Time-domain finite-element homogenisation of laminated iron cores with net circulating currents

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This paper deals with the time-domain homogenization of laminated cores in 2D or 3D finite element (FE) models of electromagnetic devices, in particular allowing for net circulating current in the laminations (which may result from imperfect or damaged insulation). The homogenization is based on the decomposition of the variation of the induction in the lamination thickness by means of a orthogonal set of polynomial basis functions, in conjunction with the magnetic vector potential (MVP) formulation. The conventional even skin-effect basis functions are linked to net flux, whereas the odd ones are now added so as to allow for net current. The approach is validated through a simple linear 2D test case, although the extension to 3D nonlinear problems is straightforward.

***Index Terms*—Eddy currents, finite-element methods, homogenization, lamination stack.**

I. Introduction

When modelling real-life electromagnetic devices comprising laminated iron cores by means of the FE method, it is for practical reasons mostly impossible to discretise each lamination separately. Dedicated numerical techniques are to be used in order to account for the induced eddy currents, the associated losses and the ensuing skin effect in a precise but computationally efficient way. The various approaches proposed in literature so far are applicable in frequency and/or time domain, account for saturation (and possibly hysteresis) or not, include perpendicularly incident flux, and are single-step or two-step algorithms [1-4]. Mostly perfect insulation of the laminations is assumed, i.e. the induced current density cancels in any cross-section of a lamination at all times. In practice net circulating current may occur due to inter-lamination insulation damage [5].

In this paper the net-current feature is added to the nonlinear time-domain homogenization method proposed in [4]. The additional treatment and equations are similar to the thin-shell technique in [6]. For sake of brevity the application example in this short paper is linear and 2D.

II. 1D lamination model

Let us consider a lamination of thickness d ($-d/2 \leq z \leq d/2$), of homogeneous isotropic material having a constant conductivity σ (resistivity $\rho = \sigma^{-1}$) and constant permeability μ (reluctivity $\nu = \mu^{-1}$). The magnetic induction $b(z,t)$ and magnetic field $h(z,t) = \nu\, b(z,t)$ are assumed along e.g. the x-axis, and the current density $j(z,t)$ and electric field $e(z,t) = \rho\, j(z,t)$ along the y-axis. The 1D eddy-current problem is governed by

$$\partial_z^2 h(z,t) = \sigma\, \partial_t b(z,t)\,. \tag{1}$$

Regarding the symmetry of $b(z,t)$ and $j(z,t)$ with respect to $z = 0$, two dual cases can be distinguished, as shown in Fig. 1, with net flux and net current in the lamination respectively.

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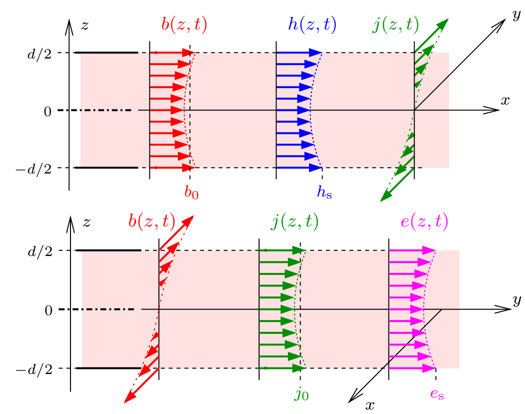

Figure 1. 1D lamination model with 2 symmetrical cases: net flux with symmetric $b(z,t)$ and $h(z,t)$ and odd $j(z,t)$ (up) versus net current with symmetric $j(z,t)$ and $e(z,t)$ and odd $b(z,t)$ (down)

For these two cases, the analytical solution of (1) in the frequency domain, at frequency f and pulsation $\omega = 2\pi f$, gives rise to the following expressions of the complex lamination-level reluctivity $\boldsymbol{\nu}$ and resistivity $\boldsymbol{\rho}$ respectively:

$$\boldsymbol{\nu} = \frac{\boldsymbol{h}_\mathrm{s}}{\boldsymbol{b}_0} = \nu\, \boldsymbol{\Gamma}(d^*) \quad \text{and} \quad \boldsymbol{\rho} = \frac{\boldsymbol{e}_\mathrm{s}}{\boldsymbol{j}_0} = \rho\, \boldsymbol{\Gamma}(d^*)\,, \tag{2}$$

where $d^* = d/\delta$ is the relative lamination thickness (with $\delta = \sqrt{2/(\omega\mu\sigma)}$ the penetration depth), $\boldsymbol{b}_0$ and $\boldsymbol{j}_0$ the average induction and current density resp., $\boldsymbol{h}_\mathrm{s}$ and $\boldsymbol{e}_\mathrm{s}$ the surface magnetic and electric field resp., and with the frequency dependence contained in $\boldsymbol{\Gamma}(d^*)$:

$$\boldsymbol{\Gamma}(d^*) = \frac{1+\jmath}{2}\, d^* \operatorname{cotanh}\Big(\frac{1+\jmath}{2}\, d^*\Big)\,, \tag{3}$$

where $\jmath$ is the imaginary unit. (Complex quantities are in bold.)

An approximate time-domain solution of (1) can be obtained through expansion of $b(z,t)$ with both even and odd polynomial basis functions $\alpha_k(z)$ up to order n:

$$b(z,t) = \sum\nolimits_{k=0}^{n} \alpha_k(z)\, b_k(t)\,. \tag{4}$$

By choosing n one may compromise between accuracy and computational cost. With $n = 0$, i.e. neglecting skin effect, low-frequency eddy-current losses are effected, at practically no additional cost. The case $n = 2$ is developed very briefly hereafter. Imposing $\alpha_k(d/2) = 1$ and orthogonality, we have: $\alpha_0(z) = 1$, $\alpha_1(z) = 2\,z/d$ and $\alpha_2(z) = -1/2 + 6\,(z/d)^2$. Thanks to $\alpha_2(z)$ and $\alpha_1(z)$ skin effect (of net flux) and net current can be considered respectively.

With a view to the incorporation in the FE equations (in terms of the magnetic vector potential) four differential equations regarding the next flux and net current result:

$$\begin{bmatrix} h_s \\ 0 \end{bmatrix} = \nu \begin{bmatrix} 1 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} b_0 \\ b_2 \end{bmatrix} + \sigma d^2 \begin{bmatrix} 1/12 & -1/60 \\ -1/60 & 1/210 \end{bmatrix} \partial_t \begin{bmatrix} b_0 \\ b_2 \end{bmatrix}, \tag{5}$$

$$j_0 = \frac{2\nu}{d} b_1 + \frac{\sigma d}{30} \partial_t b_1 \quad \text{and} \quad e_s = \frac{2\nu}{d\sigma} b_1 + \frac{d}{5} \partial_t b_1 \,, \tag{6}$$

with $h_s(t) = (h(d/2,t) + h(-d/2,t))/2$ and $e_s(t) = (e(d/2,t) + e(-d/2,t))/2$.

See for implementation issues. In the FE model, a coarse mesh can then be used in the homogenized lamination stack, and for each basic unknown herein (MVP values associated to an edge e.g.) there are $n - 1$ additional unknowns.

III. Application Example

The presented homogenization approach is applied to the 2D model shown in Fig. 2. It comprises a core of 2×10 laminations with $d = 0.5$ mm, $\sigma = 2\ 10^6$ S/m, and relative permeability $\mu_r = 2000$. The induction and magnetic field are perpendicular to the plane, whereas all current density, imposed and induced, is in the plane of the model.

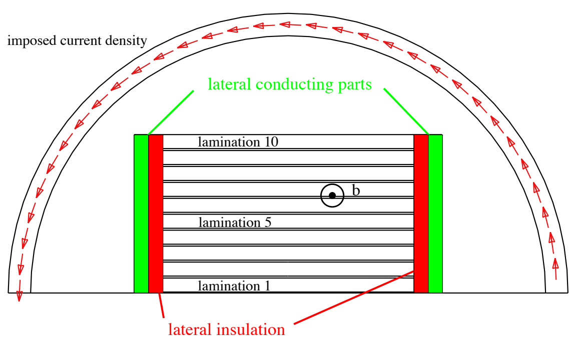

Figure 2. 2D FE model with laminated core (half cross-section of the ring core with toroidal coil considered in [4] but ignoring curvature)

Net induced currents in the laminations are possible due to the presence of the conducting layers shown in red and green in the figure. For the lateral insulation layer (in red), we assume $\mu_r = 1$ and either $\sigma_i = 0$ (perfect insulation) or $\sigma_i = 2\ 10^3$ S/m. For the layer in green we assume $\mu_r = 1$ and $\sigma_g = 5\ 10^6$ S/m.

A 1 kHz sinusoidal current is imposed with an amplitude such that the induction in the lamination would be uniform with a 1 T amplitude in absence of all induced currents ($\sigma = 0$, $\sigma_i = 0$ and $\sigma_g = 0$). In presence of induced currents in the laminations only ($\sigma_i = 0$ and $\sigma_g = 0$), there is a clear skin effect inside the lamination as $d^* = d/\delta = 1.99$, with the same induction profile along each lamination thickness. In presence of conducting insulation and lateral parts, there is also a global skin effect (as can already be seen in Fig. 3).

The 3D MVP formulation adopted is the same as in [4], with the unknowns associated to the edges and with co-tree gauging. Two different meshes of quadrangles (in the laminations) combined with triangles elsewhere are adopted: a fine one (M12) and a coarse one (M1), with 12 and just 1 layer of quadrangles per lamination resp., with a total of around 10000 and 1000 complex unknowns resp., to be used without and with homogenization resp..

With non-conducting layers and the fine mesh, the average induction in each lamination is $\boldsymbol{b}_{av} = (0.697 - \jmath\, 0.398)$ T. With the frequency-domain homogenization, using the coarse mesh and the complex $\boldsymbol{\nu}(d^*)$, we obtain $(0.683 - \jmath\, 0.405)$ T.

With conducting layers and the fine mesh, the average induction in the core is $(0.166 - \jmath\, 0.280)$ T ; see the blue dots in Fig. 3. The frequency-domain (FD) homogenization, using $\boldsymbol{\rho}(d^*)$ as well, produces $(0.168 - \jmath\, 0.281)$ T. The time-domain (TD) homogenization with $n = 2$ produces (in steady-state and after phasor calculation) $(0.163 - \jmath\, 0.280)$ T; see the red curves in the figure. The agreement is again satisfactory, and can be improved by increasing n.

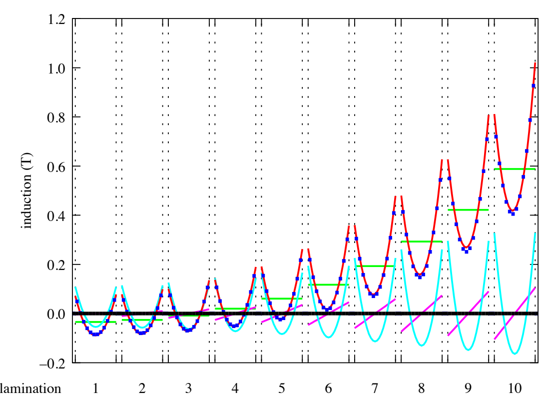

Figure 3. Variation of induction throughout core with imperfect lateral insulation (component in phase with imposed current), obtained with fine mesh (blue dots) and with TD homogenization (in red, plus components b_0, b_1 and b_2 in green, magenta and cyan resp.)

In the extended paper, the proposed method will be elaborated in detail and more results will be given and analyzed.

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