FINITE ELEMENT ANALYSIS OF A COMPOSITE CATAMARAN

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THEME

Composite - Damage & Failure Criteria

SUMMARY

In this paper, the catamaran of the AEROFLEET company is analyzed with the SAMCEF finite element code. The two hulls, the three crossbeams connecting these two hulls and the mast are made of materials like glass/epoxy, carbon/epoxy and sandwich constructions.

In this paper, it is explained how CAD and CAE tools are used to evaluate the stiffness and strength of this very particular composite structure. Proposals for the improvement of the structural design are also discussed.

First, specificities of the geometry, such as transverse panels used to stiffen the overall structure as well as the hull’s floor, are defined. Those make the catamaran acting like a composite box structure. The relevant load cases are discussed and modeled. The best way to determine the boundary conditions is also studied. The structural analysis is then carried out, first with a linear approach. This allows to identify the critical zones based on classical Tsai-Wu and Tsai-Hill criteria. The results validate the current design, and possible improvements for future catamaran constructions are discussed.

KEYWORDS

Composite, finite element linear analysis, fracture criteria, sailing boat, meshing.

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1: Introduction

Nowadays, the composite materials are increasingly used for structures due to the significant ratio between stiffness and weight. Consequently, the engineers need numerical tools to optimize and design the composite structure. This paper presents some results of the VirtualComp project of the Walloon Region in Belgium. The objective is to develop and validate new numerical tools to design some composite aeronautical structures and to model manufacturing process. In this project, the developed solutions are applied on industrial structures.

In this context, an ecological catamaran is developed. The advantage of light weight with a significant strength enables to justify the use of composite material. To reduce the weight and improve the strength of the sail, an optimisation tool is interesting to design the composite laminate and to size the shells. A first linear analysis is performed with the SAMCEF finite element code. Some classical fracture criteria are applied to show the strength of the structure. The analysis is more focused on the floats of the catamaran.

Firstly, the composite modelling is described with the failure criteria. In addition, the catamaran structure and meshing are presented. Then, this paper presents the modelling of a composite catamaran by finite element. Finally, the results of the analysis are discussed.

2: Behaviour law of composite material in Samcef

2.1 Elastic behaviour law of the laminate

The composite laminate is divided in n plies. Each ply, k, is defined by an orientation of the fibres, θ_k, the thickness, t_k and the orthotropic elastic material parameters described by the matrix $\underline{Q}$ in the orthotropic axis {xyz}:

$$\underline{\sigma}_{\{xyz\}} = \begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{Bmatrix} = \underline{Q}_{\{xyz\}}\underline{\epsilon}_{\{xyz\}} = \begin{bmatrix} mE_x & m\nu_{yx}E_x & 0\\ m\nu_{xy}E_y & mE_y & 0 \\ 0 & 0 & G_{xy}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix} \qquad (1)$$

$$\text{with } m = \frac{1}{1-\nu_{xy}\nu_{yx}} \text{ and } \nu_{yx} = \frac{\nu_{xy}E_x}{E_y}$$

where E_x is the Young’s modulus in the fibre axis, E_y is the Young’s modulus in the orthogonal fibre axis, G_{xy} is the shear modulus and ν_{xy} is the Poisson’s ratio.

The matrix $\underline{Q}_{\{123\}}$ components can be computed in a global axis {1 2 3}, with the components $\underline{Q}_{\{xyz\}}$in the orthotropic axis with the following relationship:

$$\begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{Bmatrix} = \begin{bmatrix} c^4 & s^4 & 2c^2s^2 & 4c^2s^2 \\ s^4 & c^4 & 2c^2s^2 & 4c^2s^2 \\ c^2s^2 & c^2s^2 & c^4+s^4 & -4c^2s^2 \\ c^2s^2 & c^2s^2 & -2c^2s^2 & (c^2-s^2)^2 \\ c^3s & -cs^3 & cs^3-c^3s & 2(cs^3-c^3s) \\ cs^3 & -c^3s & c^3s-cs^3 & 2(c^3s-cs^3) \end{bmatrix} \begin{Bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{xy} \\ Q_{ss} \end{Bmatrix} \tag{2}$$

$$\text{with } \mathrm{c} = \cos(\theta_\mathrm{k}) \text{ and } \mathrm{s} = \sin(\theta_\mathrm{k})$$

The laminates are thin structures therefore a Mindlin’s theory (Mindlin, 1951) is sufficient to model their mechanical behaviour. The global strain of the laminate is divided by:

$$\underline{\varepsilon} = \underline{\varepsilon}^0 + z\underline{K} \tag{3}$$

Where $\underline{\varepsilon}^0$ is the in-plain strain, $\underline{K}$ is the curvature and z is the through-thickness position.

The efforts, $\underline{N}$, and the moment, $\underline{M}$, by length unit are linked with the global strain by:

$$\begin{Bmatrix} \underline{N} \\ \underline{M} \end{Bmatrix} = \begin{Bmatrix} \int_{-\frac{h}{2}}^{\frac{h}{2}} \underline{\sigma}_{\{123\}} dz \\ \int_{-\frac{h}{2}}^{\frac{h}{2}} \underline{\sigma}_{\{123\}} z dz \end{Bmatrix} = \begin{bmatrix} \underline{A} & \underline{B} \\ \underline{B} & \underline{D} \end{bmatrix} \begin{Bmatrix} \underline{\varepsilon}^0 \\ \underline{K} \end{Bmatrix} \tag{4}$$

where $\underline{\sigma}_{\{123\}}$are the stress in the ply in the global axis and the coefficients of the previous stiffness matrix are calculated by

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} Q_{ij}(\theta_k) t_k \\ B_{ij} &= \sum_{k=1}^{n} Q_{ij}(\theta_k) t_k z_k \\ D_{ij} &= \sum_{k=1}^{n} Q_{ij}(\theta_k) \left(t_k z_k^2 + \frac{t_k^3}{12} \right) \end{aligned} \tag{5}$$

Then one can come back to the orthotropic strain in each ply with the rotation matrix

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \tag{6}$$

And finally the orthotropic stress tensor in each ply can be computed with the equation (1).

2.2 Failure criteria

Failure criteria are used to extend the use of strength data measured from tensile, compressive and shear uni-axial tests to combined stress states.

The first classical one is the Tsai-Hill criterion (Azzi & Tsai, 1965). This theory defines a general “yield-criterion” for orthotropic material. In case of plane stress state, the formulation is:

$$\begin{gathered} \left(\frac{\sigma_x}{X}\right)^2 + \left(\frac{\sigma_y}{Y}\right)^2 - \frac{\sigma_x \sigma_y}{X^2} + \left(\frac{\sigma_{xy}}{T}\right)^2 \leq 1 \\ \text{with } \mathrm{X} = \begin{cases} \mathrm{X}^\mathrm{C} \text{ if } \sigma_\mathrm{x} \leq 0 \\ \mathrm{X}^\mathrm{T} \text{if } \sigma_\mathrm{x} > 0 \end{cases} \quad \mathrm{Y} = \begin{cases} \mathrm{Y}^\mathrm{C} \text{ if } \sigma_y \leq 0 \\ \mathrm{Y}^\mathrm{T} \text{if } \sigma_y > 0 \end{cases} \end{gathered} \tag{7}$$

where X^C ,Y^C are ultimate compressive stresses , X^T and Y^T are ultimate tensile stresses and T is ultimate shear stresses in orthotropic direction.

A second one is the Tsai-Wu criterion (Tsai & Wu, 1971). This criterion takes into account coupling between stress components:

$$F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + F_{66}\sigma_{xy}^2 + 2F_{12}\sigma_x\sigma_y \leq 1$$

$$with \qquad \begin{gathered} F_1 = \frac{1}{X^T} - \frac{1}{|X^C|} \; F_{11} = \frac{1}{X^T|X^C|} \\ F_2 = \frac{1}{Y^T} - \frac{1}{|Y^C|} \; F_{22} = \frac{1}{Y^T|Y^C|} \\ F_{66} = \frac{1}{T^2} \\ F_{12} = \frac{1}{2\sigma_{45}^2}\left[1 - \left(\frac{1}{X^T} - \frac{1}{X^C} + \frac{1}{Y^T} - \frac{1}{Y^C}\right)\sigma_{45} + \left(\frac{1}{X^T X^C} + \frac{1}{Y^T Y^C}\right)\sigma_{45}^2\right] \end{gathered} \tag{8}$$

where σ_{45} is the stress applied on a sample submitted to a tensile test along 45° of the fibre direction.

3: Modelling of the catamaran

3.1 Structures of the modelled catamaran

Figure 1 shows the geometry of the studied composite catamaran. The float of catamaran is 15 m long and the mast is 20 m height. The geometry is generated by the CAO software SolidWorks according to plans. This geometry is imported in Samcef Field to generate the mesh and to convert the data for the Samcef solver.

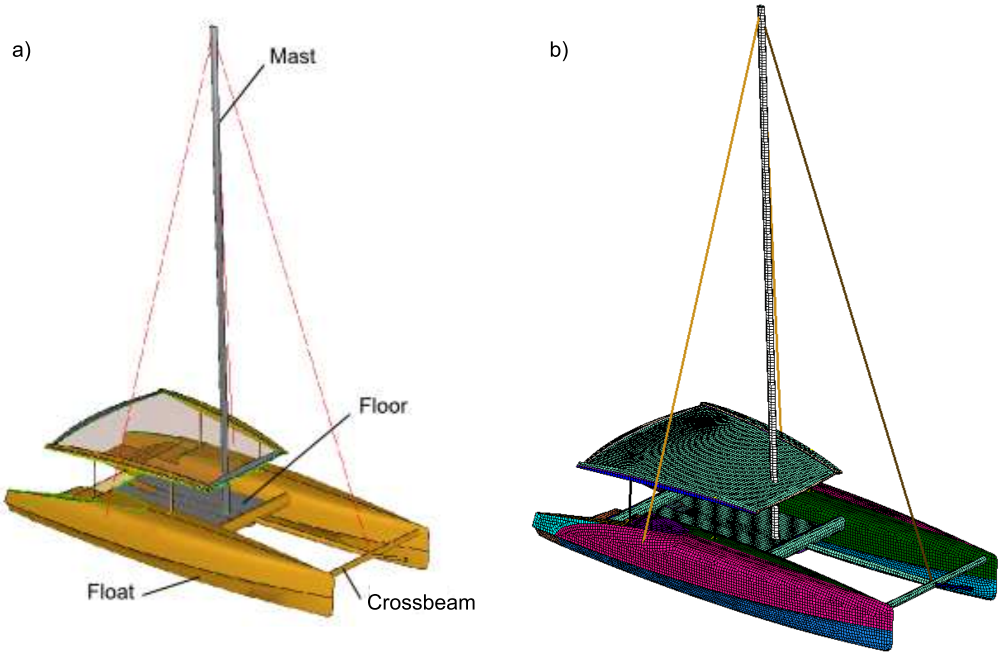

Figure 1: Presentation of the catamaran and its mesh

The study is more focused on the floats. To perform this, the float, the crossbeams and the mast's ropes are extracted to another model. In this model, the geometry was cleaned to generate the mesh. The cleaning means remove some gaps and simplify the geometry which doesn't affect the stiffness of the structure. Then, some structural elements which would affect the stiffness of the structure are added in the model. A floor has been added. Some horizontal boards are used for bed. Some stiffener panels are put along the float. Some zones of the float shells were delimited around the crossbeams. These zones are reinforced with a thicker composite laminate. The mast ropes are linked with the float by a steel plate. Epoxy foam is added at the head and the back of the float and behind the floor to avoid water intrusion in case of impact.

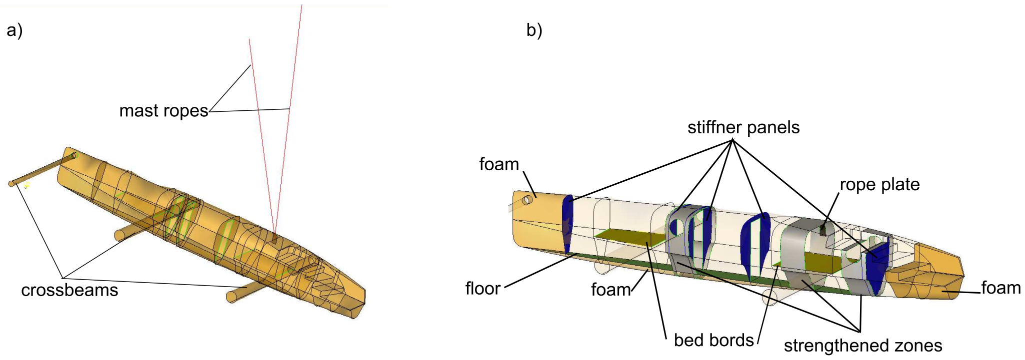

Figure 2: **Floats and its components**

The shells of the float are sandwich structures. The core is Airex linear and crosslinked PVC foams. The skin is composed of glass-epoxy. The crossbeams are made of carbon-epoxy. In the reinforced zone, the number of ply is higher than other parts. The used direction of the ply is 0° or 45°. The volume foam to protect water intrusion is epoxy foam.

3.2 Finite element model

The elements which compose the float are composite shell elements which use Mindlin's theory. The mast ropes are beam elements which are linked with the steel rope plate. The epoxy foam is model with volume 8 node elements. The model uses 51752 nodes and 52809 elements and so 332228 degree of freedom.

The loading is an acceleration applied along the mast on the mast ropes in order to apply a 7T tension. This level corresponds to critical force applied on the mast rope in service according to the catamaran manufacturer.

Three cases of boundary supports are studied (see Figure 3:) :

a) The crossbeams end nodes are fixed in all directions

b) The crossbeams end nodes are fixed in axial direction for displacement and rotation and the nodes of the mid-edge at the float bottom are fixed along the vertical and travel directions.

c) The crossbeams end nodes are fixed in axial direction for displacement and rotation and the nodes on the shell under the floor are fixed along vertical direction and advance direction.

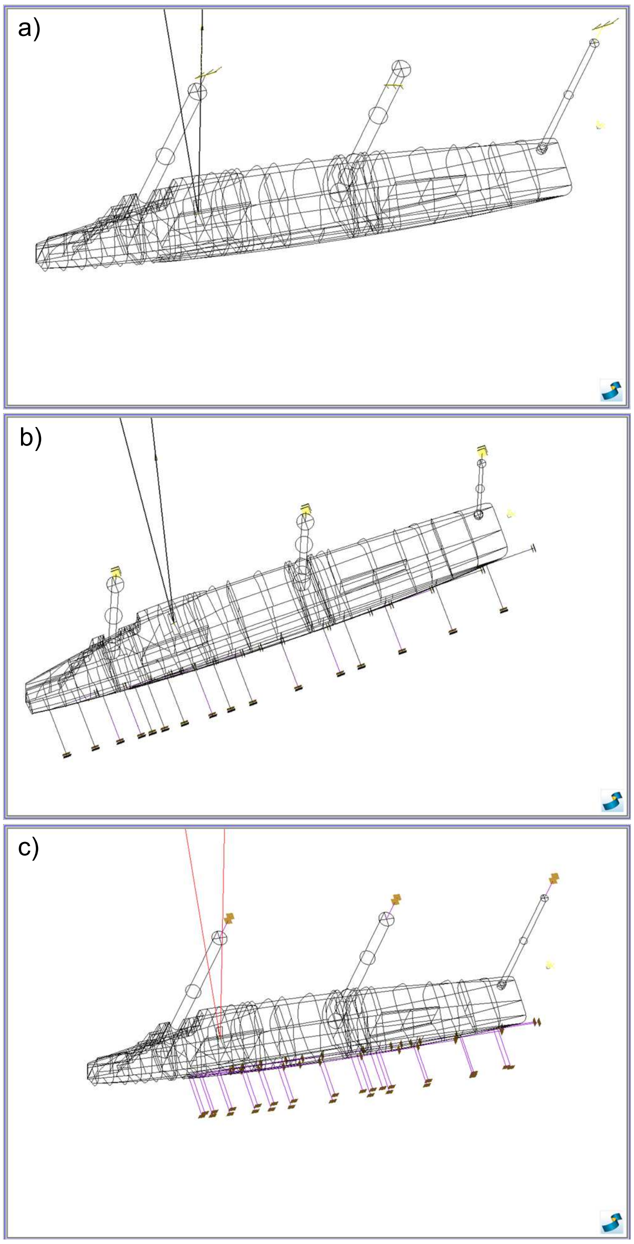

Figure 3: **Boundary supports studied**

The reality may be between these different cases.

The finite element analysis is linear. The stress field is computed. Then the post-processor applies the Tsai-Hill and Tsai-Hu criteria to check if a failure happens.

4: Results and discussion

In Figure 4 to Figure 9 the failure criterion is applied after the analysis on the float with the three kinds of support conditions. Tsai-Hill's criterion is inferior to Tsai-Wu because the formulation of the second criterion is two dimensional as contrary to the first one. The second kind of support condition shows a failure around the edge which is fixed. The cause is the support is local which is severe and not realistic. The third kind of support which is closest to the reality shows no failure happens because the support is spread on a surface and not a edge.

It can be noticed that the more solicited zone is around the mast rope plate as expected. The reinforced zone is very important to this zone.

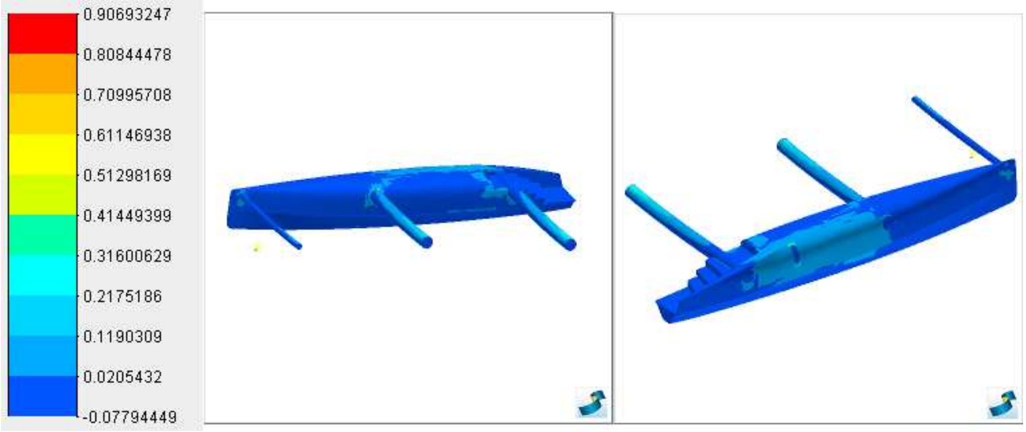

Figure 4: **Tsai-Wu's criterion with type (a) support**

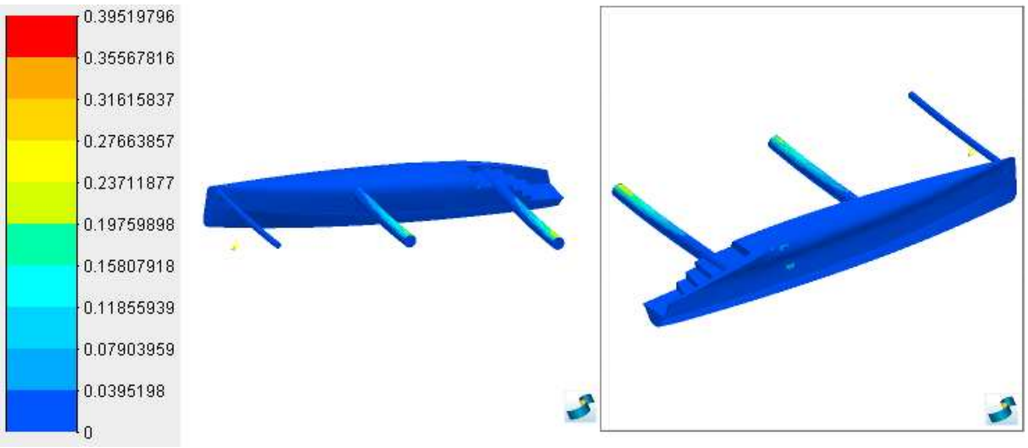

Figure 5: **Tsai-Hill's criterion with type (a) support**

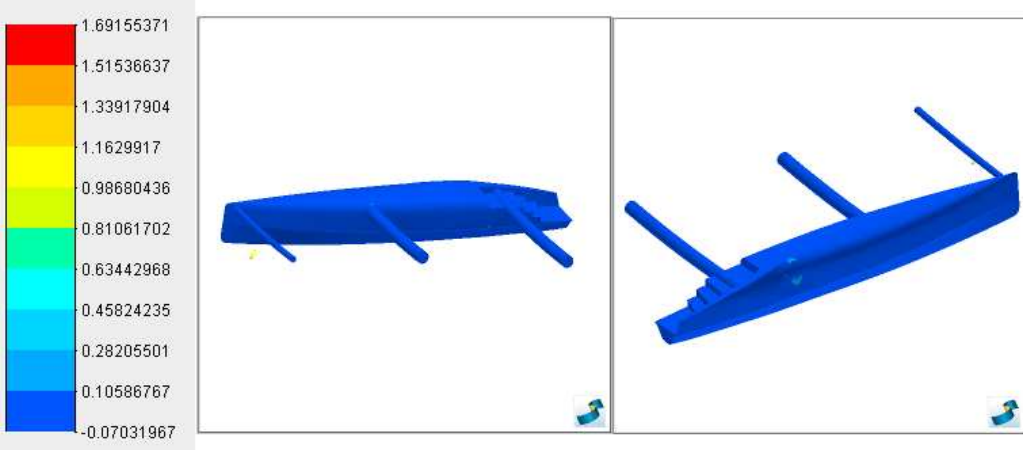

Figure 6: Tsai-Wu’s criterion with type (b) support

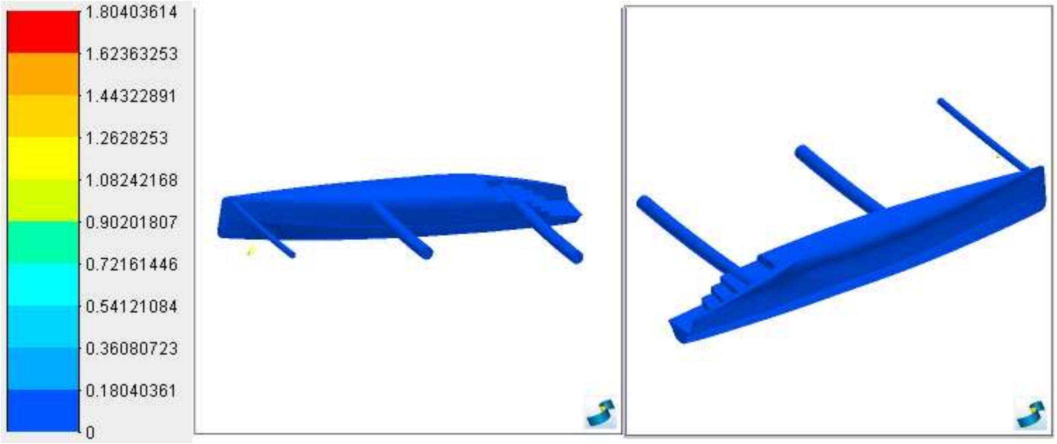

Figure 7: Tsai-Hill’s criterion with type (b) support

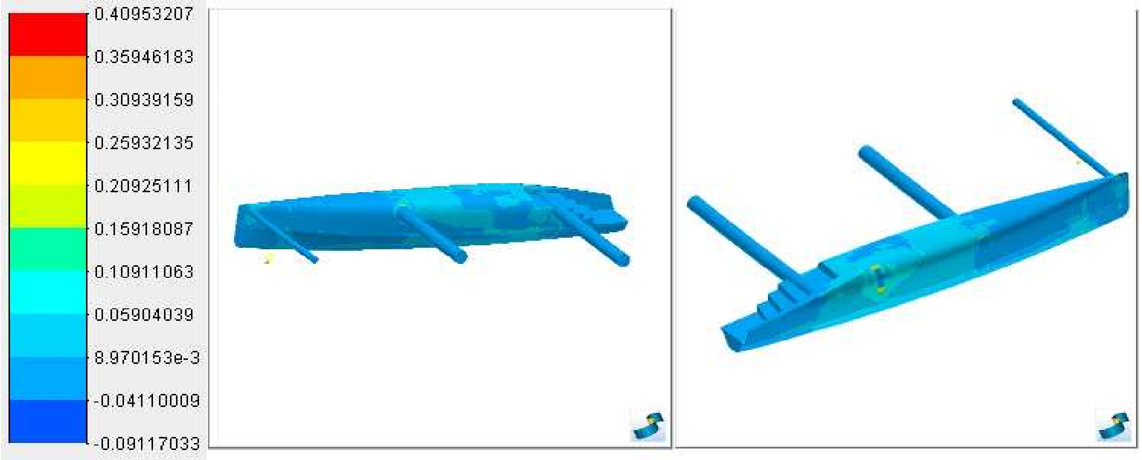

Figure 8: Tsai-Wu’s criterion with type (c) support

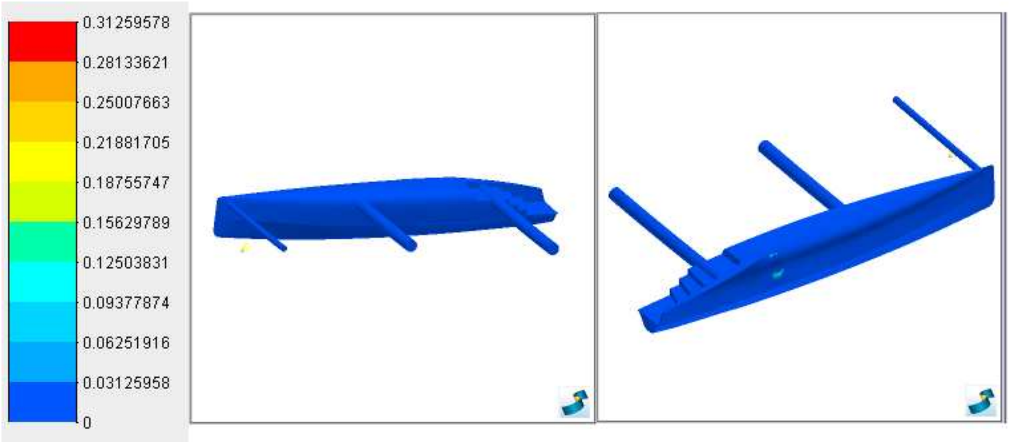

Figure 9: Tsai-Hill's criterion with type (c) support

5: Conclusions

The paper presents a linear analysis of a composite catamaran. This study opens the door for catamaran designers to reduce the weight and improve the strength of their product. They can vary the laminate by the number of ply, their fiber direction or their material components. They can add or remove some stiffener panels and observe the impact on the stiffness.

Moreover, the support conditions deserve to be improved to be closest to the real service situation. In our case, the float is link to the groove as contrary on the case with a catamaran on the water.

Moreover, the model of the failure can be improved. Samcef can model the continuous damage inside and between the plies. This kind of model can better preview the failure than static criterion. However, the models require non-linear analysis and more material parameters.

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