Subproblem Methodology for Progressive Finite Element Modeling of Transformers

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Model refinements of transformers are performed via a subproblem finite element method. A complete problem is split into subproblems with overlapping meshes, to allow a progressive modeling from ideal to real flux tubes, 1-D to 2-D to 3-D models, linear to nonlinear materials, perfect to real materials, wired to volume inductors, and homogenized to fine models of cores and coils, with any coupling of these changes. Its solution is the sum of the subproblem solutions. The procedure simplifies both meshing and solving processes, and quantifies the gain given by each refinement on both local fields and global quantities. Efficient ways to chain the refinements are proposed and tested.

***Index Terms*—Finite element method, model refinement, subproblems, transformers.**

I. Introduction

With the objective of efficient and accurate numerical modeling of electromagnetic devices, an innovative step-by-step methodology, towards more and more complex and accurate models, is developed and applied to transformers, up to accurate 3-D calculations in a large frequency range. A framework allowing a variety of refinements has been developed. It is based on a finite element (FE) subproblem (SP) method (SPM) with magnetostatic and magnetodynamic problems solved in a sequence [1]-[5].

Each step of the SPM aims at improving the solution obtained at previous steps via any coupling of the following changes, defining model refinements: change from ideal to real (with leakage flux) flux tubes [1], change from 1-D to 2-D to 3-D [2], change of material properties [1]-[3] (e.g., from linear to nonlinear), change from perfect to real materials [4], change from wired to volume inductors [4], [5], and change from homogenized [6] to fine models (cores and coils) newly developed. The magnetic circuits and their inductors (stranded or massive coils), with their complex common particular design as juxtaposition of thin or wired regions (lamination stacks in magnetic cores, foil windings, wires in stranded inductors), are given a thorough study for their accurate design. This methodology involves and couples, for the first time, numerous techniques that have been developed by the authors and, up to now, applied and presented only in particular contexts of simplified test problems [1]-[5].

The proposed step-by-step approach can also help in education with a progressive understanding of the various aspects of transformer design.

II. Progressive Models – Methodology

A. Sequence of changes

Low frequency electromagnetic systems, such as transformers, are made of magnetic regions, defining magnetic circuits, and active (connected to external electric circuits, i.e. coils) and passive (not fed by circuit, e.g., tank) conducting regions. Such systems are planned to be studied with the following methodology, defining sequences of adequate changes/corrections as these listed in the introduction.

B. Magnetic circuits

For magnetic cores with possible air gaps, the analysis can first be focused on the main flux tubes before considering the coil conductors with their details. The design of such systems requires a preliminary sizing of their components that can be done via an equivalent magnetic circuit, defining a 1-D (or 0-D) model. The progressive consideration of the actual geometry of the regions, in 2-D and 3-D, can be done via SPs of associated dimensions at two levels, first inner then outer to magnetic materials, i.e. respectively without and with leakage flux. Changes of dimensions (1-D to 2-D to 3-D) [2] and from ideal to real flux tubes [1] can be involved in such steps. They both consider changes of boundaries of domains that can be either extended or connected together. Each dimension fixes some boundaries through which particular assumptions on magnetic flux are considered via interface conditions (ICs). A higher dimension modifies such ICs via surface sources (SSs) [2].

Changes of material properties can be considered via volume sources (VSs) when adding, removing, changing or moving some regions [1]-[3].

Once the real flux tubes are considered, the actual geometry of the inductors has to be considered, instead of magnetomotive force or flux sources. Simultaneously with the IC sources allowing the flux tube to be permeable [2], the inductors are progressively defined, as hereafter.

C. Stranded and massive conductors

Changes related to active and passive conductors are of importance to allow their model refinement up to accurate local and global inductive and resistive behaviors [3]-[5]. The FE modeling of inductors (coils) can be tackled at various levels of precision. Their geometry as well as the distribution of the current they carry may be first simplified. Then, progressive refinements can be done from wire to volume FE geometries, and from stranded to massive inductors, to improve the local field distributions and to accurately render skin and proximity effects, i.e. the non-uniformly distributed fields and current densities. An accurate determination of Joule losses and force densities in inductors lies on finely calculated local fields. Models for passive conductors can similarly be improved. Considering each inductor without any other region, with some possible symmetries that do not exist anymore in the complete problem, offers advantages in mesh operations, especially in parameterized analyzes on positions and dimensions. The related source field is accurately determined in this inductor mesh or can even be defined via the Biot-Savart law [5]. It can then be used as a source for the FE calculation of the reaction fields of added magnetic and/or conducting regions.

D. Common periodic structures – Juxtaposition of thin or wired regions

The consideration of thin or wired conducting regions (lamination stacks in magnetic cores, foil windings, wires in stranded inductors) in FE analyses is a source of difficulty regarding the mesh as well as the numerical solving. An isolated thin or wired volume region can be efficiently reduced to surface or line elements satisfying the actual distributions or ICs of the fields. Nevertheless, when numerous thin or wired regions are juxtaposed and separated with insulating layers, the whole resulting region must remain a volume and its homogenization is usually the only feasible solution for a 3-D FE analysis. Homogenization models are nevertheless tainted with errors, in particular on the border of the homogenized domains [6]. Local corrections of homogenized solutions can be done via the FE SPM, in certain thin or wired regions separately, surrounded by their insulating layers and the remaining regions kept homogenized. The correction SPs use sources calculated from the homogenized solution and can perform local corrections of the fields and current densities in regions of interests, allowing to improve the determination of global quantities as well such as Joule losses, resistances and inductances. The sources concern changes of constitutive relations and ICs, so as to simultaneously express the non-trivial changes to both the actual permeability/conductivity and shape of each thin or wired conductor, with a method derived from [5].

Any of the defined changes with a significant effect on the previously solved SPs has to be considered as a source for these, which defines series of corrections on both magnetic circuits and conductor models. Nonlinear behaviors of materials can naturally be taken into account in this methodology.

III. Applications – Practical Steps

The proposed step-by-step procedure consists of the following sequence of FE SPs (calculating the associated magnetic and/or electric fields):

- Step 1 – Transformer magnetic core with no leakage flux and no losses (no eddy current, no hysteresis) (magnetic circuit as ideal flux tubes, thus with ideal windings; changes from 1-D to 2-D to 3-D).
- *Step 2* – Introducing the windings as stranded conductors (i.e., with uniform current density distributions in their cross sections) or foil windings (homogenization), at different frequencies, for leakage flux calculation (change from ideal to real flux tubes, from 2-D to 3-D); calculation of the related winding resistances and inductances; classical modeling usually stops at this step.
- *Step 3* – Refining the windings, for accurate skin and proximity effects calculations, with (e.g., conductors with rectangular cross-section, surrounded by an insulating paper layer, and together with oil ducts for cooling; which is usually unfeasible in a direct 3-D approach): 1) one local SP at full winding scale with volume homogenization, via equivalent material properties of the winding regions (change of material properties, from 2-D to 3-D); 2) several local SPs at coil turn scale (limited to one or few turns, surrounded by their insulating layers and the remaining regions (turns) kept homogenized), either from step 2 or the already improved solution 3.1, that could then be improved for the border turns (change of material properties, from 2-D to 3-D). A great care has to be given to this important step, that needs rigorous mathematical tools to assure a correct interaction between the local SPs; 3) calculation of the resulting corrections of winding resistances and inductances (to be involved in circuit coupling).
- *Step 4* – Correcting the transformer laminated core (with similar correction and homogenization tools as in step 3): eddy current losses (via homogenization and/or correction), linear to nonlinear behavior (change of material properties); and quantification of the resulting effects.
- Step 5 – Adding conducting regions (e.g. transformer tank, magnetic shield) and quantification of the resulting effects.

A quantification of the efficiency of the method and a validation of each SP step will be presented on practical problems: 1) in 2-D, via the comparison of the SP solutions with the solution of the direct (heavy) approach, consisting in solving one single full problem, e.g., with all the winding details; 2) in 3-D, via simplified test problems.

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References

[1] P. Dular, R. V. Sabariego, M. V. Ferreira da Luz, P. Kuo-Peng, L. Krähenbühl, "Perturbation finite element method for magnetic model refinement of air gaps and leakage fluxes," *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1400–1403, 2009.

[2] P. Dular, R. V. Sabariego, C. Geuzaine, M. V. Ferreira da Luz, P. Kuo-Peng and L. Krähenbühl, "Finite Element Magnetic Models via a Coupling of Subproblems of Lower Dimensions," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2827-2830, 2010.

[3] P. Dular, L. Krähenbühl, R.V. Sabariego, M. V. Ferreira da Luz, P. Kuo-Peng and C. Geuzaine. "A finite element subproblem method for position change conductor systems", *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 403-406, 2012.

[4] P. Dular, V. Péron, R. Perrussel, L. Krähenbühl, C. Geuzaine. "Perfect conductor and impedance boundary condition corrections via a finite element subproblem method," *IEEE Trans. Magn.*, vol. 50, no. 2, paper 7000504, 4 pp., 2014.

[5] P. Dular, V. Péron, L. Krähenbühl, C. Geuzaine. "Subproblem Finite Element Refinement of Inductors from Wire to Static and Dynamic Volume Models," *IEEE Trans. Magn.*, vol. 51, no. 3, paper 7402704 , 2015.

[6] L. Krähenbühl, P. Dular, T. Zeidan, F. Buret. "Homogenization of lamination stacks in linear magnetodynamics," *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 912-915, 2004.