Using a vector Jiles-Atherton hysteresis model for isotropic magnetic materials with the FEM, Newton-Raphson method and relaxation procedure

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Abstract — **This paper deals with the use of a vector Jiles-Atherton hysteresis model included in 2D finite element modelling. The Newton-Raphson algorithm is used with a relaxtion procedure, which ensures the convergence in most of the cases. We have simulated a T-shaped magnetic circuit with rotating fields and then a three-phase transformer model.**

I. INTRODUCTION

In this paper, the 2D magnetic vector potential formulation is used with the finite element (FE) method, the Newton-Raphson (NR) method and the implicit Euler scheme for time stepping. Loss computation accounting for magnetic hysteresis and eddy currents in the lamination stacks of electrical devices, such as transformers or rotating machines, is often performed with a posteriori loss models. However, when a circuit coupling exists, this approach is not valid anymore and the hysteresis model must be included in the FE equations. The Jiles-Atherton (JA) model is widely employed, because of the small number of parameters, its relative ease of implementation in a FE software and its low computation cost compared to other models such as Preisach's [1]. Some vector hysteresis JA models have been proposed, e.g., in [2-6]. Paper [6] reports stability and convergence problems and proposes some procedures to overcome these problems.

In [2][3] a model implemented with the NR method and valid for isotropic magnetic materials only is proposed. The novelty of the present paper consists in using the model of [2][3] with an algorithm to determine the relaxation factor at each step of the NR algorithm presented in [8] to ensure the NR convergence in most cases. First, the numerical aspects of the method are recalled. Then, the method is applied to the simulation of two numerical examples relative to a three-phase transformer taking into account the eddy current losses in the laminations with a low frequency model.

II. NUMERICAL METHOD

A. Magnetic vector potential formulation

With the 2D magnetic vector potential (**a**) formulation, **a** has only one non-zero z-component orthogonal to the xOy plane and the magnetic flux density is **b** = **curl a**. The weak form of Ampère's law **curl h** = **j** is:

$$\int_\Omega \mathbf{h}\cdot\mathbf{curl}\,\mathbf{a}'\,\mathrm{d}\Omega = \int_\Omega \mathbf{j}\cdot\mathbf{a}'\,\mathrm{d}\Omega\,, \qquad (1)$$

where **a**' is a test function, **h** the magnetic field, **j** the current density.

B. Vector Jiles-Atherton model

With the vector JA model described in [3][4] five coefficients are required, usually denoted by m_s, a, k, c and α. The main governing equation of the model is:

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}\mathbf{h}} = [\mathrm{Id} - \alpha\chi]^{-1}\cdot\chi \text{ with } \chi = (1-c)\frac{\mathrm{d}\mathbf{m}_{irr}}{\mathrm{d}\mathbf{h}_e} + c\frac{\mathrm{d}\mathbf{m}_{an}}{\mathrm{d}\mathbf{h}_e}, \qquad (2)$$

where **m**, $\mathbf{m}_{an}$ and $\mathbf{m}_{irr}$ are the total, anhysteretic and irreversible magnetization respectively, $\mathbf{h}_e$ is the effective field ($\mathbf{h}_e = \mathbf{h} + \alpha\mathbf{m}$), and Id the unit tensor. See [3][4] for the details of the computation of $\mathrm{d}\mathbf{m}_{irr}/\mathrm{d}\mathbf{h}_e$ and $\mathrm{d}\mathbf{m}_{an}/\mathrm{d}\mathbf{h}_e$. The differential permeability tensor is then computed with:

$$\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}\mathbf{h}} = \mu_0\left(\mathrm{Id} + \frac{\mathrm{d}\mathbf{m}}{\mathrm{d}\mathbf{h}}\right). \qquad (3)$$

It is inverted to get the differential reluctivity tensor d**h**/d**b** and the field **h** at the new time step can then be obtained. This model has been developed in the Flux® software [7].

C. Newton-Raphson method with relaxation

The NR method is applied to solve the non-linear FE system: the magnetic field **h** and a differential reluctivity tensor d**h**/d**b** are computed by the JA model at the previous iteration of the NR algorithm [3] [4]. So as to ensure the convergence of the NR method, a relaxation factor is employed, which is calculated with the method described in [8]. This relaxation factor is determined at each NR iteration so as to minimize the norm of the residual of the linearized system of equations.

D. Low frequency lamination model

In the 2D finite element model, it is possible to take into account the eddy current losses due to the magnetic flux density flowing in the lamination plane of a magnetic circuit [1]. For the sake of simplicity, we assume a stacking factor of one. The skin effect is assumed to be negligible. So we use the following relation:

$$\mathbf{h}_s(t) = \mathbf{h}_a(t) + \frac{\sigma d^2}{12}\frac{\mathrm{d}\mathbf{b}_a}{\mathrm{d}t}, \qquad (4)$$

where $\mathbf{h}_s$ is the magnetic field at the surface of a lamination, $\mathbf{h}_a$ and $\mathbf{b}_a$ the average magnetic field and flux density respectively (linked by the hysteresis model presented in subsection B, with $\mathbf{h}_a = \mathbf{h}$, $\mathbf{b}_a = \mathbf{b}$), d the thickness of the laminations and σ their conductivity.

III. NUMERICAL EXAMPLES

A. *Square region example*

We have first tested the method on a simple case: a square domain with pulsating or rotating field described in [4]. We have verified that in the case of a pulsating field, the vector JA model gives the same results as the scalar one, according to the theory, as stated in [2]. In the rotational case, the b and h loci are circular as expected.

B. *T-joint and three-phase transformer*

We have then performed simulations of two test cases concerning a three-phase transformer operating at 50 Hz: 1) a T-joint with imposed shifted currents in two coils with $I_{max} = 0.01$A or $I_{max} = 0.2$A and 2) the whole three-phase transformer described in [1] operating at no load at 100 Vrms or 230 Vrms (cf. fig. 1). The JA coefficients we used have been found in [9] corresponding to non-oriented M330-50A steel sheet: $m_s = 1.28\times10^6$A/m, a = 26.1A/m, k = 52.3A/m, c = 0.13 and $\alpha = 7.45\times10^{-5}$. 50 periods have been simulated with 200 time steps per period. The amplitude of the currents or voltages are smoothly increased from 0 s until 0.8 s by a sine step function sf(t) so as to reduce the simulation time to reach the steady state.

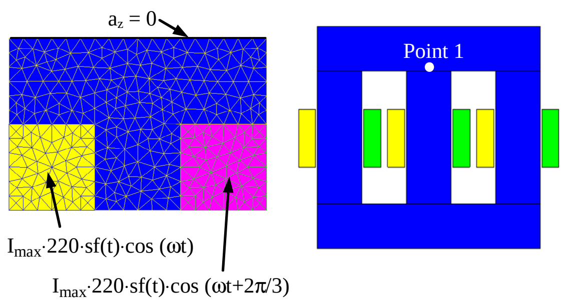

Fig. 1. T-joint (left) and the whole transformer (right).

Both test cases have been simulated with and without the lamination model (cases 1 and 2 respectively). When used, a lamination is 0.5 mm thick and has a conductivity of $2.03\ 10^6$ S/m. In the T-joint case, we have also taken into account eddy currents in the z-direction by considering that the magnetic circuit is solid (not laminated), with a 500 S/m conductivity and zero net current (case 3). With the relaxation procedure, the NR algorithm converges in all cases presented in this paper. With a current of 0.01A in the coils of the T-joint and with the transformer at 100 V, a drift of flux density and also a drift of magnetic field, however to a lesser extent, are observed after 0.7 s in case 1 or 2. It is much reduced in case 3 (with conductivity) of the T-joint. Notice that this simulation gives different results as the magnetic circuit is not any more considered laminated. With a current of 0.2A in the coils of the T-joint and with the transformer at 230 V, the simulation results do not exhibit any drift of flux density in time. An explanation can be that, in these cases, the saturation is reached at most of the points of the magnetic circuit.

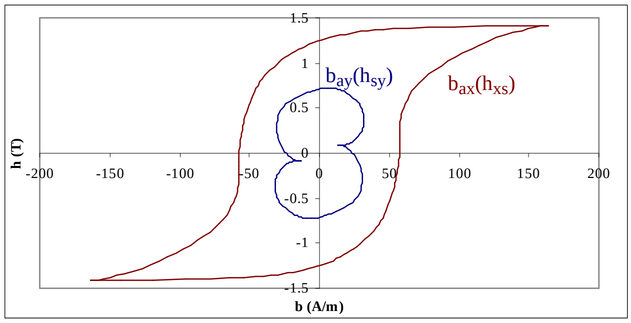

Fig. 2. $b_{ax}h_{sx}$- and $b_{ay}h_{sy}$-loops at point 1 of the transformer for a voltage of 230 V with the lamination model.

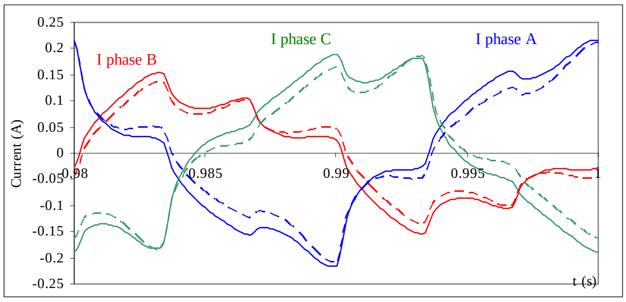

Fig. 3. Transformer phase currents for a voltage of 230 V. Dashed line: without lamination model, continuous line: with the lamination model.

IV. CONCLUSION

We have simulated a three-phase transformer model in 2D using the inverse vector JA model for isotropic magnetic materials. We have observed good convergence with the relaxation procedure used with the NR algorithm. In a future work, instability problems of the hysteresis model with reduced currents and voltages must be overcome.

REFERENCES

[1] J. Gyselinck, M. De Wulf, L. Vandevelde, and J. Melkebeek, "Incorporation of vector hysteresis and eddy current losses in 2D FE magnetodynamics", in Proc. Electrimacs'99, Lisbon, Portugal, Sept., 14–16 1999, vol. 3, pp. 37–44.

[2] A. Bergqvist, "A simple vector generalisation of the Jiles–Atherton model of hysteresis", *IEEE Trans. Magn.*, Vol. 32, No. 5, pp. 4213–4215, 1996.

[3] J. Gyselinck, L. Vandevelde, J. Melkebeek and P. Dular "Complementary two-dimensional finite element formulations with inclusion of a vectorized Jiles-Atherton model", COMPEL, vol. 23, no. 4, pp. 959—967, 2004.

[4] J. Gyselinck, P. Dular, N. Sadowski, J. Leite and J.P.A. Bastos, "Incorporation of a Jiles-Atherton vector hysteresis model in 2D FE magnetic field computations – Application of the Newton-Raphson method", *COMPEL*, vol. 23, no. 3, pp. 685—693, 2004.

[5] J. V. Leite, N. Sadowski, P. Kuo-Peng, N. J. Batistela, J. P. A. Bastos and A. A. de Espíndola, "Inverse Jiles-Atherton vector hysteresis model", *IEEE Trans. Magn.*, vol. 40, no. 4, pp. 1769–1775, 2004.

[6] K. Hoffmann, J.P.A. Bastos, N. Sadowski and J. V. Leite, "Convergence Procedures Applied for Solving 3D Vector Hysteresis Modeling Coupled with FEM", *CEFC'2014*, Annecy, France, 2014.

[7] Flux software, www.cedrat.com.

[8] K. Fujiwara, T. Nakata, N. Okamoto, and K. Muramatsu, "Method for determining relaxation factor for modified Newton-Raphson method", *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1962–1965, 1993.

[9] K. Chwastek, "The applications of fixed-point theorem in optimisation problems", *Arch. Elect. Eng.*, vol. 61, no. 2, pp. 189–198, 2012.