MONITORING THE QUALITY LOSS PERFORMANCE OF PRODUCTS

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Abstract: This paper presents a control chart based on the Taguchi (1986)'s loss function for monitoring the performance of a process and its capability in monetary form. The proposed chart monitors the loss due to poor quality to the society. The proposed control chart also allows for simultaneously monitoring both changes of the mean and variance.

Keywords: Taguchi Loss Function, Quality Cost, Shewhart Control Chart.

1. INTRODUCTION

Many quality characteristics (Q.Ch.s) are expressed in terms of original or derived measurement units, like weight, length, pressure etc. for which they are called continuous or variable. As normality is a usual assumption of control charts with continuous Q.Ch.s and independency of mean and variance is a basic assumption of normal distribution, separate control charts are prepared for monitoring the process average and process variation.

When the distribution of the process Q.Ch.s is defined as the normal distribution, we conventionally combine the $\overline{X}$ control chart and either the R or S control chart for monitoring the shift in the process mean and the changes in process variance (Faraz and Moghadam, 2006).

However, in process control it is important to acknowledge that a process can be in a state of statistical control and yet performing poorly if the mean of a process characteristic is not equal to the target value. For example, assume that a particular process characteristic should have a value of 12.0 for a product to function optimally, and the process appears to be in statistical control with $\overline{\overline{X}} = 13.3$. Having plotted points fall within 3-sigma limits is simply not good enough when process mean is far away from the target value. In traditional quality control systems, a product is accepted if its measurement meets specification requirements. At this stage it is assumed that there is no quality loss in most economic models of control and quality losses tend to be constant if the product measurements go beyond the quality specification limits.

Taguchi (1986) suggests that any deviation from a characteristic's target value results in a loss. If a characteristic's measurement is the same as the target value, the loss is zero. Otherwise, the loss can be measured using a quadratic function and actions need to be taken to systematically reduce the deviation from the target value. Till now, these said control charts consider the variation of process parameters only and ignore specification limits. Therefore, the only information that can be drawn from these charts is when detection and correction actions are to be taken. Also, process capability studies are further conducted to examine whether the process is capable of producing high quality products or not.

Based on Taguchi's philosophy of social loss of quality, there is a loss associated with any variation of Q.Ch.s distribution around their target value. Therefore, companies not only should manufacture Q.Ch.s within specifications, but also should do their best to reduce the variability of Q.Ch.s around their target values, so that they can raise their competitiveness in domestic and international markets. According to this approach and customers' view, the in-control status is valuable when it can meet customers' needs. Therefore, in statistical process control, to determine whether an in-control state of a process satisfies the customer's needs, process capability studies are carried out. In this regard, there may be in-control processes that due to low capability are not able to meet customers' needs and hence customers' satisfaction. Thus, the need of having control charts that can combine the above two approaches arises. Classical control charts should be modified so that they include this approach and thus force the process to meet customer satisfaction.

In this paper, we introduce a special control chart based on Taguchi's view of quality and his quality loss function. The proposed chart extends Taguchi's philosophy of social loss of quality from off-line to on-line activities.

2. The Control chart for monitoring the Quality loss function

Based on Taguchi's philosophy, the social loss incurred by a product is associated with any variation of a Q.Ch. under study around its target value. This variation which is due to effects of noise factors is monitored through a probability distribution. So, a reduction of dispersion of the distribution around the target value of a Q. Ch. is equivalent to higher quality and lower social loss. The Taguchi loss function for the Q.Ch X is calculated as follows:

$$L(X) = k(X - T)^2 \quad (1)$$

where k is a positive coefficient, and T is the target value. The expected quadratic loss (henceforth, EQL) under this loss function (for the i-th sample of size n) is given by

$$\text{EQL}=\text{E(L(X))} = kn[\sigma_i^2 + (\mu_i - T)^2] \quad (2)$$

If the process parameters are not known, they should be estimated based on a sample of size n as follows:

$$\hat{\mu}_i = \bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \quad (3)$$

$$\hat{\sigma}_i^2 = \frac{1}{n}\sum_{j=1}^{n}\left(x_{ij} - \bar{X}_i\right)^2 \quad (4)$$

Hence, At each sampling epoch $\mathrm{i} = 1,2, \ldots$ the estimated average quality loss for the ith sample is calculated by

$$\mathrm{EQL_i} = k \cdot n \cdot [\hat{\sigma}_i^2 + (\bar{X}_i - T)^2] \quad (5)$$

where $X \sim N(\mu_0, \sigma_0^2)$, $\hat{\sigma}_i^2 = \frac{(n-1)S_i^2}{n}$ is the maximum likelihood estimator (MLE) of the current process variance and $\hat{\mu}_i = \bar{X}_i$ is the unbiased estimator of the current process mean.

Note that the EQL due to bias is independent from the EQL due to process variation. Suppose the specification limits of quality characteristic are USL and LSL and it costs A dollars to repair a non-conforming item. In this case, the coefficient k can be calculated by $k = \dfrac{A}{(USL - LSL)^2}$.

Figure 1 illustrates Taguchi's loss function given in (1). The loss increases when the process deviates from the target value. This is in concordance with main principle of the six-sigma approach in which the main object is to reduce deviations (σ) from the target.

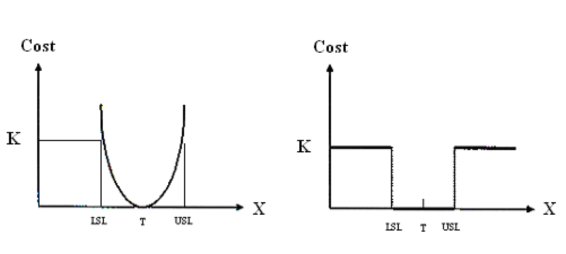

Figure 1. Taguchi loss function and traditional loss function

In the following sections, we establish a control chart to monitor the EQL statistic. The distribution of the sample loss function $\frac{\mathrm{EQL_i}}{k\sigma_0^2}$ follows a central chi-squared distribution with n degrees of freedom (χ_n^2) when the process mean is centered on the target. Thus, the (1-$\alpha/2$)th and $\alpha/2$th percentiles of this distribution can be applied as upper and lower control limits, respectively. That is:

$$(1-\alpha) = \Pr\left(\chi^2_{n,\frac{\alpha}{2}} \le \frac{\mathrm{EQL_i}}{\mathrm{k}\sigma_0^2} \le \chi^2_{\mathrm{n},1-\frac{\alpha}{2}}\right)$$

$$= \Pr\left(k\sigma_0^2 \cdot \chi^2_{n,\frac{\alpha}{2}} \le \mathrm{EQL_i} \le \mathrm{k}\sigma_0^2 \cdot \chi^2_{\mathrm{n},\alpha}\right) \quad (6)$$

$$LCL = k\sigma_0^2 \cdot \chi^2_{n,1-\frac{\alpha}{2}} \;;\; \mathrm{UCL} = \mathrm{k}\sigma_0^2 \cdot \chi^2_{\mathrm{n},\frac{\alpha}{2}} \quad (7)$$

Since $\mathrm{E}\left(\frac{\mathrm{L_i}}{\mathrm{k}\sigma_0^2}\right) = \mathrm{n}$,

$$\mathrm{CL} = \mathrm{k}\sigma_0^2 \cdot \mathrm{n} \quad (8)$$

The performance of a control chart is evaluated by means of its statistical properties. In particular, the most widely used measure of performance is the ARL (Average Run Length), which represents the average number of samples to be taken before the control chart signals an out-of-control condition (out-of-control ARL) or issues a false alarm (in-control ARL). Of course, quality practitioners want to implement control charts which immediately react to the occurrence of an out-of-control condition (i.e., the desired out-of-control ARL should be as small as possible); conversely, the expected number of samples taken between two successive false alarms issued by the control chart should be large, (i.e., the desired in-control ARL should be as large as possible). The in-control ARL of the Taguchi control chart is $\mathrm{ARL} = \frac{1}{\alpha}$. To compute the out-of-control ARL of the Taguchi control chart, we assume that only one assignable cause occurs to the process and shifts its in-control mean μ_0 or increases its standard deviation σ_0.

If an out-of-control condition occurs to the process which shifts its mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$, then the distribution of the statistic $\mathrm{EQL_i}$ changes as follows:

$$\frac{\mathrm{EQL_i}}{\mathrm{k}\sigma_0^2} = \frac{(\mathrm{n}-1)\mathrm{S_i^2}}{\sigma_0^2} + \frac{\mathrm{n}\cdot(\bar{\mathrm{X}}_\mathrm{i}-\mathrm{T})^2}{\sigma_0^2} \sim \chi^2_{(n,\lambda)} \quad (9)$$

where the non-centrality parameter $\lambda = n\delta^2$. The probability of Type II error in this case is equal to:

$$\beta = \Pr(LCL \le \mathrm{EQL_i} \le \mathrm{UCL} | \lambda > 0) \quad (10)$$

If the occurrence of the special cause increases the process dispersion from σ_0 to $\sigma_1 = \tau \cdot \sigma_0$, then the distribution of the Taguchi control chart statistic changes as follows:

$$\frac{EQL_i}{k\sigma_1^2} = \frac{(n-1)S_i^2}{\sigma_1^2} + \frac{n\cdot(\bar{X}_i - T)^2}{\sigma_1^2} \sim \chi_n^2 \quad (11)$$

In this case, the probability of a Type II error is equal to:

$$\beta = \Pr\left(\frac{1}{\tau^2}\cdot\chi^2_{n,\frac{\alpha}{2}} \le \frac{EQL_i}{k\sigma_1^2} \le \frac{1}{\tau^2}\cdot\chi^2_{n,1-\frac{\alpha}{2}} \middle| \tau > 1\right) \quad (12)$$

The out-of-control ARL of the Taguchi control chart in the presence of a shift ($\delta > 0$) in the process mean or the process standard deviation ($\tau > 1$) can be immediately computed as follows:

$$\mathrm{ARL} = \frac{1}{1-\beta} \quad (13)$$

3. RESULTS AND DISCUSSION

3.1. The Taguchi Control Chart and Six Sigma

From a manufacturer's point of view, it is always economical to reduce unit-to-unit performance variation around the target value even if the products are within specification limits. Continuous quality improvement and cost reduction are necessary to remain competitive. This is taken into account by applying Taguchi's philosophy to Statistical Process Control (SPC) while traditional SPC control charts fail to do so.

In six sigma all quality improvement activities aim at reducing process variations. The proposed chart translates the quality improvement activities into monetary forms. For example, assume the process mean is on its target value and the process is at the three sigma level. That is,

$$\Delta = USL - LSL = 6\sigma_1$$

$$EQL_1 = kn\sigma_1^2$$

if the sigma level increases to four, we will have:

$$\Delta = USL - LSL = 8\sigma_2$$

$$EQL_2 = kn\sigma_2^2$$

and

$$EQL_1 - EQL_2 = kn(\sigma_1^2 - \sigma_2^2) = \frac{7}{16}kn\sigma_1^2$$

This means that by increasing the level of sigma from three to four and assuming that the process mean is on its target, it is expected that the quality loss decreases by 43.75%.

In spite of this merit, some managers may not want to use six sigma until the cost of their poor quality products and lost opportunities along with the costs of a six sigma implementation are precisely calculated. The Taguchi control chart serves as a very useful tool to identify and detect processes that need to be improved. Reducing the totality of errors in such detected processes yields a basic step toward six sigma level. The advantages of using Taguchi control charts include: (1) encouraging quality as a business mark and guideline, (2) prediction of process conformity with specifications, (3) helping designers and manufacturers to improve and refine processes, (4) selecting potential suppliers, (5) facilitating performance measurement and improvement activities, (5) creating motivational goals, (6) expressing the importance of quality problems in monetary measures, (7) identifying existing opportunities for increasing customers' satisfaction and decreasing the price of products, (8) measuring the performance of quality improvement activities, (9) evaluating and grading proposed process quality improvement projects.

3.2. Illustrative Example

Consider a forging process which produces piston rings for an automotive engine. We want to establish statistical control of the inside diameter of the rings manufactured by this process using a Taguchi control chart. The inside diameter *X* has target value $T = 74$ mm, specifications limits $USL = 74.05$ mm and $LSL = 73.95$ mm . The replacement cost of nonconforming piston rings is $A_0 = 10$ \$. Therefore, it results that $k = \frac{A_0}{(74.05-73.95)^2} = \frac{10}{0.05^2}\frac{\$}{mm^2} = 4000\frac{\$}{mm^2}$. Twenty-five samples, each of size *n*=5, have been taken when the process was in control and after data analysis the process variation estimated 0.0000968. Consider the following new samples shown in Table 1. The control limits for the Taguchi control chart with at level of $\alpha = 0.005$ are calculated as follows:

$$UCL = k\sigma_0^2 \cdot \chi^2_{5,1-0.005/2} = \$7.12$$

$$LCL = k\sigma_0^2 \cdot \chi^2_{5,\frac{0.005}{2}} = \$0.12$$

Table 1 shows a set of simulated samples collected from the process and the corresponding EQL statistic. Samples #1 − #13 have been generated by assuming $N(74.001, 0.00984^2)$. To simulate the occurrence of assignable causes, samples #14 − #18 have been generated from $d \sim N(74.021, 0.00984^2)$ and samples #19 − #23 have been generated from $N(74.001, 0.01476^2)$. Figure 2 shows the Taguchi control chart for the samples collected in Table 1.

Table 1. Samples of n = 5 measures and corresponding values of the sample EQL for the forging process example

	x_1 $[mm]$	x_2 $[mm]$	x_3 $[mm]$	x_4 $[mm]$	x_5 $[mm]$	$\bar{X}_i$ $[mm]$	$S2_i[mm]$	EQL_i [$]
1	74,00745	74,00814	74,01256	73,99596	73,97368	73,99956	0,00025	3,99222
2	73,99592	74,02013	74,01055	74,01089	74,00177	74,00785	0,00009	2,32586
3	73,99663	73,99368	73,99837	74,00764	73,99410	73,99809	0,00003	0,68477
4	73,99782	73,98148	74,02020	74,00739	74,00696	74,00277	0,00021	3,34434
5	74,02765	74,01887	73,99780	74,00380	74,01362	74,01235	0,00014	4,82671
6	73,99339	74,00267	74,02227	73,99968	74,01545	74,00669	0,00014	2,89514
7	74,00551	73,99351	73,99827	73,99956	74,01064	74,00150	0,00004	0,71562
8	73,99988	74,00045	73,99555	74,01762	74,00238	74,00318	0,00007	1,23709
9	74,00359	74,00495	73,99363	73,99207	73,98805	73,99646	0,00006	1,29597
10	74,01584	73,99500	73,97828	74,00132	73,97907	73,99390	0,00025	5,01372
11	74,00708	74,00938	73,98841	73,98914	73,99172	73,99715	0,00010	1,96906
12	74,00152	74,03112	73,99776	73,98824	73,98887	74,00150	0,00031	4,91234
13	73,99802	73,98391	74,01294	74,00108	73,99067	73,99732	0,00012	2,20100
14	74,00683	74,01658	74,03076	74,01408	74,02760	74,01917	0,00010	8,16499
15	74,02328	74,00782	74,02663	74,00870	74,04282	74,02185	0,00021	12,03386
16	74,02829	74,02285	74,02573	73,99649	74,01238	74,01715	0,00017	7,93539
17	74,03049	74,01589	74,01414	73,99970	74,03149	74,01834	0,00017	8,78184
18	74,01472	74,00798	74,04227	74,01027	74,01380	74,01781	0,00019	8,76062
19	74,01438	73,98625	73,97822	73,99807	73,97990	73,99136	0,00023	5,47804
20	74,02127	74,00610	73,99841	73,99383	73,98059	74,00004	0,00023	3,64583
21	73,98978	74,02586	74,03509	74,01542	73,98180	74,00959	0,00053	9,92829
22	73,99440	74,01107	74,00795	73,98825	73,96830	73,99399	0,00029	5,70172
23	73,99731	73,98808	73,99321	73,97693	73,98778	73,98866	0,00006	3,98069

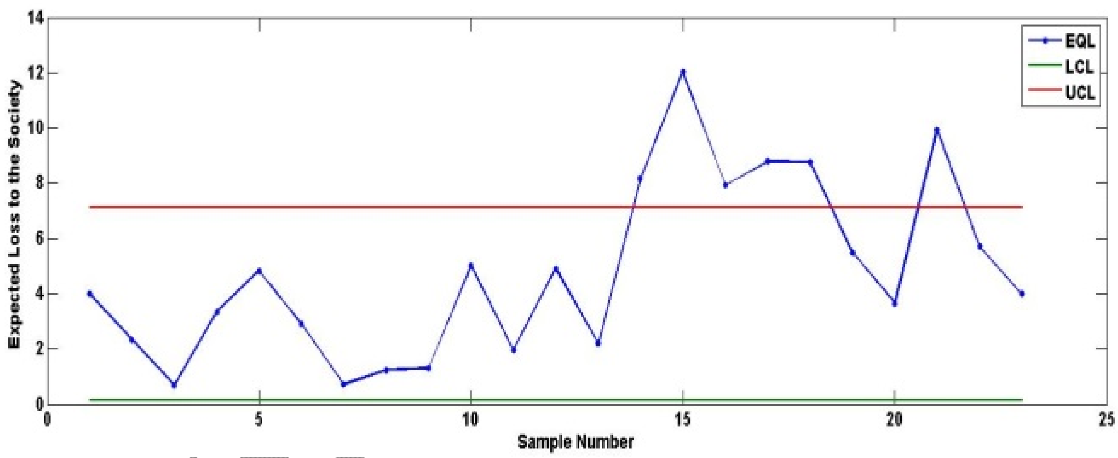

Figure 2. The Taguchi control chart for the forging process example

As we recognize, the chart shows an out-of-control signal for the first time in subgroup 14 which indicates an imposed loss of approximately $12. The process variation in subgroup 19 goes out of control and the Taguchi control chart detects that shift in subgroup 21. Hence, from a Taguchi control chart point of view, a process is out of control if only the quality loss imposed upon customers exceeds the expected limit.

CONCLUSIONS

In this paper we illustrate how the Taguchi loss function can be used as a control chart to monitor process loss to the society. In this chart, the process is monitored in monetary form, which clearly shows the amount of loss that the process is causing at its current level. Quality improvement activities then can be supervised to reduce the amount of the loss to the final customers. Moreover, benchmarking can be applied in different companies in order to determine best practices which help to reduce loss. The proposed control chart forces manufacturers to continuously reduce variation in their processes around the target value. Moreover, the chart statistic clearly shows that reducing process variation is not enough to ensure product quality to be consistent and it is also important to bring the process mean to its target value in order to minimize the loss encountered by society.

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