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Research Paper

Numerical modeling of the long term behavior of Municipal Solid Waste in a bioreactor landfill

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ABSTRACT

This paper presents a thermo-hydro-biochemo-mechanical model for simulating the long term behavior of Municipal Solid Waste (MSW) in a bioreactor landfill, in which the multi-physics coupling mechanism plays a dominant role. In the model, a two-stage anaerobic biochemical model based on McDougall's formulation is incorporated into a fully coupled thermo-hydro-mechanical models originally developed for unsaturated porous medium. The mechanical model is a modified Camclay model allowing for biochemical hardening/softening, while the thermal model is described by a classical energy balance equation with a source term accounting for the heat generation from the biodegradation of organic matter. The hydraulic model is an unsaturated flow model using Richard's equation. The derived coupled model is implemented into an in-house built multi-physics finite element code. Finally, numerical simulations were performed to illustrate the capability of the proposed model for estimating long-term settlement of a bioreactor landfill and its aptitude as a landfill management tool for optimizing the landfill operation.

1. Introduction

Recent technological advances in the operation of modern landfills for Municipal Solid Waste (MSW) disposal have led to the emergence of new generation "bioreactor landfill technologies" [54]. The fundamental principle of this new technology is to operate a landfill as a bioreactor to accelerate the decomposition and stabilization of the biodegradable organic waste constituents, which can be done by the recirculation of generated leachate or the addition of water into the landfill [38]. In comparison to the conventional "dry-tomb" landfills, the bioreactor landfills have proven to have many advantages and benefits, which include rapid stabilization of waste, improvement of landfill gas production rate and total yield, significant gain in landfill space and considerable reduction in time and cost of post-closure monitoring and maintenance e.g. [53,6,71]. To optimize the design and operation of the bioreactor landfill, it is essential to be able to accurately assess and predict the landfill gas generation and settlements in both short and long-terms [20]. This study is concerned with those aspects which focus on the long-term settlement behavior of a bioreactor landfill.

Accurate prediction of bioreactor landfill settlement is a great challenge because of the complexity of the landfill system, in which the biochemical reactions of degradable wastes, coupled with thermal, hydraulic and mechanical related processes, play an important role [43,13]. Therefore, the key step to develop any predictive tool for estimating the landfill settlement is to understand the dominant biochemical reactions occurring during the waste decomposition process and thereby create mathematical models to describe those reactions. So far, the former aspect has been widely investigated through both laboratory and field experiments (e.g. [52,1,57,30,21,4,2,37]). Those studies have identified the biodegradation process into two main phases: an aerobic phase followed by an anaerobic phase including four sub-phases: hydrolysis, acidogenesis, acetogenesis and methanogenesis [57]. The former lasts for a short period and can be neglected in most cases, whereas the latter dominates the amount of degraded wastes and gas production. Furthermore, both the aerobic and the anaerobic reactions are affected by a number of environmental factors such as moisture content, pH, temperature, among which the most important is moisture content [38].

With taking into account the two-stage biochemical reactions and associated environmental influencing factors, a great number of mathematical models have been proposed to describe the biodegradation process, such as (i) multi-phases degradation models using Monod kinetics (e.g. [19,27,67]), (ii) two-stage anaerobic digestion models incorporating hydrolysis/acidogenesis and methanogenesis phases, which are correlated by using the primary

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substance of Volatile Fatty Acid (e.g. [43,24,56,11]). In contrast to the formers requiring a great number of parameters, the latters are capable of capturing the dominant biochemical reactions in a simplified manner and thereby have been increasingly adopted in biodegradation modeling.

Both laboratory and field experiments have shown that long-term bioreactor landfill settlement is mainly attributed to mechanically – and biologically – induced compression as well as related creep effects (e.g. [49,7,50,31,26,3,4,61]). With accounting for these mechanisms, a variety of numerical models have been proposed to estimate long-term landfill settlement. These existing models can be divided into two main categories as inspired by Liu et al. [40]; Simões [58]: (i) empirical time-dependent models with or without taking into account decay of waste ([25,60,68,18,8,51]) of which the most common approach is based on Terzaghi's one-dimensional consolidation theory; (ii) integrated analysis models, which incorporate the aforementioned biodegradation models into classical mechanical and/or hydraulic models developed in soil mechanics [28,29,43,26,56,12,66,13,22]. Although the empirical models are useful in practice due to their simplicity, they have very limited capabilities for accurately estimating long-term landfill settlement. In contrast, the integrated analysis models bring an insight into the fundamental multi-physics processes occurring in bioreactor landfills, leading to better prediction of the long-term behavior of the landfills. Up to date, most of existing integrated analysis models are developed for isothermal conditions despite the fact that the considerable variation in waste temperature occurs under field conditions, which can be mainly attributed to the aerobic reaction and leachate recirculation [38]. Although El Fadel et al. [19]; Gholamifard et al. [24] took into account the thermal effects on biological reaction in their hydro-thermo-biological model, the model is incapable to calculate landfill settlement. Additionally, few models [43] employed sophisticated constitutive models within elasto-plastic framework to describe the chemo-mechanical behavior of solid wastes. More importantly, all the existing coupled models were solved in a decoupled manner, which may result in numerical instability as for such highly non-linear system. This study attempts to bring the current integrated analysis a step closer to full thermo-hydro-biochemo-mechanical coupling.

The objective of this study is to incorporate the two-stage anaerobic biodegradation model adopted by McDougall [43] into a fully coupled thermo-hydro-mechanical framework for unsaturated porous media, which has been developed in University of Liege over the last three decades [10,14]. In the derived thermo-hydro-biochemo-mechanical (THBCM) model, the biodegradation model was linked to both the governing mass balance equations for VFA and methanogen biomass and the energy balance equation through source term. The mechanical model adopted is a simplified version of the chemo-hydro-mechanical (CHM) model presented by Liu et al. [41], which has been originally introduced to simulate the behavior of unsaturated clay in presence of chemicals in the pore fluid. The THBCM model was implemented into an in-house built multi-physics FEM code for large deformation simulation (LAGAMINE) [10]. Finally, the simulation was done on a bioreactor landfill with water injection as a boundary value problem to show the performance of the proposed model in terms of predicting the long-term behavior of the landfill. The potential to use this model for optimizing the operation of bioreactor landfills was also explored and discussed.

2. THBCM multi-physics model

The THBCM framework comprises four main ingredient models describing thermal, hydraulic, mechanical and biodegradation behavior. The interdependencies of the different models are schematically shown in Fig. 1. The sub-models will be detailed in the coming sub-sections with emphasis on biochemical model.

2.1. Bio-chemical model

The microbiological activity within the landfills is responsible for the mineralization of the organic content and the production of biogas. This process modifies the hydromechanical properties of the waste and has to be considered if one wishes to perform a rigorous analysis of the MSW behavior.

The biodegradation can be split into two main stages [57], which are quickly described in the following sections.

2.1.1. Aerobic stage

The aerobic phase is the first step of the biodegradation and it begins just after the wastes are landfilled. It lasts at most a few weeks since the deposit of subsequent layers of waste will deprive the previous ones of any oxygen and will cut the aerobic stage short. During this process, the organic content is degraded into macromolecules by bacteria. It is a very exothermal reaction leading to an important temperature raise sometime reaching over 60 °C.

2.1.2. Anaerobic stage

The anaerobic stage begins as soon as the aerobic one ends. It can last 40 to 50 years. This stage consists of the four reactions defined below [57]:

- Hydrolysis: the macromolecules are decomposed by hydrolytic bacteria into smaller molecules (lipid into fatty acid; polysaccharide into monosaccharide; protein into amino acid).
- Acidogenesis: the products of the hydrolysis are transformed into ethanol, organic acid and Volatile Fatty Acid (VFA).
- Acetogenesis: the products of the hydrolysis are consumed and transformed into acetyl acid and hydrogen.
- Methanogenesis: during this last step, the acetyl acid is consumed to produce carbon dioxide and the hydrogen is consumed to produce methane and carbon dioxide.

The two-stage biochemical model described by McDougall [43] was adopted in this study to describe the hydrolysis/acidogenesis and methanogenesis reactions. Fig. 2 presents the structure of this model and different phases of biodegradation as well as associated products: volatile fatty acids (VFA) and methanogenic biomass (MB).

VFAs are intermediate products linking the two stage biochemical reaction and serve as a precursor for methanogen biomass. However, high VFA concentration has inhibitory effects on those reactions [24], which was also taken into account in the model through an inhibitor factor. It is worth noting that the aerobic phase occurring during the early period of biodegradation was neglected due to its short duration [72] and considering the focus of this study on long-term behavior of bioreactor landfills. The formulation of the biochemical model and the governing mass balance equations for organic matter, VFA and MB are presented in the following subsections.

2.1.3. McDougall's formulation for biodegradation

McDougall's formulation neglects the aerobic stage but since the aerobic waste decomposition represents a minor part of the landfill life-time it is, therefore, less significant than anaerobic decomposition [72]. This formulation is used to determine the growth/decay term for the internal variable characterizing the biodegradation. These terms describe the reaction rate and are

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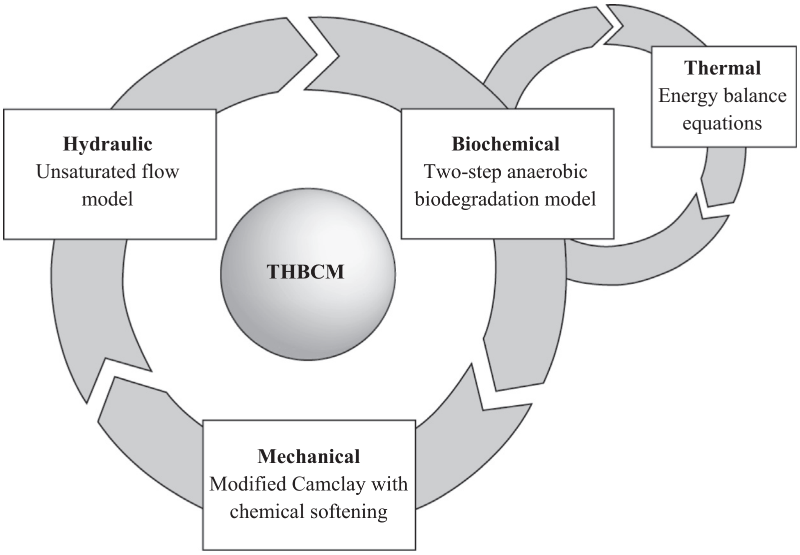

Fig. 1. Sub-models interdependencies within the THBCM framework.

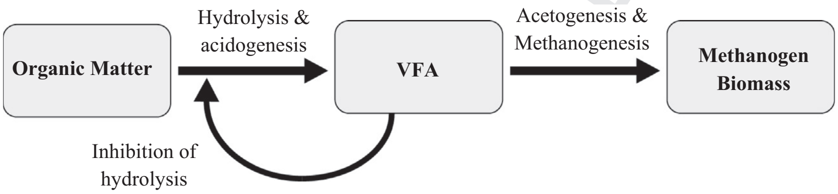

Fig. 2. Two steps biodegradation process.

expressed in $[(\mathrm{g/m^3_{aqueous}})\ \mathrm{s^{-1}}]$. This unit highlights the dependency of the reaction on the moisture content.

In the following, the Organic content will be referred by the Org variable, the VFA concentration in water by the c variable and finally the methanogen biomass by the variable m. Having these three chemical species, McDougall's model provides the corresponding balance equations.

2.1.3.1. Hydrolysis and acidogenesis. Hydrolysis and acidogenesis is the first stage of the biodegradation which represents the depletion of the organic content and its transformation into VFA. The accumulation of VFA may in turn, inhibit the hydrolysis. This is accounted for by using the modified enzymatic hydrolysis equation proposed by McDougall [43] and thus the reaction rate $[(\mathrm{g/m^3_{aqueous}})\ \mathrm{s^{-1}}]$ can mathematically be described as:

$$r_g = b\theta_e\phi P \tag{1}$$

where four governing factors are taken into account:

- (i) $\theta_e = \frac{\theta-\theta_{res}}{\theta_{sat}-\theta_{res}}$ [–] is the effective moisture content, which can be affected by both water flux and mechanically and biologically-induced compression.
- (ii) $b\ [(\mathrm{g/m^3_{aqueous}})\ \mathrm{s^{-1}}]$ is the maximum VFA growth rate under the most favorable environmental conditions, which normally occurs at the early stage of hydrolysis reaction.
- (iii) $\phi = 1-\left[\frac{\mathrm{Org_0}-\mathrm{Org}}{\mathrm{Org_0}}\right]^{\xi}$ [–] is the relative digestibility describing the decrease of the depletion rate of organic matter and in turn of the VFA growth rate from the maximum value b. In the equation, Org $[\mathrm{kg/m^3}]$ is the organic content, $\mathrm{Org_0}$ $[\mathrm{kg/m^3}]$ is the initial organic content, ξ[–] is a parameter.
- (iv) $P = e^{-k_{VFA}\cdot c}$ [–] is the inhibition factor to account for the inhibitory effect of high VFA concentration, in which $k_{VFA}(\mathrm{m^3/g})$ is an inhibition constant and $c(\mathrm{g/m^3})$ is the VFA concentration.

2.1.3.2. Acetogenesis and methanogenesis. The second stage of the biochemical reactions occurring in the MSW transforms the VFA generated from the hydrolysis/acidogenesis reactions to methanogen biomass. Note that the biogas was not taken into account in current model for the sake of simplicity. The MB reaction rate $r_j[(\mathrm{g/m^3_{aqueous}})\ \mathrm{s^{-1}}]$ is calculated through a Monod kinetic Eq. (2) [5] and the consumption rate $r_h[(\mathrm{g/m^3_{aqueous}})\ \mathrm{s^{-1}}]$ is directly linked to the methanogen biomass accumulation through a substrate yield coefficient Y.

$$r_j = \frac{k_0\frac{c}{\theta}}{k_{MC}+\frac{c}{\theta}}\frac{m}{\theta} \tag{2}$$

$$r_h = \frac{r_j}{Y} \tag{3}$$

where k_0 $[\mathrm{s^{-1}}]$ is the maximum growth rate, k_{MC} $[(\mathrm{g/m^3_{aqueous}})]$ is the half saturation constant, $m\ [(\mathrm{g/m^3})]$ is the methanogen concentration.

2.1.3.3. Methanogen decay. The MB decay over time is given by the first order decay equation as follow:

$$r_k = k_2\frac{m}{\theta} \tag{4}$$

where r_k $[(\mathrm{g/m^3_{aqueous}})\mathrm{s^{-1}}]$ stands for the decay rate while k_2 $[\mathrm{s^{-1}}]$ is the methanogen death coefficient.

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2.1.4. Governing balance equations

The degradation rate of organic matter and accumulation rate of VFA and MB can be incorporated as sink or source terms into classical advection–dispersion equation, leading to advection–dispersion–reactive transport models expressed by Eqs. (5)–(7), respectively for VFA, MB and organic matter. In the derived models, Org, c and m denote organic matter content in MSW, VFA concentration and MB concentration. Note that all the governing balance equations are expressed in bulk volume. Hence, all the parameters associated with the reactions were multiplied by the volumetric water content.

$$\mathrm{div}(\underline{u}\cdot c) - \mathrm{div}\left(\underline{\underline{D_h}}\nabla c\right) + [r_g - r_h]\theta = \frac{\partial c}{\partial t} \tag{5}$$

$$\mathrm{div}(\underline{u}\cdot m) - \mathrm{div}\left(\underline{\underline{D_h}}\nabla m\right) + [r_j - r_k]\theta = \frac{\partial m}{\partial t} \tag{6}$$

$$-\theta Z r_g = \frac{\partial \mathrm{Org}}{\partial t} \tag{7}$$

where $\underline{u} = \frac{\underline{f}_w}{S_{r,w} n_e}$ is the actual average velocity of the liquid flow, n_e is the effective porosity, $\underline{f}_w$ is the water Darcy's velocity and Z is a conversion factor accounting for the fact that most of the degradable organic content in the landfill in most cases is cellulose (molar mass: 162 g/mol) and VFA are mostly acetic acid (molar mass: 60 g/mol). According to the stoichiometric ratio for the reaction, 162 g of organic content are consumed to produce 60 g of VFA.

On the left side of Eqs. (5) and (6), the first terms represent advective flux, linking to the actual average velocity of liquid flow ($\underline{u}$). The second terms describe diffusive flux, which combines mechanical dispersion and molecular diffusion and the thirds are the source terms describing the generation or degradation of VFA or MB. The terms on the right side are the growth rate of VFA and MB, respectively. In contrast to the Eqs. (5) and (6), the mass balance Eq. (7) for organic matter doesn't include any transport terms because the organic matter is considered as a part of solid skeleton and no erosion of organic matter occurs during the leachate recirculation or water injection.

It is worth noticing that the above reactive transport models were derived with no consideration of immobile water phase. These models are more suitable for the MSW with low organics matter content while not for the MSW with high organics matter content. This is because the organic matter can retain high content of immobile water, which tends to transform to mobile water during the biodegradation reaction [13]. To account for the transformation from immobile water to mobile water, Chen et al. [13] recently proposed a new hydro-bio-mechanical model to simulate the long-term behavior of MSW with high kitchen waste content in bioreactor landfills, in China. They also indicated that landfilled MSW in developed countries, as this study is devoted to, deal with low kitchen waste content and in turn a low organic matter content. Therefore, the proposed models is applicable to assess the settlement of European landfills. However, it would still be interesting to go a step further to take into account the transformation between the different water phases in the current models.

2.2. Hydraulic model

The MSW is considered to behave like an unsaturated porous medium, partly saturated by liquid water and air. The water mass balance equation is based on Richard's equation [55]:

$$\frac{\partial(\rho_w n S_{r,w})}{\partial t} + \mathrm{div}\left(\rho_w \underline{f}_w\right) = Q \tag{8}$$

where ρ_w [kg/m^3] is the water density, n [–] the porosity, $S_{r,w}$ [–] the water saturation degree, t [s] the time, Q [kg/s] is the injected flux and $\underline{f}_w$ [m/s] is the Darcy's flow given by the following equation:

$$\underline{f}_w = -\frac{k_w(S_{r,w})}{\mu_w}\left(\underline{grad}(p_w) + \rho_w.g.\underline{grad}(y)\right) \tag{9}$$

In which $k_w = K_w \cdot \mu_w/\rho_w g$ is the intrinsic permeability, K_w is the hydraulic conductivity, p_w is the water pressure, g is the gravity acceleration, μ_w is the water viscosity and y is the upwards vertical spatial coordinates. The intrinsic water permeability k_w can be determined based on degree of saturation and hydraulic model proposed by van Genuchten [65] which is presented as follows.

2.2.1. Water permeability and water retention curve

For an unsaturated porous medium, the intrinsic water permeability is a function of the water saturation and can be calculated using a relative permeability curve k_{rel} and the intrinsic saturated permeability k_{sat} as follows:

$$k_w(S_{r,w}) = k_{rel}(S_{r,w})k_{sat} \tag{10}$$

where $k_w(S_{r,w})$ is determined using the well-known formulation suggested by van Genuchten [65]:

$$k_{rel} = \sqrt{S_{r,w}}\left[1 - \left(1 - S_{r,w}^{\frac{1}{m_{vG}}}\right)^{m_{vG}}\right]^2 \tag{11}$$

where m_{vG} is a model parameter.

The retention curve links the water saturation as a function of the capillary pressure. The formulation proposed by van Genuchten [65] is used in our model:

$$S_{r,w} = S_{res} + (S_{sat} - S_{res})\left[\left(1 + \frac{p_c}{\alpha}\right)^{n_{vG}}\right]^{-m_{vG}} \tag{12}$$

in which n_{vG}, m_{vG} are model parameters, S_{res} is the residual water saturation, S_{sat} is the maximal water saturation and p_c is the capillary pressure.

2.3. Thermal model

Due to the exothermal nature of the early stage of biodegradation reactions (so-called aerobic reactions) and the general heat transfer associated with boundary conditions, a significant temperature increase of MSW normally takes place in bioreactor landfills [7]. The variation of the MSW temperature can be further affected by leachate recirculation or water injection [36]. To be able to simulate the temperature evolution within the landfill, we incorporate a simple source term describing the heat generation due to the biodegradation reaction into classic governing energy balance equation, leading to the following heat transport equation:

$$\dot{S}_T + \mathrm{div}(V_T) - Q = 0 \tag{13}$$

where S_T is the heat storage, V_T is the heat flux and Q is the heat production term.

In the derived transport Eq. (13), the heat storage term can be expressed as follows:

$$S_T = nS_r\rho_w c_{p,w}(T - T_0) + nS_a\rho_a c_{p,a}(T - T_0) + (1 - n)\rho_s c_{p,s}(T - T_0) \tag{14}$$

In which, $c_{p,w}$, $c_{p,a}$, $c_{p,s}$ are respectively the water, the air and the solid waste specific heats, ρ_w, ρ_a, ρ_s are respectively the water, the air and the solid waste densities.

The heat flux consists of a conduction term proportional to the thermal conductivity of the MSW and a convective term related to the heat transported by water flows:

$$V_T = -\Gamma\nabla T + c_{p,w}\rho_w \underline{f}_w(T - T_0) \tag{15}$$

where Γ is the waste thermal conductivity, $c_{p,w}$ is the water specific heat, T_0 is the initial temperature and T is the temperature.

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The thermal conductivity of the MSW is estimated by integrating the thermal conductivities of different phase components of the MSW, including the water, air and the solid phase, as denoted by Γ_w, Γ_a and Γ_s, respectively:

$$\Gamma = nS_r\Gamma_w + nS_a\Gamma_a + (1-n)\Gamma_s \tag{16}$$

The heat production term (Q) is derived by empirical consideration of the energy release from exothermal biochemical reactions occurring in the MSW, which is similar to the formulation proposed by El Fadel et al. [19]:

$$Q = \frac{\Delta \mathrm{Org}(t)}{\Delta t} Q_m \tag{17}$$

where Org is the organic matter content, t is the time and Q_m is the quantity of heat produced by the degradation of one kilogram of waste.

2.4. Mechanical model

McDougall's work suggests that the total settlement can be calculated based on three contributions: elastic and plastic load induced strain, creep and biodegradation strain. The first part is predicted using a modified Camclay model. The second and parts are considered through a variation of the void ratio leading to a modification of the pre-consolidation pressure and thus of the yield surface. Biodegradation is taken into account by a void change-related parameter allowing for the determination of hardening or softening with decomposition. In fact, the three strain parts are considered within a chemo-hydro-mechanical (CHM) framework.

The adopted mechanical model is a simplified version of the chemo-hydro-mechanical (CHM) model presented by Liu et al. [41] which is based on previous works of Hueckel and Hueckel et al. [32–34]. The CHM has been originally developed to simulate the behavior of unsaturated clay in presence of chemicals in the pore fluid. Through a "concentration" parameter (α) related to organic matter content, the model used for this study is able to simulate the effect of the biodegradation on the mechanical behavior. This "concentration" parameter (α) has to vary between $[0,1]$. The closer the value approaches to one, the more chemical softening is induced. Using the updated organic content, the mechanical model determines the softening/hardening and thereby the MSW settlement.

2.4.1. Stress state variables

Bishop's effective stress has been chosen to describe the stress–strain relation because it directly incorporates the effect of the suction. It is expressed as:

$$\sigma'_{ij} = \sigma_{ij} - p_g\delta_{ij} + S_{r,w}(p_g - p_w)\delta_{ij} \tag{18}$$

where σ'_{ij} is the effective stress tensor, σ_{ij} is the total stress tensor, $S_{r,w}$ is the water saturation and δ_{ij} is Kronecker's tensor. p_g and p_w denotes respectively gas and water pressure.

2.4.2. Constitutive equations

They are expressed to relate the strain to the stress and the organic content – through the concentration factor, α.

The strain rate can be expressed as the sum of an elastic reversible part and a plastic irreversible part. The elastic part can also be decomposed into mechanical and chemical components. These components will be detailed below:

$$\dot\varepsilon_{ij} = \dot\varepsilon^e_{ij} + \dot\varepsilon^p_{ij} = \dot\varepsilon^{e,m}_{ij} + \dot\varepsilon^{e,c}_{ij} + \dot\varepsilon^{p,m}_{ij} \tag{19}$$

The three main strain components are presented as follows:

(i) The elastic strain–stress law is a classical one:

$$\dot\sigma_{kl} = C^e_{klij}\dot\varepsilon^e_{ij} \tag{20}$$

where $\dot\sigma_{kl}$ is the elastic stress tensor and C^e_{klij} is the elastic strain (Hooke's tensor) defined as:

$$C^e_{klij} = 2G\delta_{ij}\delta_{kl} + \left[K - \frac{2}{3}G\right]\delta_{ij}\delta_{kl} \tag{21}$$

$$K = \frac{1+e}{3*\kappa}\sigma_{mm} \tag{22}$$

$$G = \frac{3(1-2\nu)}{2(1+\nu)}K \tag{23}$$

where K is the bulk modulus, e is the void ratio, G is the shear modulus and ν is the Poisson's ratio.

(ii) The chemical elastic strain is defined as:

$$\dot\varepsilon^{e,c}_{i,j} = -\frac{1}{3}\beta\dot\alpha\delta_{ij} \tag{24}$$

$$\beta = -F_0\beta_0 e^{\beta_0(1-\alpha+\ln\alpha)}\left(\frac{1}{\alpha} - 1\right) \tag{25}$$

where F_0 and β_0 are material constants dependent on the soil and the chemical.

(iii) The plastic strain rate is defined within the boundaries of the yield criterion:

$$f(\sigma_{ij},\kappa) \leqslant 0 \tag{26}$$

where κ is an internal variable depending on plastic strain and organic content.

The yield function needs to satisfy the consistency condition:

$$\dot f = \frac{\partial f}{\partial\sigma_{ij}}\dot\sigma_{ij} + \frac{\partial f}{\partial\kappa}\dot\kappa = 0 \tag{27}$$

where the variation of the internal variable is described as:

$$\dot\kappa = \frac{\partial\kappa}{\partial\varepsilon^p}\dot\varepsilon^p + \frac{\partial\kappa}{\partial\alpha}\dot\alpha \tag{28}$$

Three plastic yielding mechanisms are implemented into the chemo-hydro-mechanical (CHM) model by Liu et al. [41]: pore collapse, frictional-cohesive failure and tensile failure. They are represented respectively by the following equations:

$$f_1 \equiv q^2 + M^2(p+p_s)(p-p_0) = 0, \quad p \geqslant (p_0 - p_s)/2 \tag{29}$$

$$f_2 \equiv q - M(p-p_s) = 0, \quad \sigma_t < p \leqslant (p_0 - p_s)/2 \tag{30}$$

$$f_3 \equiv p + \sigma_t = 0 \tag{31}$$

where p_0 is the pre-consolidation pressure, p_s is a parameter related to the cohesion, M is a parameter defining the slope of the frictional cohesive failure in the deviatoric plane, σ_t is the limit tensile strength, $p = \sigma_{kk}/3$ and $q = \sqrt{\frac{3}{2}s_{ij}s_{ij}}$ are the stress invariants and s_{ij} is the deviatoric part of the stress tensor.

Theses criterions define the yield surface presented Fig. 3:

2.4.3. Chemical hardening/softening

The large deformation due to the biodegradation of organic matter fraction of MSW is considered in our proposed constitutive model by introducing chemical softening, which is controlled by a so-called "concentration" parameter (α). This parameter is defined as a function of updated and initial organic matter contents:

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$$\alpha = 1 - \frac{\mathrm{Org}}{\mathrm{Org}_0} \quad (32)$$

This equation shows that the concentration parameter varies from zero to one with degradation of the organic matter from initial content to zero, leading to an increase in chemical softening induced.

This "concentration" parameter (α) is linked to both the pre-consolidation pressure and the cohesion parameter in order to describe the biodegradation effect on pore collapse mechanism and frictional-cohesive failure mechanism, respectively.

(i) Effect on pore collapse mechanism
The effect of the "concentration" parameter (α) on the pre-consolidation pressure is described by

$$p_0(\alpha) = p_0^* S(\alpha) \quad (33)$$

where p_0^* is the pre-consolidation pressure for initial organic content ($\alpha = 0$) and $S(\alpha) = \exp(-a\alpha)$ is the chemical softening function, where $a$stands for a constant governing the decrease of the pre-consolidation pressure with the increase in "concentration" parameter.

(ii) Effect on frictional cohesive failure mechanism
The parameter p_s, controlling cohesion, is assumed to vary as a linear function of the concentration parameter (α)

$$p_s = p_s^* + k_\alpha \alpha \quad (34)$$

where p_s^* is the value of the parameter for initial organic content ($\alpha = 0$) and k_α is a model constant.

3. Implementation

The THBCM model described above was solved by using finite element method (FEM). An in-house built FEM code LAGAMINE was used. This code has been developed at University of Liège over the last three decades [10,14] and is capable of dealing with complex multi-physics and large deformation problems in both 2D and 3D. To solve the proposed THBCM model mainly requires implementation of the biodegradation model and mechanical model into the 2D version of LAGAMINE, in which the six primary variables at each node are chosen as two displacements of porous medium skeleton, water pressure, gas pressure, temperature and VFA concentration. In comparison to the existing numerical resolutions adopted for solving the coupled models in the literature (e.g. [43,24,14]), the main part (THM) of the proposed THBCM model was solved in a fully coupled manner, which has proven to be more efficient and lead to a more stable numerical solution [15].

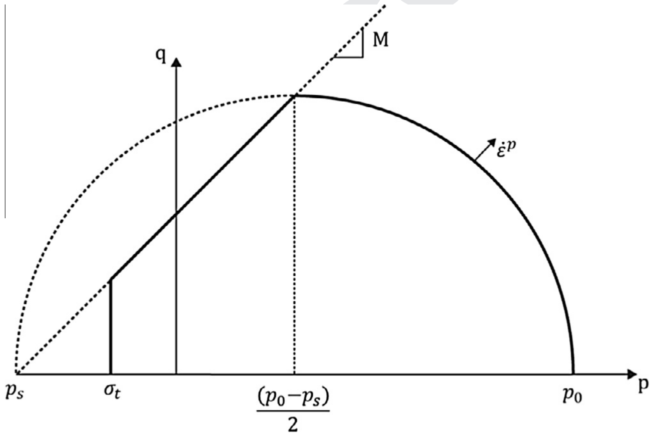

Fig. 3. Yield surface Liu et al. [41].

3.1. Problem formulation

To converge to an accurate solution, we need to satisfy all the governing balance equations derived in the previous section as well as the mechanical equilibrium. It has to be mentioned that for the sake of simplicity, the degree of freedom for gas pressure is completely fixed for all the simulations performed in this study although the implemented model is capable of calculating its evolution. In other words, gas transport and associated effects are neglected despite of the necessity to model biogas recovery. This is because gas pressure variation may be negligible if we assume that the pipe line system for biogas recovery is properly designed and installed. In addition, the focus of this study is to predict the long-term settlement of bioreactor landfills and not biogas recovery. To properly simulate the biogas transport in bioreactors requires considering the coupled processes leading to generation of methanogen biomass and biogas. This is clearly out of the scope of this study and will correspond to future developments. Therefore, five governing equations for the five primary variables considered in this study are rewritten as follows:

$$\text{Static equilibrium} \quad \mathrm{div}\left(\underline{\sigma}\right) + \rho g = 0 \quad (35)$$

$$\text{Liquid mass conservation} \quad \dot{S}_w + \mathrm{div}(V_w) - Q_w = 0 \quad (36)$$

$$\text{Heat conservation} \quad \dot{S}_T + \mathrm{div}(V_T) - Q_t = 0 \quad (37)$$

$$\text{VFA mass conservation} \quad \dot{S}_c + \mathrm{div}(V_c) - Q_c = 0 \quad (38)$$

From a numerical point of view, since organic matter is not subjected to any transport phenomena, its evolution is directly linked to water content and VFA concentration. It is thus unnecessary to consider organic matter as a degree of freedom. Its value depends only on local quantities. In contrast, the methanogen biomass should be subjected to transport phenomena. However, a simplification has been considered by neglecting those phenomena. This is a reasonable simplification since our study focuses on the long term settlement of waste landfill. Under these assumptions, the MB is not considered as a degree of freedom in the current model. It would be interesting to further include these aspects into the future model developments, especially in the case where sewage sludge is landfilled.

These equations are locally defined. To be able to apply them to large domain without having to verify them at every point, the principle of virtual work has to be used to attain a global equilibrium [14]. Consider δv a virtual velocity field respecting the solid continuum and the boundary conditions, δW_E the external virtual work and δW_I the internal virtual work which are defined as:

$$\delta W_E^m = -\int_V \rho g \; \delta \underline{v} \; dV + \int_A t \; \delta \underline{v} \; dA \quad (39)$$

$$\delta W_I^m = \int_V \underline{\underline{\sigma}} \delta \dot{\underline{\underline{\varepsilon}}} \; dV = \int_V \underline{\underline{\sigma}} \frac{1}{2} \left(\frac{\partial \delta v_i}{\partial x_j} + \frac{\partial \delta v_j}{\partial x_i} \right) dV \quad (40)$$

The balance Eqs. (36)–(38) present the same form: a storage term ($\dot{S}$), a transport term (V) and a generation term (Q). Thus the principle of virtual work will only be applied to a reference equation of the form:

$$\dot{S} + \mathrm{div}(V_q) - Q = 0 \quad (41)$$

The external virtual work and the internal virtual are defined as:

$$\delta W_E^q = \int_V Q \; \delta q \; dV + \int_A q \; \delta q \; dA \quad (42)$$

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$$\delta W_I^q = \int_V \dot{S}\,\delta q - V_q \frac{\partial \delta q}{\partial x_i}\,dV \tag{43}$$

Those Eqs. (39) and (40) and (42) and (43) are the weak form of the static equilibrium (35) and of the mass balance Eqs. (36)–(38), respectively.

3.2. Spatial discretization

In LAGAMINE, the element used for the spatial discretization to solve the aforementioned Eqs. (40)–(43) is an 8-noded bi-dimensional large strain finite element. It possesses six nodal degrees of freedom (DOF): horizontal and vertical displacements, liquid water pressure, gas pressure, temperature and the VFA concentration.

This element is isoparametric which means that the same shape function is used to discretize all the DOF. If a parabolic shape function is used, strain (spatial derivative of the position) and, in elasticity, stress are linear. This difference in the degree of interpolation can lead to less accurate results. To counter this, the element is modified to allow for a parabolic discretization of the coordinate and a linear one of the other DOF.

The different DOF are expressed in function of their nodal value thanks to the interpolation function:

$$\begin{aligned} &x_1 = N_L X_{L1} \quad x_2 = N_L X_{L2} \quad p_w = N_L P_{wL} \\ &p_g = N_L P_{gL} \quad T = N_L T_L \quad c = N_L C_L \end{aligned} \tag{44}$$

where $L = 1, 2, \ldots, M$ (number of nodes) and N_L the interpolation function.

The virtual velocity can also be expressed in nodal value:

$$\begin{aligned} &v_i = N_L V_{Li} \quad q = N_L Q_L \\ &\frac{\partial \delta v_i}{\partial x_j} = \frac{\partial N_L}{\partial x_j}\delta V_{Li} \quad \frac{\partial \delta q}{\partial x_i} = \frac{\partial N_L}{\partial x_i}\delta Q_L \end{aligned} \tag{45}$$

Then by applying those discretizations to the expressions of the internal work (40) and (43) and using Gauss integration, these equations can be expressed as:

$$\delta W_I^m = \sum_{IP} \sigma_{ij} \frac{1}{2}\left(\frac{\partial N_L}{\partial x_j}\delta V_{Li} + \frac{\partial N_L}{\partial x_j}\delta V_{Lj}\right) h|J|W_{IP} \tag{46}$$

$$\delta W_I^q = \sum_{IP} \dot{S} N_L - \left(V_i \frac{\partial N_L}{\partial x_i}\delta Q_L\right) h|J|W_{IP} \tag{47}$$

where h is the thickness of the element, J is the Jacobien of the transformation from global $(x_1; x_2)$ to local $(\xi; \eta)$ (deformed configuration) coordinates and W_{IP} is the weight of the integration point.

Those expressions can themselves be expressed as equivalent nodal forces, $F^{Int}_{L,i}$ (related to each DOF):

$$\delta W_I^m = F^{Int}_{L,i}\,\delta V_{L,i} \tag{48}$$

$$\delta W_I^q = F^{Int}_{L,q}\,\delta Q_L \tag{49}$$

And then:

$$F^{Int}_{L,1} = \sum_{IP}\left(\sigma_{11}\frac{\partial N_L}{\partial x_j} + \sigma_{12}\frac{\partial N_L}{\partial x_j}\right) h|J|W_{IP} \tag{50}$$

$$F^{Int}_{L,2} = \sum_{IP}\left(\sigma_{12}\frac{\partial N_L}{\partial x_j} + \sigma_{22}\frac{\partial N_L}{\partial x_j}\right) h|J|W_{IP} \tag{51}$$

$$F^{Int}_{L,pw} = \sum_{IP} \dot{S}_w N_L - \left(V_{w1}\frac{\partial N_L}{\partial x_1} + V_{w2}\frac{\partial N_L}{\partial x_2}\right) h|J|W_{IP} \tag{52}$$

$$F^{Int}_{L,T} = \sum_{IP} \dot{S}_T N_L - \left(V_{T1}\frac{\partial N_L}{\partial x_1} + V_{T2}\frac{\partial N_L}{\partial x_2}\right) h|J|W_{IP} \tag{53}$$

$$F^{Int}_{L,c} = \sum_{IP} \dot{S}_c N_L - \left(V_{c1}\frac{\partial N_L}{\partial x_1} + V_{c2}\frac{\partial N_L}{\partial x_2}\right) h|J|W_{IP} \tag{54}$$

Now that the internal nodal forces are defined, the problem consists in finding a new configuration where the out of balance forces $\underline{F}^{ob} = \underline{F}^{Int} - \underline{F}^{Ext}$ vanish. This can be achieved by solving the following equation (see Fig. 4):

$$\underline{F}^{ob} = \underline{K}\begin{bmatrix} u_1 \\ u_2 \\ \Delta p_w \\ \Delta T \\ \Delta c \end{bmatrix}$$

Quadratic convergence can be reached if the stiffness matrix, $\underline{K}$, is correctly defined. It associates five DOF per node and is shown at Fig. 5.

Where the diagonal terms K_{ii} are the stiffness matrices of the different phenomena involved and K_{ij} represent the coupling of the i mechanism with the j mechanism.

Classically, K_{ii} takes into account the effect of the displacement on nodal forces, of the water pressure on the water flow, of the temperature on the heat flux, and of the VFA concentration on the VFA flux.

It also considers a certain number of couplings: (i) The effect of the water pressure on the nodal force through Bishop's formulation of the effective stress (18) is considered as well as the effects of the

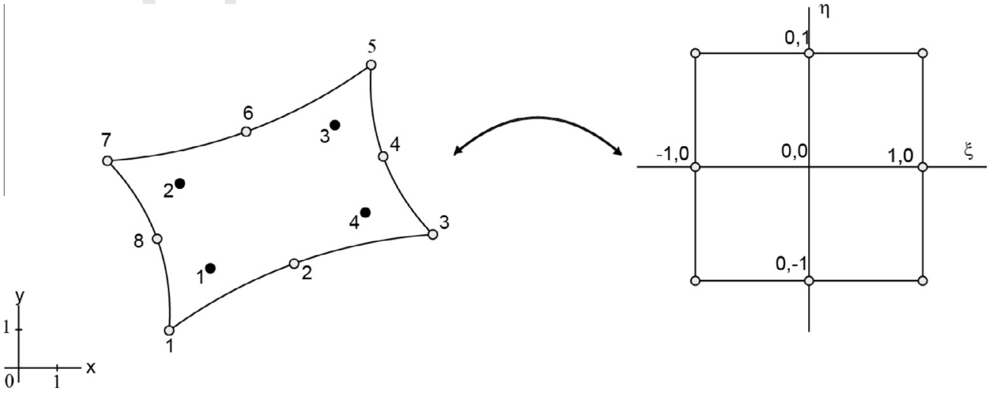

Fig. 4. Isoparametric element.

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$$\begin{bmatrix} K_{MM} & K_{WM} & K_{TM} & K_{CM} \\ K_{MW} & K_{WW} & K_{TW} & K_{CW} \\ K_{MT} & K_{WT} & K_{TT} & K_{CT} \\ K_{MC} & K_{WC} & K_{TC} & K_{CC} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial p_w} & \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial c} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial p_w} & \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial c} \\ \frac{\partial F_{p_w}}{\partial x_1} & \frac{\partial F_{p_w}}{\partial x_2} & \frac{\partial F_{p_w}}{\partial p_w} & \frac{\partial F_{p_w}}{\partial T} & \frac{\partial F_{p_w}}{\partial c} \\ \frac{\partial F_T}{\partial x_1} & \frac{\partial F_T}{\partial x_2} & \frac{\partial F_T}{\partial p_w} & \frac{\partial F_T}{\partial T} & \frac{\partial F_T}{\partial c} \\ \frac{\partial F_c}{\partial x_1} & \frac{\partial F_c}{\partial x_2} & \frac{\partial F_c}{\partial p_w} & \frac{\partial F_c}{\partial T} & \frac{\partial F_c}{\partial c} \end{bmatrix}$$

Fig. 5. Stiffness matrix.

displacement on the water flows by the means of a modification of the storage capacity. (ii) The effect of the temperature on water flows (modification of the densities and viscosities of the water) and the effect of the displacements and water pressure on the heat flux (modification of the storage capacity) is also added. No numerical couplings with the VFA are directly considered in the stiffness matrix though they are physically included (38). This is an approximation acceptable because, after the establishment of the steady state following the hydraulic equilibrium, the water flux doesn't change much from the start to the end of the time step.

3.3. Biodegradation

Numerical solution for VFA mass balance equation becomes complicated since the time scales for the degradation reaction and the VFA transport may vary over large magnitudes. The smallest time scale needs to be resolved to obtain a stable numerical solution, likely leading to a significant increase in simulation time. To tackle this issue, the operator-splitting approach as suggested by Wessling et al. (2008) is applied to the VFA mass balance Eq. (38). To do so, the calculation of VFA accumulation can be split into a transport-only part (V) and a production-only part (Q), which can be solved using a sequential scheme. In other words, the latter is solved after the former has been calculated. The two operators ($\underline{V}_c$ and Q_c) can be expressed separately by combing the Eqs. (1)–(3) into (38) as follows:

$$\underline{V}(c) = \underline{u}\nabla c - \nabla(D_h \nabla c) \quad (55)$$

$$\begin{aligned} Q(c) &= r_g - r_h \\ &= b\theta_e \left(1 - \left[\frac{\mathrm{Org}_0 - \mathrm{Org}}{\mathrm{Org}_0}\right]^{\xi}\right) e^{-k_{VFA} \cdot c} - \frac{k_0 c}{k_{MC} + c} \frac{m}{Y} \end{aligned} \quad (56)$$

where all the notations can be found in the section detailing the biodegradation model (Section 2.1).

In the FEM code, each operator is evaluated at the end of a given time step Δt. When large time step can be used for an accurate evaluation of the transport operator, the precision of the production operator may decrease rapidly with the time step. Indeed this second operator is highly nonlinear with the VFA concentration and assuming a constant production rate over the time step may become problematic.

At low effective moisture content (θ_e^t) and low organic content (Org^t) or high VFA concentration, the VFA production takes place over large timescales. In such case, large time step can be used for the computation of the two operators. In contrast, $\Delta t = \sum_n \Delta t_{ch}$ needs to be employed in the opposite cases meaning that high effective moisture content and high organic content or low VFA concentration. Therefore, the global time step Δt for solving the whole coupled system is considered, while Δt_{ch} is used as the sub time step for the biodegradation reaction.

To obtain the smallest sub time step to ensure the numerical convergence, Δt_{ch} is defined by Eq. (57) to be inversely proportional to the VFA production rate while considering a reduction factor of 0.1. For the sake of simplicity, the VFA production rate is considered as the r_g with ignoring the r_h. This is a reasonable simplification since the former predominates the latter in most cases. In addition, the VFA concentration (c) is added into the formulation to satisfy the unit coherence with time. This modification is acceptable since the sub time step Δt_{ch} tends to increase with accumulation of VFA, which leads to an increase in the VFA concentration.

$$\begin{aligned} \Delta t_{ch} &= 0.1 * \frac{1}{\text{VFA production rate}} \\ &= 0.1 * c \left[\frac{\mathrm{g}}{\mathrm{m}^3_{\text{aquous}}}\right] * \frac{1}{r_g} \left[\frac{\mathrm{m}^3_{\text{aqueous}}\ \mathrm{s}}{\mathrm{g}}\right] \end{aligned} \quad (57)$$

4. Boundary value problem

Numerical simulation was performed on a hypothetical bioreactor landfill to illustrate the capabilities of the proposed THBCM model, with emphasis on the long-term settlement prediction and the effects of leachate recirculation. The geometry, initial and boundary conditions and associated model parameters for the illustrative simulation are respectively presented in Fig. 6.

4.1. Geometry

Neglecting the boundary effects near the external slope of the disposal, the behavior of a typical bioreactor landfill is considered as a one-dimensional problem, despite of the capability of the FEM code to deal with much more complex 2D problems such as waste heterogeneity or anisotropic hydraulic conductivity. As presented in Fig. 6, it is a 30 m high waste column sitting on a 1 m thick drainage layer. Underneath the drainage of the landfill is a 5 m thick geological clay layer. An aquifer is present at the bottom of the clay layer. This is a configuration that can be observed on the site of Hallembaye in Belgium. The column mesh is composed of 300 elements for the waste, a single element for the drainage layer and 10 elements for the clay layer.

4.2. Initial conditions

Initial conditions were uniformly set for all the elements, including the temperature of 20 °C, the moisture content of 30%, the organic content of 30% expressed as a mass concentration of the solid mass, corresponding to $\mathrm{Org}_0 = 300\ \mathrm{kg/m}^3$, the VFA concentration of $300\ \mathrm{g/m}^3_{\text{aqueous}}$ and the MB concentration of $2.5\ \mathrm{g/m}^3_{\text{aqueous}}$.

The water pressure is uniformly fixed at 90 kPa. As shown in Figs. 9, 14 and 17, a hydraulic steady state is reached with a uniform water saturation degree on the whole medium, for the different water injection flux. Considering that the studied waste column may be subjected to a continuous rainfall water flux (mean annual rainfall flow is $2.5 * 10^{-5}\ \mathrm{m}^3/\mathrm{s}$ in Belgium) during the landfilling period, the medium will present a uniform water pressure at the beginning of the post closure phase. This latter phase corresponds to the beginning of our modeling.

It is worth noting that the initial organic content was a typical value in a bioreactor landfill in Europe as suggested by Olivier [49], while the initial VFA and MB concentrations were determined following the recommendations by McDougall & Philp [44]. In fact, a sensitivity analysis done by McDougall & Philp [44] (Fig. 7) showed that those values served for initiating the biodegradation

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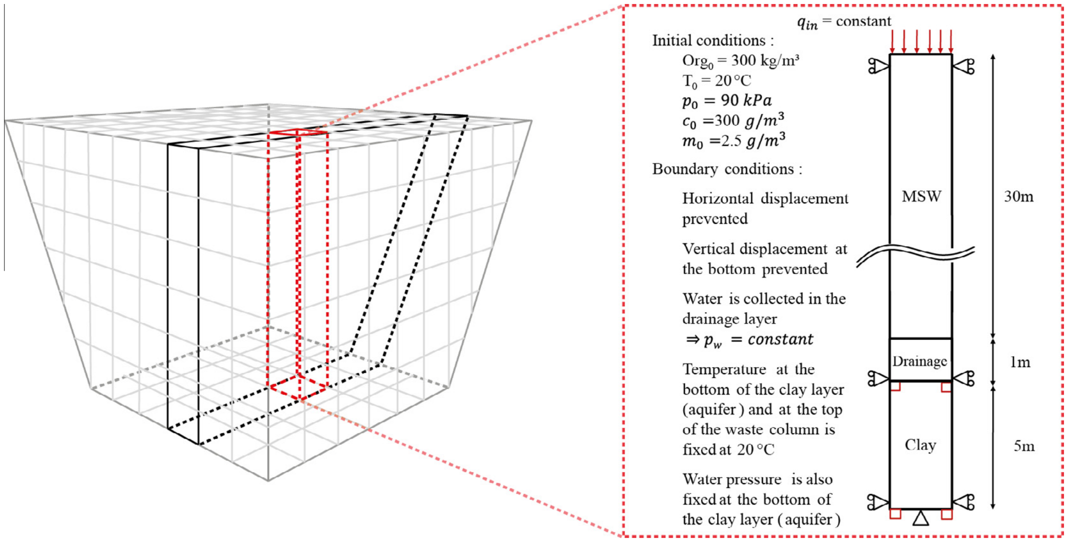

Fig. 6. Geometry and boundary conditions.

reaction while having no influence on the generated VFA and MB concentrations in long term.

The initial effective vertical stress is calculated using Bishop's effective stress formulation:

$$\sigma'_v = \sigma_v - S_r * p_w - S_a * p_a \tag{58}$$

$$\sigma_v = \rho g h \tag{59}$$

$$\sigma'_h = K_0 \sigma'_v \tag{60}$$

$$\rho = \rho_s(1-n) + \rho_w S_r n + \rho_a S_a n \tag{61}$$

where σ'_v and σ'_h are respectively the vertical and the horizontal vertical stresses, σ_v and is the vertical stress, K_0 is the coefficient of earth pressure at rest.

4.3. Boundary conditions

The boundary conditions need to be imposed for five primary variables (degree of freedoms):

(i) Lateral displacements on both side walls of the column and vertical displacements at the bottom of the column were prevented.
(ii) A water inflow flux of $2.5 * 10^{-5}$ kg/m³ m/s was imposed at the top of the column to simulate the operation of the assumed bioreactor landfill. This water flux is determined based on the mean annual rainfall in Belgium. The water pressure at the bottom of the drainage was set to a constant value of 85 kPa, which was determined as low enough to ensure the efficiency of leachate collection. The drainage layer below the waste column is made of granular material (coarse sand) having a very low air entry value, leading to the low value of water saturation. Therefore the value of water pressure was fixed to ensure that the suction would remain higher in the drainage than in the waste column at any time.
(iv) Temperature values at the top of the waste column and at the level of the aquifer were set to 20 °C (Dirichlet type).
(ii) VFA concentrations at the bottom of the column was set to zero in order to account for the efficient leachate collection and recirculation in an optimal landfill operation condition.

4.4. Parameters values

Since there is no available experimental data from a single laboratory or field test covering all the parameters involved in the proposed THBCM model, the determination of the parameters for the different ingredient models is based on the experimental results from several different studies. To minimize the inconsistency of the results used, the experimental data reported by the LIRIGM group at University of Grenoble were used as a main source and those from other studies as a complement. Bearing in mind that the focus of this study is to first propose the THBCM model and then to show its modeling capability through the simulation on a synthetic bioreactor landfill. Indeed, an extension of this work is to apply our proposed coupled model to simulate the long term behavior of some real bioreactor landfills such as the one recently reported by Chen et al. [13].

All the model parameters used in the different sub-models are presented below in table form. The table is structured in four columns: the data source, the model parameter, the value adopted and the corresponding unit. The model parameters used for hydraulic, biochemical, mechanical and thermal models are summarized in Tables 1–4, respectively. In addition, the relative water retention and relative permeability curves when using the model parameters presented in Table 1 are shown in Fig. 8.

5. Simulation results and discussion

On basis of the simulation set-up presented in the previous section, two aspects of simulation were conducted to illustrate the capability and performance of the proposed model: (i) the so-called reference simulation on some simple problem and (ii) a comprehensive sensitivity study on several key model parameters associated with the water retention curve and the water injection flux, which are supposed to be the most influencing elements governing the biodegradation rate as shown by Thonart et al. [64]. More importantly, the objective of the sensitivity analysis on the water injection flux is to show how the developed model may be used for optimizing the operation of a bioreactor landfill, for which the main controlling parameter to be designed is the estimation of dosage and frequency of water or leachate to be injected into the landfill and of the prediction of the associated long term settlements of MSW. Note that the results are presented in the landfilled

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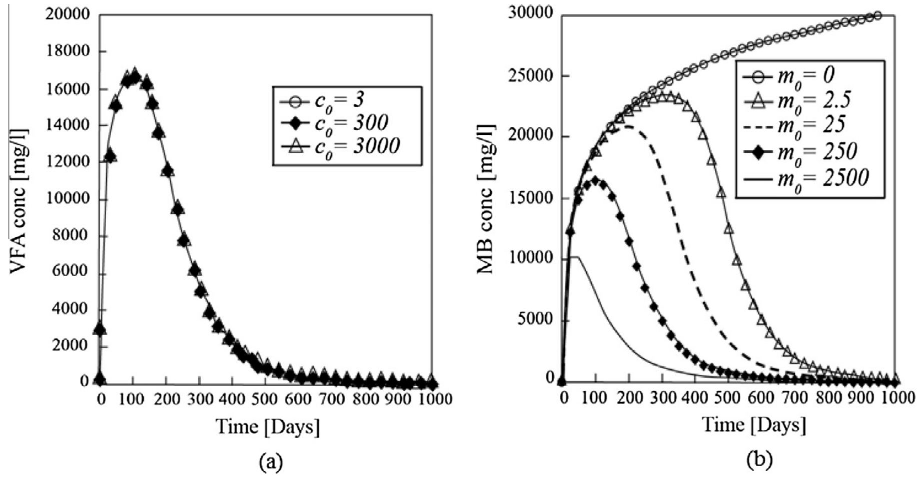

Fig. 7. Dependency of the long term VFA concentration on the initial VFA concentration (left) and initial methanogen biomass concentration (right) Invalid source specified Mcdougall and Philp [44].

part of the geometry. Some results are eventually shown in the drainage/clay layer only if they are required to understand the behavior observed in the waste column.

5.1. Reference simulation

Fig. 9 presents the evolution of the saturation profile with time. We can observe that the saturation quickly shifts from its initial value of 60% to an equilibrium position around 18%. This indicates that a steady state for water flow was progressively reached in the waste under the given water inflow flux. The final equilibrium value mainly depends on the water retention curve, the relative permeability curve and the value of the water injection flux. This is not surprising since the governing equation is Richard's Eq. (8) for water flow in an unsaturated porous medium.

Fig. 10 shows the evolution of the VFA concentration profile with elapsed time. It can be noticed that the VFA concentration gradually decreases from the initial value of 300 g/m^3 and reaches an equilibrated value of around 15 g/m^3. This can be explained by the two stage biochemical reactions, in which the VFA is a product from the first stage reaction and simultaneously consumed by the second stage reaction generating biomass. At the beginning of the reaction, the VFA concentration increases significantly because of the initial high value of the saturation degree leading to important degradation. With the decrease in the saturation degree to its equilibrium value and the high VFA concentration, the VFA consumption predominates over its production. As a consequence, the depletion of organic matter is inhibited and the VFA concentration decreases to a stable value indicating an equilibrium state. In other words, the VFA production rate becomes equal to its consumption rate as also shown by McDougall [43].

Table 1
Summary of the hydraulic parameters.

References	Parameters	Values	Units
Hydraulic parameters			
Manassero et al. [42]	k_{sat}	$[10^{-8};\ 10^{-4}] \Rightarrow \mathbf{10^{-5}}$	[m/s]
Olivier and Gourc [50] Staub et al. [62]	n	[0.48; 0.51] ⇒ **0.5**	[–]
Van Genuchten's formulation			
Feng and Zhang [22]	S_{res}	0	[–]
	S_{sat}	1	[–]
	α_{vg}	10	[kPa]
	m_{vg}	0.75	[–]
	n_{vg}	4	[–]

Table 2
Summary of the biodegradation reaction parameters.

References	Parameters	Values	Units
Transport parameters			
Domenico and Schwartz [17]	d_{lat}	0.002	m
	d_{long}	0.02	m
Cooke and Rowe [16]	d_{mol}	1.73610^{-9}	m^2/s
McDougall's formulation parameters			
McDougall [43]	b	0.029	$\left[g/m^3_{aqueous}\ s^{-1}\right]$
	Org$_0$	300,000	[g/m^3]
	ξ	0.36	[–]
	k_{VFA}	0.0002	[$m^3_{aqueous}/g$]
	Y	0.08	[–]
	k_0	$5.7 * 10^{-6}$	[s^{-1}]
	k_{MC}	4200	[$g/m^3_{aqueous}$]
	k_2	$2.3 * 10^{-7}$	[s^{-1}]
	Z	2.7	[–]

Table 3
Summary of constitutive law parameters.

Parameters	Values[a]	Units
λ	0.0648	[–]
κ	0.00792	[–]
a	3.45	[–]
$OCR = \frac{p_0}{p}$	1.01[b]	[–]
ρ	1000	[kg/m^3]
c	20	[kPa]
φ	35	[°]

[a] The values of the parameters have been assessed to fit to the response of the Sowers' model [59].

[b] The OCR is taken just over 1 to account for the compaction of the waste during the burial.

Table 4
Summary of parameters of the thermal properties.

References	Parameters	Values	Units
Yoshida et al. [69]	c_s	1939	[J/kg K]
	ρ_s	1000	[kg/m^3]
	c_w	4185	[J/kg K]
	ρ_w	1000	[kg/m^3]
	c_a	1004	[J/kg K]
	ρ_a	1.2	[kg/m^3]
Boukpeti (2004)	Q_m	632	[kJ/kg]

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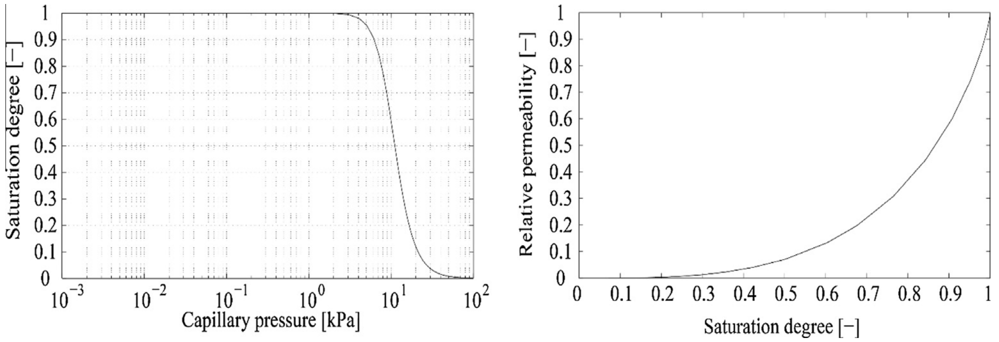

Fig. 8. Water retention and relative permeability curves.

Fig. 11 presents the evolution of the organic matter content profile with elapsed time. We can observe that the organic matter content progressively decreases with the injection of water. Almost two third of the organic content is degraded over the period of 15 years, which is half of life span of a typical bioreactor landfill. The organic content profile can be easily explained considering the saturation degree profile. The degradation is initially more important at the bottom of the column, where the water content remains high for longer period (up to six months). Then, after the hydraulic steady state is reached, since the biodegradation phenomenon is dependent on the remaining organic content through the digestibility term in Eq. (1), the degradation is slightly more intense at the top of the column.

Fig. 12 presents the evolution of the temperature profile with elapsed time. It can be observed that the temperature increases in the zone where the organic content decreases and reaches almost 70 °C near the bottom of the column. The temperature tends to be higher near the bottom of the column because of the thermal convection effect due to water infiltration. In the long term, the temperature starts to decrease because the available organic content is decreasing, which in turn inhibits the biodegradation process.

Fig. 13 presents the evolution of the landfill settlement profile with elapsed time. The displacements are related to the biodegradation of the organic matter, which modifies the mechanical properties of the MSW and induces bio-chemical softening, resulting in an increase in settlement under constant load. The final settlement is considerable and reaches over one meter in 15 years. One peculiar feature is that the wastes initially swell, due to the decrease in water content and in turn the bulk density of the medium.

That value may not seem that important compared to value obtained by other models, some settlement reaching 50% of the original fill thickness [47,59]. One of the explication to the difference overserved comes from the fact we consider the initial load induced settlement (filling phase) has already occurred and we only consider post closure behavior. Also, the vertical strain is very dependent on the set of parameters chosen (water and organic matter content, injected flux, etc.) and the one used for the reference simulation is very generic and based on the experimental results from several different studies. If the goal was to model a chosen landfill, the parameters would have to be fitted based on the composition of said landfill.

5.2. Sensitivity study

As aforementioned, the sensitivity analysis was performed only on different water retention curves and water injection flux, which are presented, respectively.

5.2.1. Sensitivity to the injected flux

The effect of the injected flux on the global behavior is obvious. This latter flux greatly modifies the hydraulic equilibrium at steady state and thus the importance of the biodegradation process. This simulation was conducted with an injected flux of $q = 2.510^{-4}\mathrm{kg/m^3\ m/s}$. All other parameters were kept identical to the reference simulation. In our simulations, water evaporation at the top boundary is not taken into account although this phenomenon can be modeled with specific boundary condition element [23].

The higher injected flux results in a saturation degree shifting from 60% to an equilibrium position around 37%. The results obtained are shown in Fig. 15. Since, the moisture content at the equilibrium is higher, it can be predicted that the degradation process will be more intense. Indeed, the effect of the biodegradation is impressive: almost all of the organic matter is degraded after 15 years (Fig. 15b); and the settlement exceeds 2 meters (Fig. 15d). The temperature actually reaches a lower maximum value than the one obtained in the reference case (Fig. 15c). This is caused by the heat convection effect which, related to a higher water flux, is more predominant.

5.2.2. Sensitivity to the water retention curve

Since the hydraulic conditions have a dominant effect on the whole THBCM process, the choice of the water retention curve is not trivial. In order to conduct the sensitivity analysis on the WRC, several WRCs collected from the literature were adopted, of which all the model parameters are summarized in Table 5 and the corresponding WRCs are also presented in Fig. 16.

The curve used in the reference simulation is curve A. Since this latter curve predicts very low saturation degree for capillary pressures higher than 40 kPa, the sensitivity study has been conducted using the curve H. With this curve the water saturation remains higher than 80% for the same range of capillary pressures. Otherwise, every other parameter is kept to the reference simulation value.

We can observe that the saturation goes from the initial condition at 86% to an equilibrium position around 97% (Fig. 17). The injected water flux is the same as the one used in the reference simulation while the retention curves are different. In consequence, at an equilibrium state, the saturation degree is much higher in case of using the WRC-H. This is not surprising when comparing the two retention curves, showing that the material with the WRC-A has a much lower water retention capacity than the others. Since the biodegradation process is directly linked to the moisture content, it is expected to be much higher in case of using the WRC H. In fact, the degradation process is directly linked to the effective saturation degree:

$$S_e = \frac{S_{R,w} - S_{Res}}{S_{Sat} - S_{Res}} = 0.95 \qquad (62)$$

The “usable” effective saturation is thus of 95% instead of 97% and the effect of the retention curve is just a little bit less direct than the injected flux. The other results show, as one could guess,

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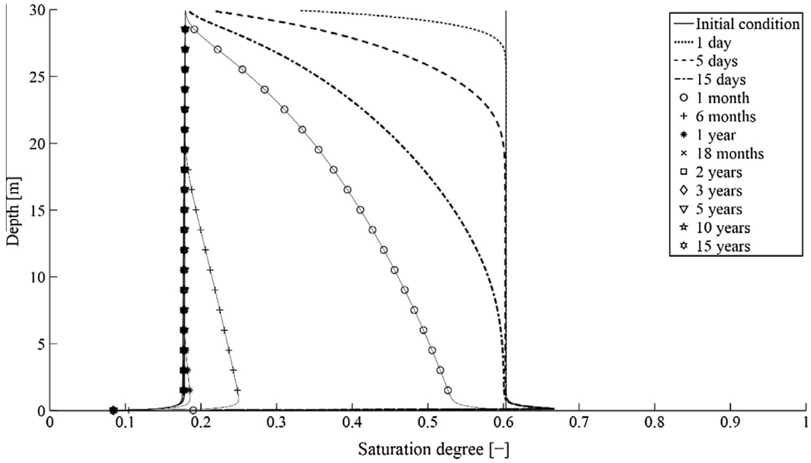

Fig. 9. Evolution of the water saturation profile with time.

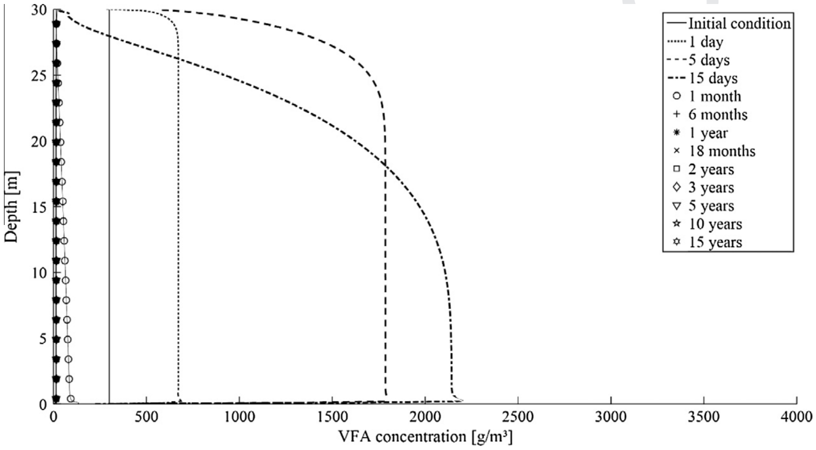

Fig. 10. Evolution of the VFA concentration profile with time.

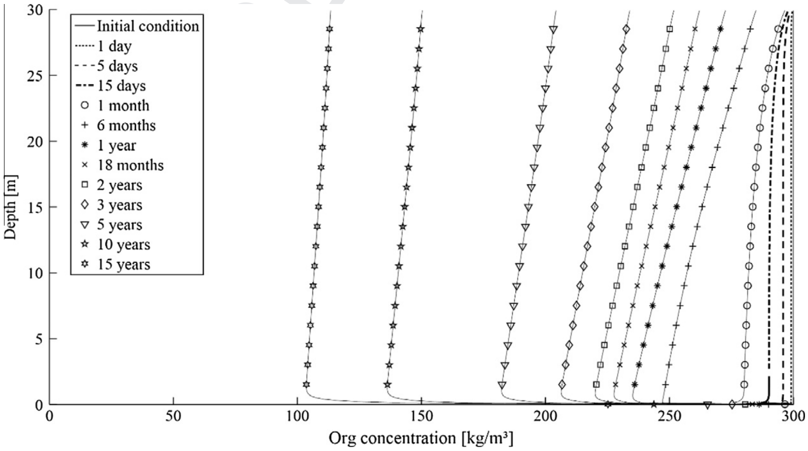

Fig. 11. Evolution of the organic content profile with time.

a biodegradation phenomenon more intense than the one exhibited in the reference simulation (Fig. 18). The global behavior is similar to the one explained for the reference simulation but the scale of it is very different. The initial VFA increase reaches much higher values (more than twice as much) (Fig. 18a), all of the organic matter is consumed after less than 5 years (Fig. 18b), temperature reaches 90 °C (Fig. 18c) and the settlement almost reaches 2.5 m (Fig. 18d).

After just slightly less than 5 years, all the organic matter is completely degraded and thus the temperature starts to significantly

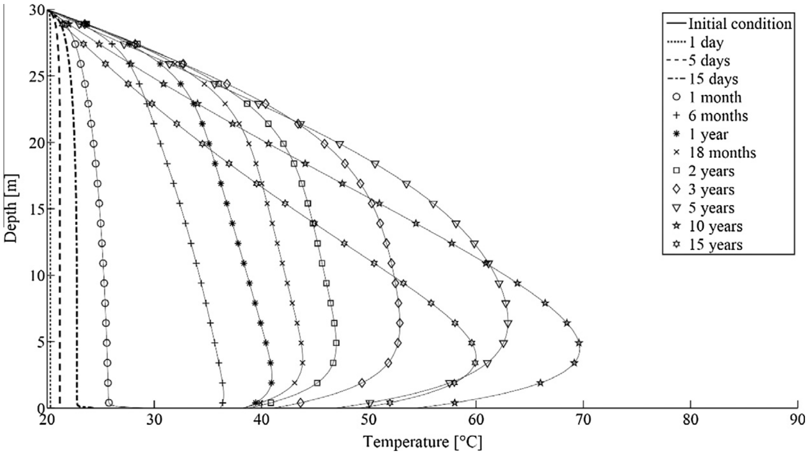

Fig. 12. Evolution of the temperature profile with time.

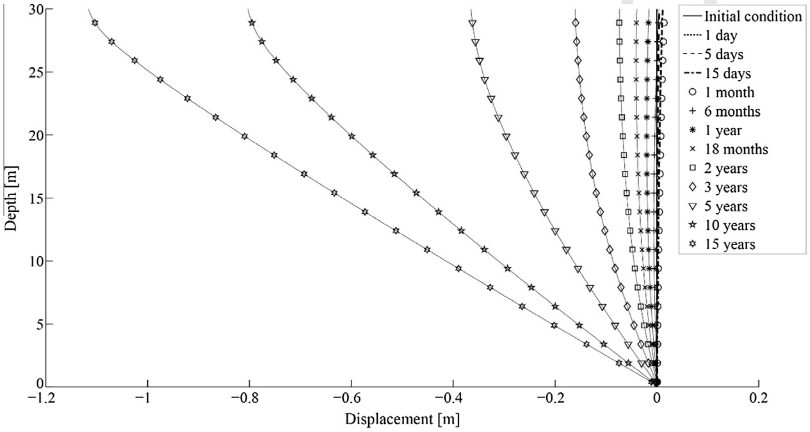

Fig. 13. Evolution of the vertical displacement profile with time.

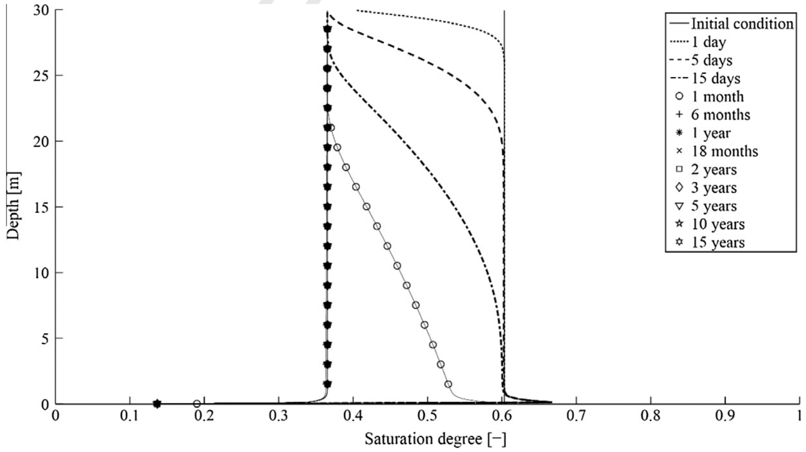

Fig. 14. Evolution of the water saturation with the injected flux $q = 2.510^{-4}\,\mathrm{kg/m^3\ m/s}$.

drop since the heat loss becomes dominant while no heat production takes place. At the same time, the settlement induced by the degradation of organic matter also tends to stabilize.

5.3. Comparison of the different simulations

Figs. 19 and 20 compare the evolution of main variables with elapsed time at the middle of the waste column (15 m in depth as shown in Fig. 6) obtained from different simulations previously displayed: (Fig. 19) degree of saturation, (Fig. 20a) VFA concentration, (Fig. 20b) organic matter content, (Fig. 20c) temperature and (Fig. 20d) displacement. Note that compared with the reference simulation, the modified water injected flux simulation used a 10 times higher flux value and the modified WRC one adopted a water retention curve H (Fig. 16) leading to a much higher degree of saturation at the equilibrium state, while keeping all the other param-

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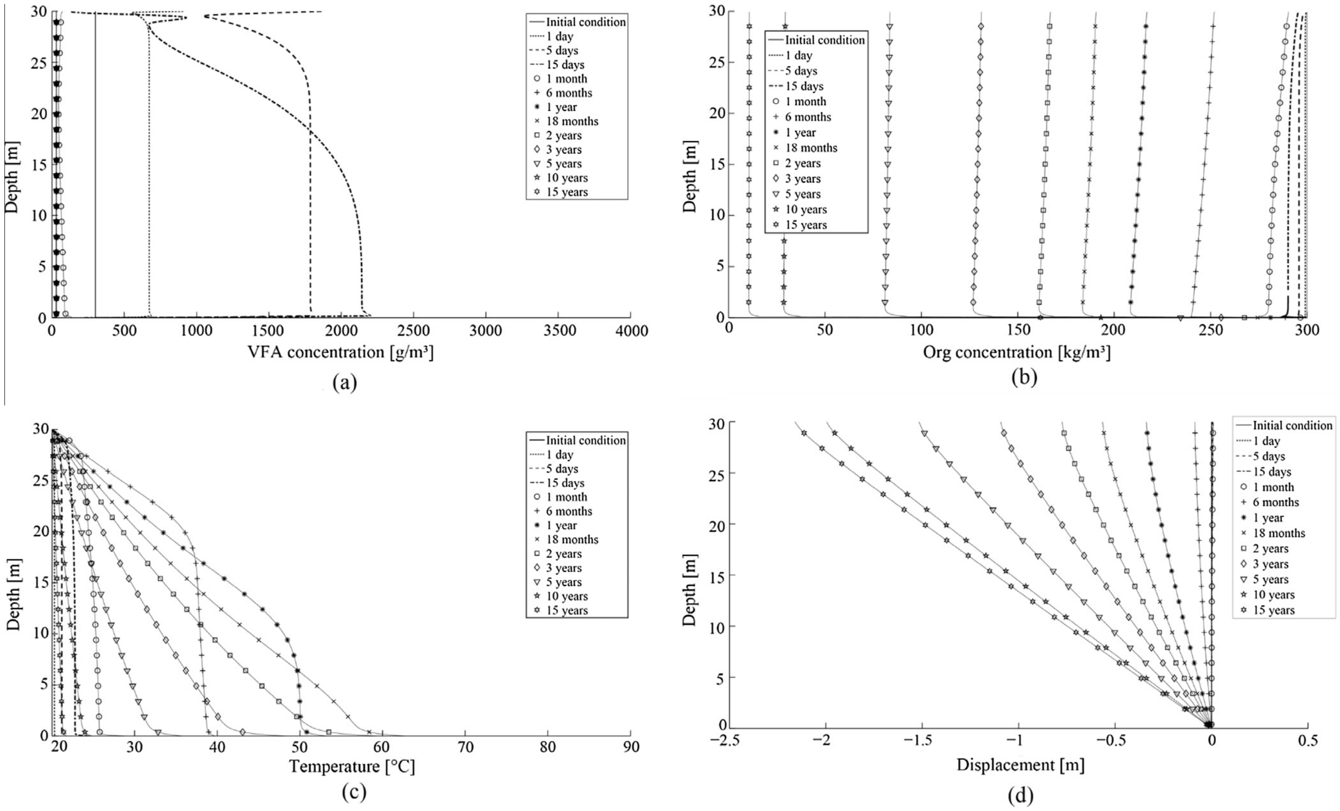

Fig. 15. Simulation results for different injected water flux: (a) VFA concentration, (b) organic concentration, (c) temperature and (d) displacement.

Table 5
Summary of parameters for water retention curves.

Curves	References	S_{res}	α [kPa]	n_{vG}
A	Feng and Zhang [22]	**0**	**10**	**4**
B	Stoltz et al. [63]	0.29	3	1.64
C		0.32	2.9	1.56
D		0.34	2.3	1.47
E		0.44	0.57	1.33
F		0.55	2	1.33
G		0.27	3	1.19
H		**0.29**	**2**	**1.12**
I		0.22	3.5	1.37
J	McDougall [43]	0	1.4	1.6

eters the same. The reason for choosing the given depth is that the effects of enhanced water injection and modified WRC on the performance of waste column are similar at different depths for different simulation cases. The behavior of most results is almost homogeneous with depth, only a small time lag, as can be seen on the previous results.

Compared with the reference simulation, it is expected that the increase in water injection flux result in an increase in degree of saturation of waste column at the steady state and a decrease in the time required for reaching the steady state (Fig. 19). In contrast to the other two cases, using the WRC-H led to a significant increase in degree of saturation, which is close to 1.0 at the steady state (Fig. 19). The difference in the evolution of degree of

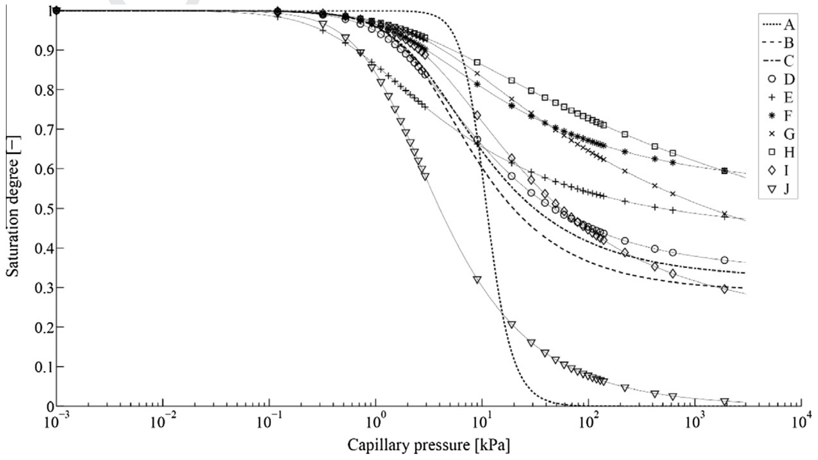

Fig. 16. Water retention curves reviewed.

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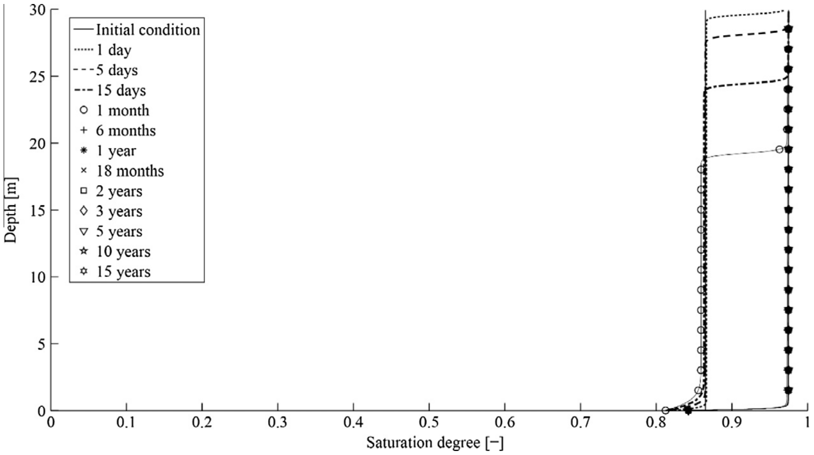
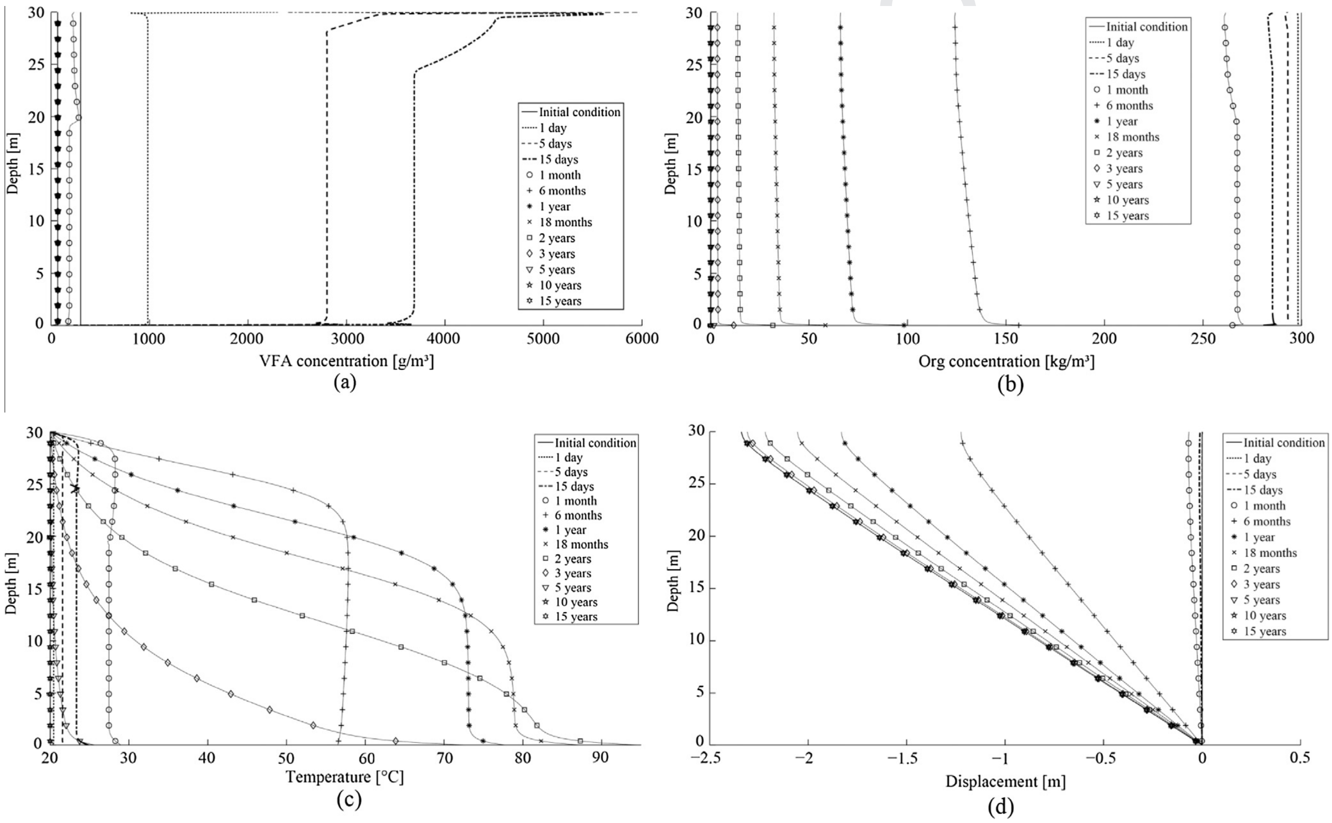


Fig. 17. Evolution of the water saturation profile with the retention curve J.



(a) (b) (c) (d)

Fig. 18. Sensitivity to the chosen retention curve: (a) VFA concentration, (b) organic matter concentration, (c) temperature and (d) displacement.

saturation between those simulations directly translates to the difference in their long-term behaviors. As expected, the higher the degree of saturation of waste column, the faster the degradation of organic matter takes place (Fig. 20b). In consequence, the settlement of waste column occurs more rapidly (Fig. 20d). It can be noticed that both the decrease in organic matter concentration and the increase in displacement follow a steep curve in case of the modified WRC simulation, whereas they exhibit a progressive evolution in the other two simulations. They also all exhibit an increase in degradation and settlement rate at the very beginning of the simulation which correspond to the decrease in VFA concentration (after the pike value) and the decrease in inhibitory effect induced by it.

In contrast, the VFA concentration and temperature evolve in two-stages: an initial increase phase followed by a decreasing phase towards stable values (Fig. 20a and c). These results can be explained by the different mechanisms governing the degradation and transport process. For the former, the initial increase is due to the early stage of considerable bio-degradation reactions, which mainly depend on the initial degree of saturation. The higher degree of saturation resulted in more intense bio-degradation of organic matter and in turn in more important production of VFA. Furthermore, the accumulation of VFA led to its high concentration, leading to the considerable transformation of VFA to methanogen biomass and the important inhibitory effect on VFA consumption. In consequence, the VFA consumption predominates

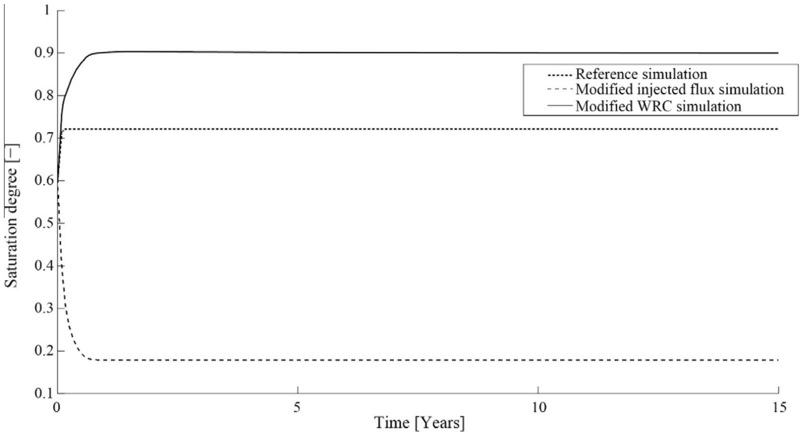


Fig. 19. Comparison of the saturation degree for the three different simulation cases.

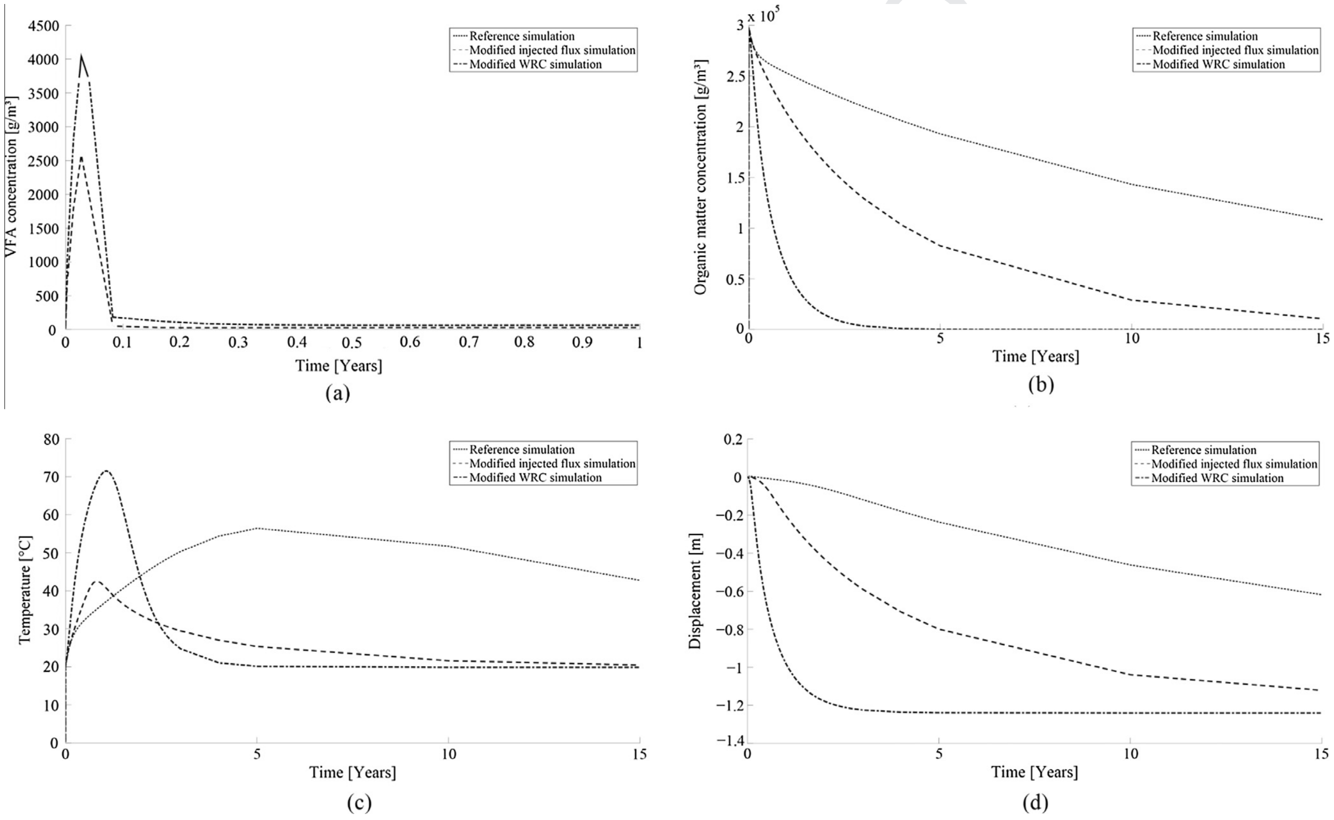


Fig. 20. Comparison of the temporal variation of main variables at the middle of waste column between three different simulation cases: (a) VFA concentration, (b) organic matter concentration, (c) temperature and (d) displacement.

over its production, resulting in the subsequent decrease in VFA concentration until an equilibrium state at which the VFA production rate is equal to its consumption rate as shown in Fig. 7a.

When looking at the evolution of the temperature in the reference simulation, it shows a progressive increase phase up to 5 years followed by a gradual reduction phase until the end of simulation time (Fig. 20c). The former phase is directly related to the bio-degradation of organic matter, which is responsible for heat production. The latter phase suggests that the heat transport through water convection and thermal diffusion slightly predominates over its production in long-term. This temperature evolution can be significantly modified by either increasing water injection flux or using a different WRC in the simulation (Fig. 20c), which dramatically enhanced the bio-degradation. In the case of the increased water injection flux, it is also followed by a huge increase in heat convection leading. The maximum temperature occurs when one third of the initial organic matter concentration has been degraded. This demonstrates that despite of continuous occurrence of bio-degradation reaction producing heat energy, the heat production rate is less important than the heat loss through water convection and thermal diffusion, leading to a progressive decrease in temperature. In contrast, for the reference simulation and the modified water injection simulation, the maximum temperature closely corresponds to the complete degradation of organic matter.

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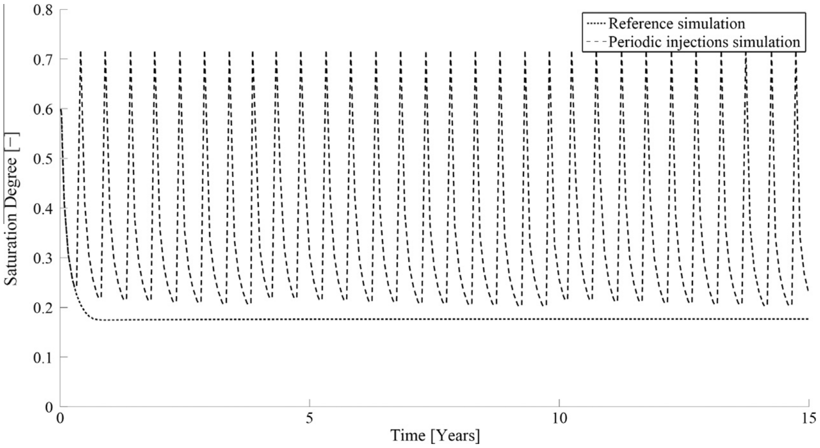

Fig. 21. Evolution of the water saturation profile with time.

This means that the subsequent reduction in temperature is mainly due to the heat loss while no considerable heat is produced, which is related to the very low availability of organic matter.

5.4. Operation optimization

To show the potential capability of the proposed model for optimizing the operational management of landfill, a fourth simulation is performed with periodic injection of water while keeping all the other parameters identical to the reference one. A water flux of $2.510^{-3}\ \mathrm{m^3/s}$ is injected during a whole month, every 6 month (as shown in Fig. 21). The variations of main variables at the middle of waste column with elapsed time are all summarized in Fig. 22, including VFA concentration (a), organic matter concentration (b) temperature (c) as well as displacement (d).

In contrast to organic matter concentration and displacement, the VFA concentration evolves in a similar pattern as the degree of saturation (Fig. 22a). It exhibits a very high first peak followed by much lower peaks. The first peak mainly depends on the initial degree of saturation while the following lower peaks can be due to the important transport phenomena through water convection associated with the significant injection flux. At equilibrium state, the VFA production and consumption rate at peaks remain rela-

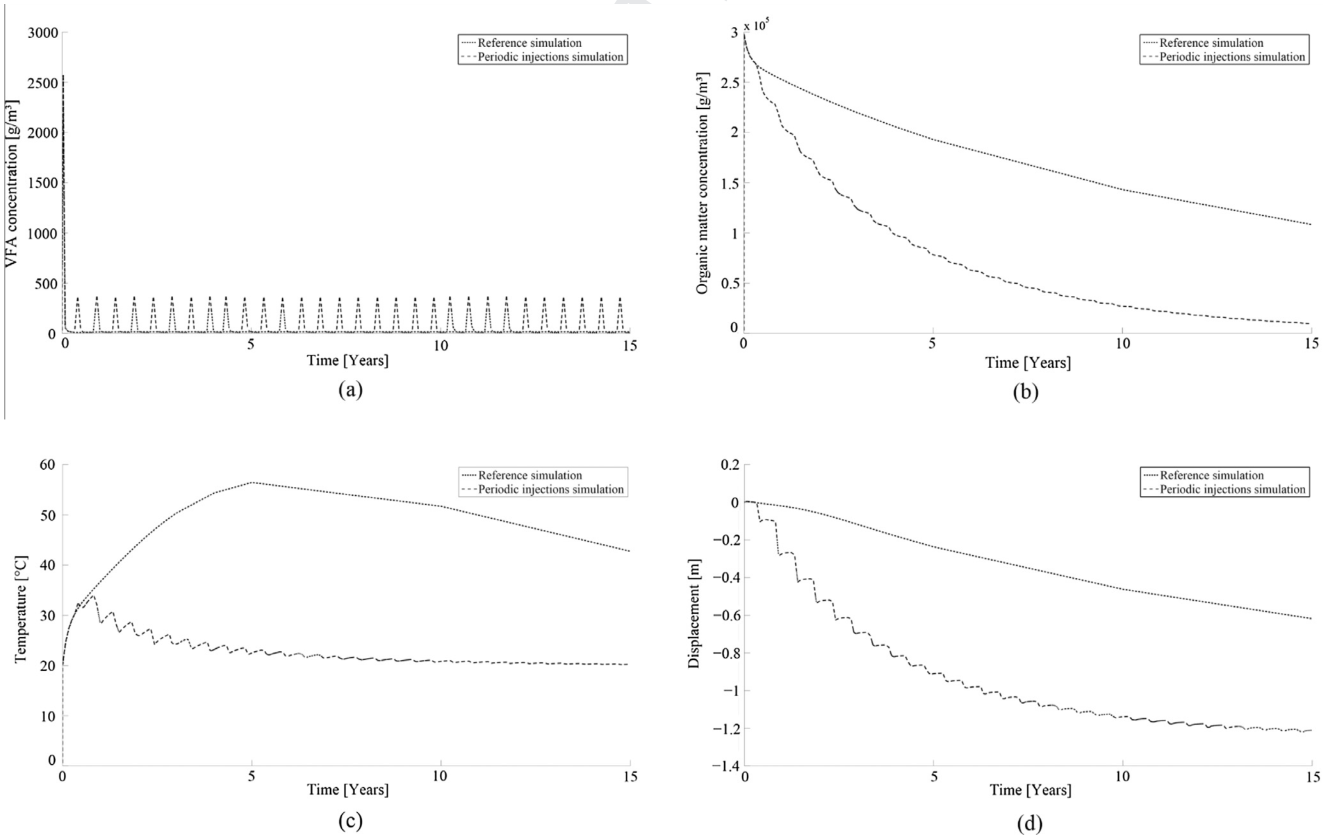

Fig. 22. Comparison of the temporal variation of main variables at the middle of waste column between three different simulation cases: (a) organic matter concentration, (b) displacement, (c) VFA concentration and (d) temperature.

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tively constant, leading to quasi constant peak values. Looking at the temperature evolution (Fig. 22c), it presents a first increasing phase, corresponding to the first peak degree of saturation (Fig. 21), which is followed by a progressive decreasing phase with certain degree of oscillation. Again, this evolution can be explained by the predominance of heat production and dissipation each other. At the beginning of water injection, the heat production predominates over its dissipation, leading to a significant increase in temperature. With the continuous bio-degradation reaction, the heat dissipation due to the important water convection and thermal diffusion becomes more important in long-term.

The results displayed in Fig. 22 clearly present the potential to determine the period after which water injection is not worth it anymore. This may help to decide if we need to stop water injection. In practice, using the proposed model, we can perform different simulations with different water injection dosage and frequency to minimize the effort required for landfill management. Using the predicted settlement, we may also determine the suitable time to close the landfill. This suggests that the proposed model can be a good tool for optimizing the landfill management, allowing for shorter maintenance period and thus less expensive operation.

6. Conclusion and perspectives

This paper has presented the formulation of a thermo-hydro-bio-chemo-mechanical model for the study and analysis of long term behavior of bioreactor landfill. The main focus of this study was the assessment of the long-term settlement associated with different leachate/water injection, in order to optimize the operation and the management of bioreactor landfills.

The particularity of the formulation for the THBCM model is the multi-physics coupling accounting for the inherent complexity of the problem involved in bioreactor landfills. On the basis of the multi-physics (thermal-hydro-mechanical-chemical) modeling framework developed at University of Liege over the last three decades, the effort was made to incorporate the two stage anaerobic biodegradation process into the framework. To do so, a two stage biochemical model proposed by McDougall [43] was adopted as a biodegradation sub-model to describe the depletion of organic matter associated with the production of VFA and MB. The biodegradation effect was furthermore taken into account into a simplified chemo-mechanical model developed by Hueckel [33] in order to predict the settlements induced by both mechanical and biochemical effects. This was achieved by linking the parameter controlling chemical softening to the ratio of organic matter content after degradation relative to initial content. In addition to that, the hydraulic model was based on Richard's equation for water flow in unsaturated soils, whereas the thermal model is a classical energy balance equation with a source term taking into account the heat generation during the biochemical reaction.

The implementation of the formulation in a FEM code was thoroughly explained and all the numerical algorithms used were introduced. In particular, to implement the biodegradation model into the in-house built multi-physics FEM code, a sub-stepping approach was adopted to overcome the numerical problems related to a large magnitudes variation of timescales associated with the transport phenomena and biochemical reaction.

Finally, numerical simulations were conducted on a one-dimensional problem encountered in a synthetic bioreactor landfill in order to show the performance and capability of the proposed THBCM model. The simulation results showed that the model is capable of reproducing the biodegradation effects on the long term settlement of MSW and on the heat generation. Further sensitivity analysis was performed on water retention curve and injected water flux. Results demonstrated the potential of the developed model to be used to optimize the operation of a bioreactor landfill, for which the main controlling parameter to be designed is the estimation of dosage and frequency of water or leachate to be injected into the landfill and of the prediction of the associated long term settlements of MSW.

The results from the periodic water injection simulation suggest that the model could actually be used to optimize landfill management. Based on those results, it is possible to determine the optimal water injection frequency and dosage to shorten the degradation process. The settlement obtained using the predicting model can also be used to determine a suitable time to close the landfill. All those allow for shorter monitoring period and more efficient management leading to space and monetary gain.

Going forward, there are still a few improvements that could be brought into the proposed model. One of them is to consider the effect of the temperature on the degradation kinetic. Indeed, there are favorable temperatures at which the degradation of organic matter is more intense [39]. In addition, it would be interesting to take into account gas production and transport for purpose of energy extraction.

7. Uncited references

[9,35,45,46,48,70].

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