mathbf{A_j}$ (12)–(4), with $\frac{\partial}{\partial t}$ replaced by $j\omega$, with ω the angular frequency. The goal of this application is to compute source $\mathbf{A_j}$ in $\Omega_{red} = \Omega_s$, with the aim to obtain the same results as with the source in the complete domain Ω.



(a) With $\mathbf{A_j}$ computed on Ω_s

(b) With $\mathbf{A_j}$ computed on Ω

Fig. 2. Isovalues of $\mathbf{B}$ for $\mathbf{A}-V$ formulation on the induction machine

TABLE I
Computation times (h:m:s) - $\mathbf{A}-V$ form. - Induction machine

Domain of source computation	Ω_s	Ω
Pre-resolution (for sources)	00 : 20 : 27	00 : 28 : 18
$\mathbf{A}-V$ formulation resolution	01 : 05 : 14	00 : 49 : 36
Total time of resolution	01 : 25 : 41	01 : 17 : 54

In Fig. 2, isovalues of $\mathbf{B}$ for $\mathbf{A}-V$ formulation are represented on the plane xy at $z = 0$, with source $\mathbf{A_j}$ computed on either Ω_s or Ω. Figs. 2a and 2b are identical, which points out that $\mathbf{B}$ does not depend on the computation domain of $\mathbf{A_j}$.

Times for the computation of source $\mathbf{A_j}$ with conjugate gradient on domain Ω_s or Ω and the solving of $\mathbf{A}-V$ formulation with a direct solver are given in Table I for the machine. The transformation of the terms (3b)–(3c) of $\mathbf{A}-V$ formulation into term (12b) makes the system not compatible [6]. Then $\mathbf{A}-V$ formulation (12)–(4) needs a gauge condition, defined here by a tree of edges [7]. According to Table I, computation times of source $\mathbf{A_j}$ are similar on the reduced domain Ω_s and on the complete domain Ω. Finally, it is possible to compute the source $\mathbf{A_j}$ in a reduced domain.

VI. Conclusion

In [3], the discretized source vector potential $\mathbf{A_j}$ was chosen to compute source for non-meshed coils with the $\mathbf{A}-V$ formulation. In this paper, it is described how to reduce the computation of source $\mathbf{A_j}$ to a part of the domain, in movement or fixed.

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