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DAMAGE MODELING OF COMPOSITES AND RELIABILITY ANALYSIS

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ABSTRACT

In this paper, the intra-laminar damage of laminated composites is modelled and analysed with the SAMCEF finite element code. A continuum damage mechanics model is used, and its parameters are identified based on physical tests conducted at the coupon level. Considering the variability from the tests and the sensitivity of the computed mechanical response depending on the values of the model parameters, a reliability analysis is considered to evaluate the most sensitive parameters in the model.

1. INTRODUCTION

In order to propose predictive simulation tools, it is important to use material models able to represent the different modes of degradation of the plies forming the laminated composite structure. Sometimes, damage at the interface between the plies, that is delamination, must also be taken into account in the model. In this paper, laminated composites made of unidirectional plies are considered. Damage inside the plies alone, that is intra-laminar damage, is considered in this paper.

The intra-laminar damage model used here is based on the continuum damage mechanics and was initially developed by the Ladevèze's team in Cachan [1]. Damage variables impacting the stiffness of the ply are associated to the different failure modes, representing the fibre breaking, the matrix cracking and the de-cohesion between fibres and matrix. Plasticity in the matrix is also taken into account. Such an advanced damage model includes lots of parameters, which must be identified based on test results at the coupon level. Test results present some variability, even when they are conducted on coupons coming from the same plate, because of the presence of small defects arising from the manufacturing process. It is therefore important to take into account such dispersions of the material properties and the influence they may have on the mechanical response of the composite.

In this paper, problems at the coupon level are addressed for laminates made up of unidirectional plies. The intra-laminar damage model available in the SAMCEF finite element code is first presented. The set of parameters is provided and the basics of the parameter identification procedure of such material models are briefly explained. The damage model is then used in a reliability analysis based on the polynomial chaos expansion approach [2,3], in order to determine the most sensitive parameters as well as the probability of failure depending on the parameters variability. The polynomial chaos expansion (PCE) is an efficient numerical method for performing a reliability analysis. It relates the output of a nonlinear system with the uncertainty in its input parameters using a multidimensional polynomial approximation (the so-called PCE). Numerically, such an approximation can be obtained by using a regression method with a suitable design of experiments. The cost of this approximation depends on the size of the design of experiments. If the design of experiments is large and the system is modelled with a computationally expensive FEA model, the PCE approximation becomes infeasible. In papers [2,3], an algorithm is proposed to generate efficiently a design of experiments of a size defined by the user, in order to make the PCE approximation feasible in computational time. It is an optimization algorithm which seeks to find the best design of experiments in the D-optimal sense for the PCE. This algorithm is a coupling between genetic algorithms and the Fedorov exchange algorithm.

2. DAMAGE MODEL OF UD PLIES FOR LAMINATES

The ply damage model relies on the development proposed in Ladeveze and Le Dantec [1]. For intra-laminar damage, the following potential (1) with damage, named e_d, is used, where d_{11}, d_{22} and d_{12} are the damages related to the fibres, the transverse and the shear directions, respectively.

$$e_d = \frac{\sigma_{11}^2}{2(1-d_{11})E_1^0} - \frac{\nu_{12}^0}{E_1^0}\sigma_{11}\sigma_{22} + \frac{\langle\sigma_{22}\rangle_+^2}{2(1-d_{22})E_2^0} + \frac{\langle\sigma_{22}\rangle_-^2}{2E_2^0} + \frac{\sigma_{12}^2}{2(1-d_{12})G_{12}^0} \tag{1}$$

These damage variables allow considering damage associated to the fibre direction, cracks in the transverse direction and de-cohesion between fibres and matrix. The thermodynamic forces represent the effect of the loading in the corresponding mode. These thermodynamic forces are derived from the potential and manage the evolution of the damages via relations of the form $d_{11} = g_{11}(Y_{11})$, $d_{22} = g_{22}(Y_{12},Y_{22})$ and $d_{12} = g_{12}(Y_{12},Y_{22})$. For instance, the thermodynamic force associated to shear is given in (2). Finally the model can be coupled to plasticity with isotropic hardening. The yield criterion is defined as a function of the effective stresses impacting the matrix behaviour (3).

$$Y_{12} = \frac{\sigma_{12}^2}{2(1-d_{12})^2 G_{12}^0} \tag{2}$$

$$f(\tilde{\sigma}, p) = \sqrt{\tilde{\sigma}_{12}^2 + a^2 \tilde{\sigma}_{22}^2} - R_0 - R(p) \le 0 \tag{3}$$

$$\tilde{\sigma}_{ij} = \sigma_{ij} / (1 - d_{ij})$$

$$R(p) = Kp^{\gamma}$$

In (3), R_0 is the initial yield stress. The non-linear behaviours taken into account in this model are illustrated in Figure 1: non linearity in the fibre direction, non-linearity including plasticity in the matrix.

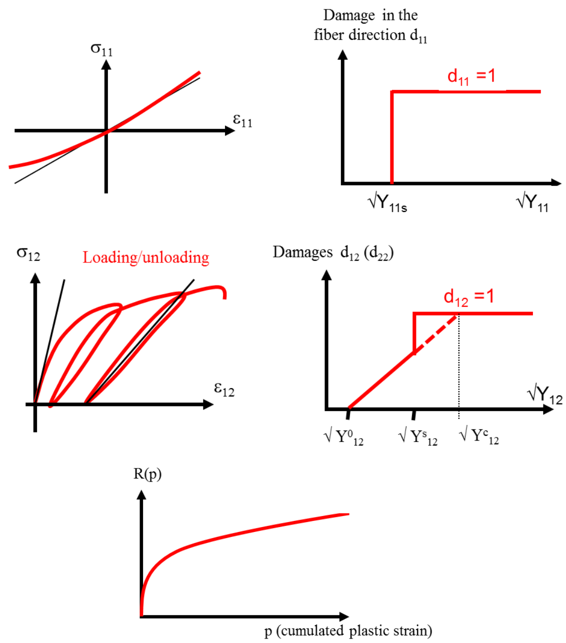

Figure 1. Non-linear behaviour of the damage model

In this paper, the evolution of the shear damage variable d_{12} as a function of the thermodynamic force is given by the following equation:

$$d_{12} = \frac{\sqrt{Y} - \sqrt{Y_{12}^0}}{\sqrt{Y_{12}^C} - \sqrt{Y_{12}^0}} \quad \text{if} \quad Y \le Y_{12}^S$$

$$d_{12} = 1 \text{ otherwise}$$

The equivalent thermodynamic force Y makes a link between the transverse and shear effects, via:

$$Y = Y_{12} + bY_{22}$$

where b is a coupling coefficient.

3. DAMAGE MODEL PARAMETERS IDENTIFICATION

The value of some parameters must be provided in order to feed the damage model. These parameters are the elastic properties (E^0_1, E^0_2, G^0_{12}, ν_{12}), the coefficients of the plastic law (R_0, K, γ, a), as well as the parameters associated to the damage law (e.g. b, Y^0_{12}, Y^C_{12}, Y^S_{12}; other parameters are also used for the behaviour along the fibre direction). When only traction is considered (and not compression), the parameter identification procedure is based on physical tests conducted on coupons made up of 3 different stacking sequences. One of these sequences is $[\pm 45]_{ns}$. Quasi-static cyclic loading is conducted (Figure 1): the initial stiffness is determined, as well as the evolution of the damage variable linked to the decrease in the stiffness during the cyclic loading. The procedure is explained in [1,4]. From the test results, it is clear that some variability exists in the mechanical response of the laminates. This dispersion can be reproduced by varying the value of the material parameters in the finite element model.

4. INFLUENCE OF THE PARAMETERS ON THE DAMAGE MODEL RESPONSE

The parameters of the damage model are determined at the coupon level (Figure 2), based on test results. It is clear that some variability appears in the tests results, and that most of the time some "mean" values are used to feed the model.

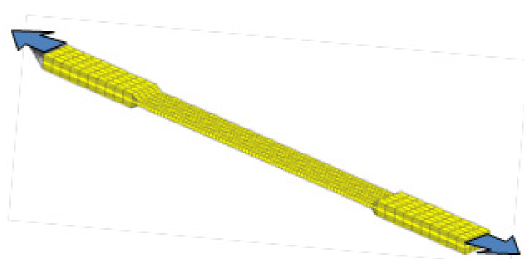

Figure 2. Model of the coupon used for the parameters identification

In Figure 3, the mechanical response of a coupon made up of a $[\pm 45]_{2s}$ laminate is illustrated. It is clear that the global mechanical behavior is non-linear, that permanent deformation is observed after an unloading, together with damage associated to the decrease in the material stiffness. The reference values (from one test) are superposed to the results of the simulation, for the identified parameters of the damage model. A very good agreement is observed. It is clear that a variability exists at the physical test level, even when the tested coupons are machined from the same composite plate. This is also concluded when the values of the identified parameters are modified, as illustrated in Figure 4. In that case, the simulation and test results are no more in good agreement.

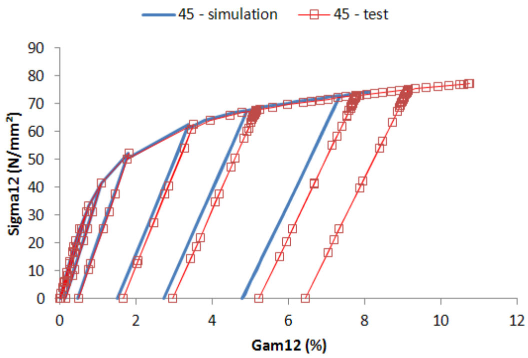

Figure 3. Mechanical response of the $[\pm 45]_{2s}$

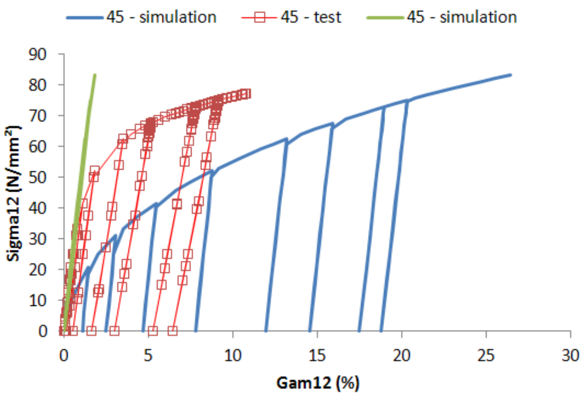

Figure 4. Mechanical response of the $[\pm 45]_{2s}$ for modified values of the identified model parameters

In Figure 5, the (reference) values of the parameters are given by: G^{0}_{12} = 4500 MPa, K = 400 MPa, Y^{C}_{12} = 6 MPa and θ = 45° in the $[\pm\theta]_{2s}$ laminate. In Figures 6 to 9, the values of these parameters are changed a little bit, and their influence on the mechanical response of the laminate can be estimated. The evolutions of d_{12} and of the hardening law are given in Figures 10 and 11, respectively.

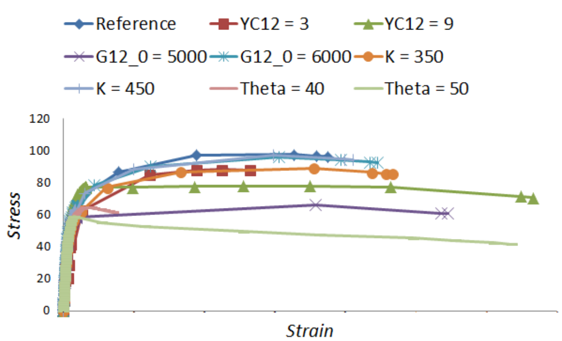

Figure 5. Influence of the parameters variation on the non-linear mechanical response of the coupon

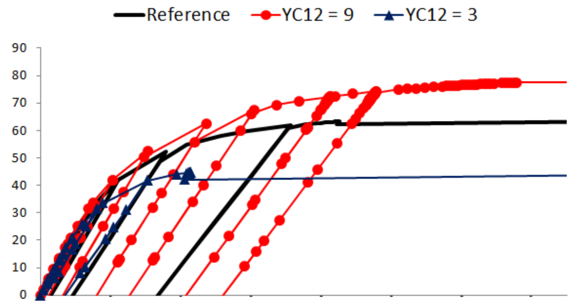

Figure 6. Influence of the damage evolution law on the mechanical response

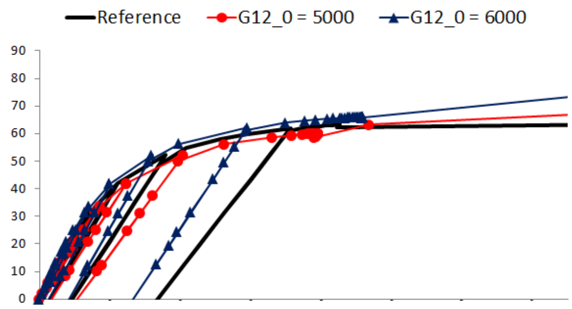

Figure 7. Influence of the initial shear stiffness on the mechanical response

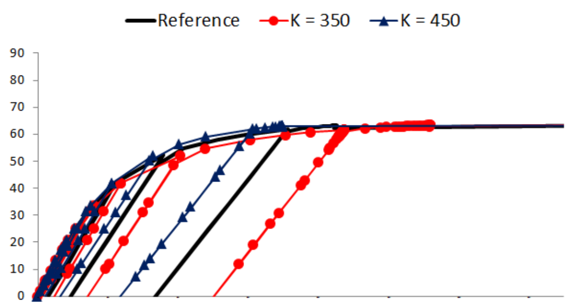

Figure 8. Influence of the hardening law on the mechanical response

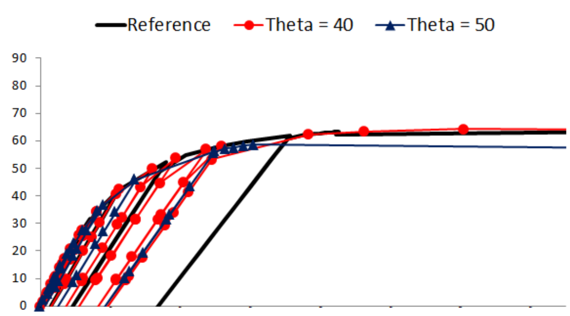

Figure 9. Influence of the fibre orientation on the mechanical response

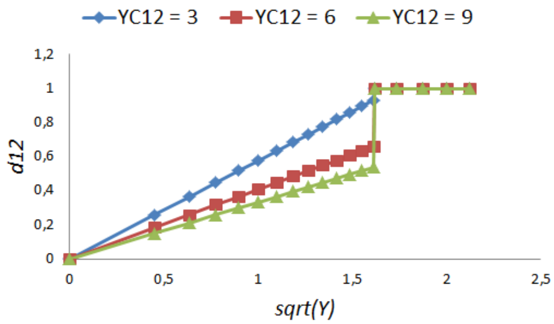

Figure 10. Damage law for different values of the Y^C_{12} parameter

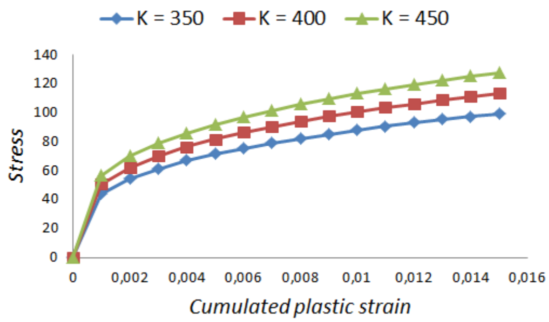

Figure 11. Hardening law for different values of the K parameter

The following questions then arise: 1/ how can we quantify the validity of the identified model parameters knowing that some variability exists at the tests level?; 2/what is the most sensitive parameter of the model, which should be identified in a very accurate way? 3/ how can we be sure that the predicted strength is in an acceptable range?

5. RELIABILITY ANALYSIS USING THE POLYNOMIAL CHAOS EXPANSION

Some variability exists in the mechanical properties of composites. The design paramerter x_i is defined as an independent gaussian random variable defined as follows:

$$x_i = \bar{x}_i + \sigma_i \xi_i ,$$

where $\bar{x}_i$ is its mean value, σ_i is its standard deviation and ξ_i a normal Gaussian variable. Note that x_i can be any of the mechanical parameters cited above. This uncertainty on the parameters is propagated through the mechanical system and its response, for example the stress, also becomes a random variable. The reliability analysis consists in computing the mean value and the variance of the response of the mechanical system. It also consists in computing the probability that the response does not exceed a certain threshold. This probability is referred to as probability of failure. Let S be the response of the mechanical system. S is approximated using the Polynomial Chaos Expansion as follows (PCE) up to degree p:

$$\tilde{S}(\zeta) = \sum_{i=0}^{n} a_i \Psi_i(\zeta)$$

where $\tilde{S}$ is the approximation of S about its nominal value, Ψ_i are the multivariate Hermite polynomial, a_i are the PCE coefficients and ζ is the vector of normal Gaussian random vairables of . The number n of terms in the PCE is: n=(N+p)!/N!p! where N is the number of uncertain parameters. The PCE coefficients are computed using the proposed method in [2]. A D-optimal design of experiments is generated and the regression method is used to compute the PCE coefficients. According to the orthogonal property of the PCE, the mean value and variance of S are approximated by

$$a_0 \text{ and } \sum_{i=1}^{n} a_i^2$$

respectively. One can also easily deduce the Sobol's indices from these coefficients (see [3]). The variance of S is the sum of variances due to the perturbation of each parameter and the joint perturbations of parameters. The Sobol's index is the part the perturbation of a parameter or a set of parameters with respect to the total variance. The probability of failure defined by Pr[S>q], the probability that S exceed a threshold q is computed by using the PCE approximation of S and the Monte Carlo Method. M^t samples of ζ are drawn randomly and the corresponding values $\tilde{S}(\zeta)$ are computed. If M^q samples correspond to $\tilde{S}(\zeta) > q$,

$$\Pr[S > q] \approx \frac{M^q}{M^t} .$$

6. APPLICATIONS

6.1 $[\pm 45]_{2s}$ laminate

The problem is run with the following values of the parameters:

Table 1. The parameters and their variability

Parameter	*Mean value*	*Standard deviation*
G^0	5500 MPa	150 MPa
K	400 MPa	15 MPa
θ	45°	1.5°
Y^C	60 MPa	0.33 MPa

The ratios between the mean value and the standard deviation are quite similar, so there is no unbalanced variable uncertainty in the problem, which would bias the sensitivity analysis using the Sobol's indices. A DOE of 30 experiments is conducted, in order to initialize the population. Two functions are considered in the problem: the first one concerns the value of the shear strain in the ply, calculated for an applied force equal to 2000N, and the second one is related to the maximum force the composite can sustain. The relative sensitivities of these functions with respect to the parameter uncertainties are expressed in terms of Sobol's indices. From Table 2, it is concluded that the angle deviation is the most sensitive parameter when both functions are considered.

Table 2. Results of the first application

Sensitive variables	**Sobol's index of ε_{12}**	**Sobol's index of F_{max}**
G^0_{12} alone	0.14533	0.107495
G^0_{12} and K	7.86508e-06	0.000291474
G^0_{12} and θ	0.000954293	0.000405614
G^0_{12} and Y	5.37728e-05	0.00183905
K alone	0.060854	0.000871712
K and θ	0.00139441	7.77772e-05
K and Y	3.43177e-05	0.000610869
θ alone	<u>0.744552</u>	<u>0.421117</u>
θ and Y	0.00100739	0.00250115
Y	0.0458122	<u>0.464791</u>
Sum of all the contributions	1	1

6.2 $[\pm 67.5]_{2s}$ laminate

The methodology is applied on a plate made up of a $[\pm 67.5]_{2s}$ laminate. The same parameters are used, with the same variability as in Table 1. In Figure 12, the mechanical responses of the plate are plotted, for the reference values and for some perturbations. It is clear that the variability in the fiber orientation is here the most important parameter. This result is confirmed by the robust analysis, as reported in Table 3.

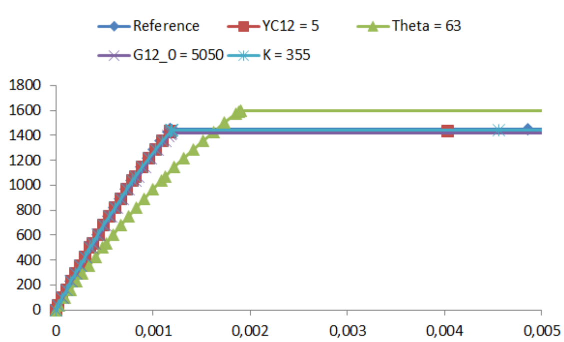

Figure 12. Mechanical response (Force/strain) of the plate

Table 3. Results of the second application

Sensitive variables	**Sobol's index of F_{max}**
G^0_{12} alone	0.0549
G^0_{12} and K	5.37e-5
G^0_{12} and θ	0.004
G^0_{12} and Y	1.48e-7
K alone	0.00139
K and θ	3.29e-5
K and Y	8.19e-5
θ alone	<u>0.923045</u>
θ and Y	0.00081
Y	0.01504
Sum of all the contributions	1

7. CONCLUSIONS

In this paper, the intra-laminar damage model for laminated composites available in SAMCEF was presented. It is a continuum damage mechanics model, whose parameters are identified based on physical tests conducted at the coupon level. Considering the variability from the tests and the sensitivity of the computed mechanical response depending on the values of the model parameters, a reliability analysis was considered to evaluate the most sensitive parameters in the model.

8. REFERENCES

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