

OPTIMIZATION OF COMPOSITE STRUCTURES WITH CURVED FIBER TRAJECTORIES

Etienne Lemaire, Samih Zein, Michaël Bruyneel

SAMTECH (A Siemens Industry Software Company), Liège Science Park, rue des chasseurs-ardennois 8, B4031 Angleur, Belgium, Email: {Etienne.Lemaire, Samih.Zein, Michael.Bruyneel}@lmsintl.com

ABSTRACT

This paper, presents a new approach to generate parallel fiber trajectories on general non planar surfaces based on Fast Marching Method. Starting with a (possibly curved) reference fiber direction defined on a (possibly curved) meshed surface, the new method allows defining a level-set representation of the fiber network for each ply. This new approach is then used to solve optimization problems, in which the stiffness of the structure is maximized. The design variables are the parameters defining the position and the shape of the reference curve. The shape of the design space is discussed, regarding local and global optimal solutions.

1. INTRODUCTION

The use of composite materials in aerospace, automotive and ship industry allows manufacturing lighter and more efficient mechanical structures. Indeed, proper use of the orthotropic properties of these materials enables further tailoring of the structure to the loadings than when using isotropic materials. However, this comes at the cost of a more complicated design and sizing process firstly because of the orthotropic behavior of composite materials but also because of the manufacturing process which induces specific constraints in the use of these materials.

From the mechanical point of view, one of the most important restrictions resulting from the practical manufacturing of mechanical parts is the orientation of the reinforcement fibers resulting from the layup process. These orientations directly determine the orthotropy axes and cannot be chosen arbitrarily in any point of a given part but rather result from the draping of the reinforcement material over the part. Several models have been developed in order to predict the orientations of the reinforcement fibers after the draping process depending on the properties of the reinforcement materials (see [1] for a review).

One of the first of these models is due to Mack and Taylor [2]. Often called the 'pin-jointed' model [3], it is based on a geometric model of the woven and it is well suited to predict the fiber orientation resulting from hand layup of dry woven fabrics. Later, more complex models relying on a finite element mechanical modeling of the reinforcement have been developed for the forming of preimpregnated fabrics as for instance by Cherouat and Bourouchaki [4].

Besides the manufacturing of composites part by hand layup of large pieces of reinforcement material, another group of methods is gaining interest since its first introduction in the 1970s. These methods rely on the robotized layup of bands of unidirectional reinforcement

material allowing more accurate and more repeatable manufacturing process [5]. In this group, two main methods can be identified Automated Tape Layup (ATL) and Automated Fiber Placement (AFP). ATL makes use of a robotic arm to layup tapes (up to 300mm wide) of unidirectional prepreg and benefits from high productivity for large and simple flat parts. But ATL main limitation comes from the relatively high minimum curvature radius (up to 6m) that can be applied to the prepreg tape without wrinkling. With AFP, this minimum curvature radius is decreased to 50cm by subdividing the tape into several tows which can be cut and restarted individually. Therefore the manufacturing of more complicated parts can be handled by AFP but with a lower productivity than ATL.

For ATL and AFP processes, one of the manufacturing issues is the determination of successive courses trajectories. Indeed, for these processes, it is crucial that there are no overlaps and no gaps between adjacent courses in order to ensure maximal strength for the final part. In other words, this means that successive layup courses have to be equidistant.

A few researchers have studied the optimal design of ATL/AFP parts. A first group of methods consists in defining an initial course which is then simply shifted over the part to define subsequent course as proposed by Tating and Gürdal [6, 7]. Secondly, the courses can be defined as geodesic paths, constant angle paths, linearly varying angle paths or constant curvature path [8]. However, these two approaches do not result in equidistant paths.

Alternatively, the subsequent courses can be obtained by computing actual offset curves from an initial curve. This approach is more difficult but leads to equidistant courses and has been investigated by Waldhart [8], Shirinzadeh et al [9] and Bruyneel and Zein [10] with different numerical schemes. The two first groups of authors propose an approach based on a geometrical description of the part while the third one developed an algorithm able to work on a mesh of the layup surface.

The goal of the present paper is to demonstrate further the capabilities of the method proposed by Bruyneel and Zein [10] by using it for optimal design of composite parts.

This paper starts with a brief introduction describing the method developed by Bruyneel and Zein to determine equidistant courses for ATL/AFP process. Next, several optimization problems with growing complexity are studied in order to illustrate the interest of the method.

2. FIBER PLACEMENT MODELING

2.1. Fast Marching Method

Bruyneel and Zein [10] first proposed the use of Fast Marching Method (FMM, see [11]) to solve the problem of determining equidistant courses on an arbitrary layup surface. The Fast Marching Method aims at solving the Eikonal equation:

$$\begin{aligned} |\nabla T(x)| &= f, & x \in \Omega \setminus \Gamma, \\ T(x) &= 0, & x \in \Gamma \subset \Omega. \end{aligned} \quad (1)$$

The problem given in Eq. (1) consists in finding a scalar field $T(x)$ such that the norm of its gradient is constant over the domain Ω and that the value of T is equal to zero on a curve Γ of Ω .

As illustrated in Fig. 1, this differential equation can be interpreted as the one characterizing a front propagation at a constant speed where $T(x)$ denotes the time at which the front passes through point x . At time $T = 0$, the front coincides with the curve Γ , therefore, all points located on the curve Γ will have a value of T equal to 0. Then as time increases, the front propagates at a constant speed equal to $1/f$ over Ω . The position of the front Γ_1 at any time T_1 corresponds to the set of point lying on the isovalue $T(x) = T_1$. Since the front speed norm is uniform over the domain, every point of Γ_1 is equidistant from Γ . The set of equidistant curve can therefore be obtained by selecting appropriate isovalues of $T(x)$ over Ω .

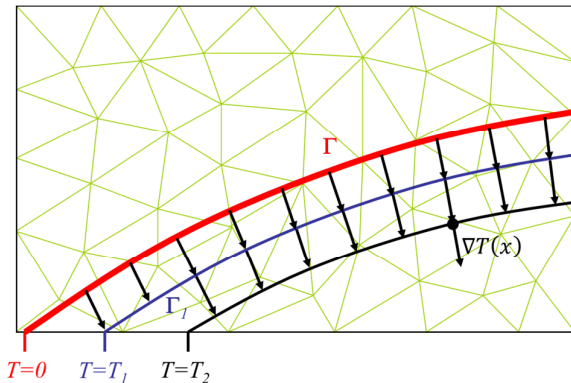


Figure 1. Front propagation interpretation of Eikonal equation.

Based on a triangular mesh of the layup surface, the developed procedure allows computing fiber orientation on each element of the mesh. At first one needs to define the initial front position on the layup surface. This curve corresponds to the reference course and the definition procedure is presented in next subsection. Secondly, the Fast Marching Method is used to solve the Eikonal equation and to compute the time T at any point of the mesh. The function $T(x)$ is supposed to be piecewise linear by element. Starting from initial values defined by the reference curve, the value of T is progressively computed on the domain by solving the Eikonal equation locally on each triangle of the mesh. For further details about the Fast Marching Method, the interested reader may refer to [10, 11]. Finally, the fiber orientations on each element are defined by computing

the direction of the isovalues of $T(x)$ over the considered element. Since those isovalues are equidistant from the reference course, the computed orientations correspond to a gap-less and overlap-less (i.e. constant thickness) layup obtained by ATL or AFP.

2.2. Reference course tracing

The definition of the reference course plays a major role in the context of the present work since the orientation or the control points of the corresponding curve are used as design parameter of the optimization problem.

Because the definition of a curve on a general 3D surface may be a difficult task, we have chosen to resort to an 'artificial' 2D space to define the reference curve and next to map this curve onto the layup surface to obtain the reference course.

This process is illustrated in Fig. 2. The reference curve is defined in the 2D space such that it passes through the axes origin. A seed point is defined on the 3D surface and the triangle containing this point is mapped in the 2D 'artificial' space such that the seed point lies on the origin. The intersection between the reference curve and the mapped triangle is approximated by a line segment which is transposed to the 3D space to define the first segment of the reference course. Next, this process is repeated for the next triangle that is intersected by the initial course (e.g. the triangle adjacent to edge AC in Fig. 2) until the boundary of the 3D surface is met. At the end of this process the initial course is obtained on the layup surface mesh as a piecewise linear curve.

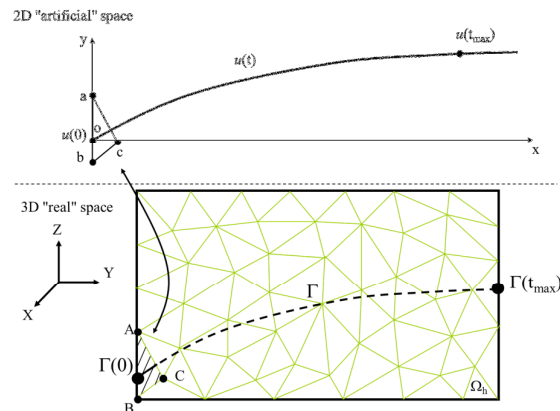


Figure 2. Reference course mapping.

3. STRAIGHT COURSE OPTIMIZATION

3.1. Optimization problem

The first illustration of the method is a very simple optimization problem consisting in minimizing the deformation energy of a square plate presented in Fig. 3. The left side of the plate is clamped and a point force is applied downwards at the lower right corner. For more convenience and since it is possible for the present 2D structure, the reference curve is presented directly on the structure (even if the mapping process described in previous section is used). The initial course is supposed to be straight and the sole design variable of the

problem is θ the angle made by this course and the horizontal. The seed point is fixed at the half of the left side boundary of the plate.

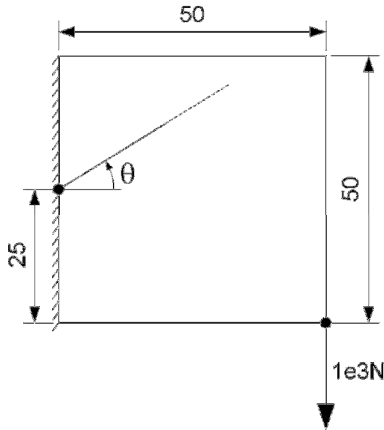


Figure 3. Straight fiber path optimization problem.

For the present application, layup courses are obviously very simple as the planar geometry of the structure and the choice of a rectilinear initial course always results in uniform fiber orientations over the structure. However, the purpose of this application is to validate the optimization approach and also to provide a reference solution for further examples.

The optimization problem is mathematically formulated in Eq. (2) where c stands for the deformation energy. An upper bound and a lower are imposed on the angle such that all possible orientations are covered.

$$\min_{\theta} c(\theta), \quad (2)$$

$$s. t. \{-90^{\circ} \leq \theta \leq 90^{\circ}.$$

The material properties used for all numerical applications are those of a typical composite used for ATP made of unidirectional carbon fibers. The properties are listed in Tab. 1.

Table 1. Material properties.

E_{11}	181 GPa
E_{22}, E_{33}	5.18 GPa
ν	0.25
G_{12}, G_{13}	2.57 GPa
G_{23}	2.07 GPa

3.2. Solution

The numerical model is created by meshing the plate with 40x40 elements. The optimization is carried out using BOSS QUATTRO [11] optimizers. Gradient based optimization [12] will be used for this application. The required sensitivities can be computed using finite difference since due to the small number of design variable it does not lead to a prohibitive computational overhead.

To initiate the optimization process a course perpendicular to the left edge (i.e. $\theta=0^{\circ}$) is selected which gives an initial value of the objective function equal to 11919 J. After a few iterations, the optimization method converges to an optimum for $\theta=-42.9^{\circ}$ and $c=8718$ J which means a decrease of 27%

with respect to the initial value. The resulting fiber orientations and courses are presented in Fig. 4. In this figure, the red line represents the initial course centerline and each color strip corresponds to a layup course. Resulting elementary fiber orientations are represented by a small segment on each element.

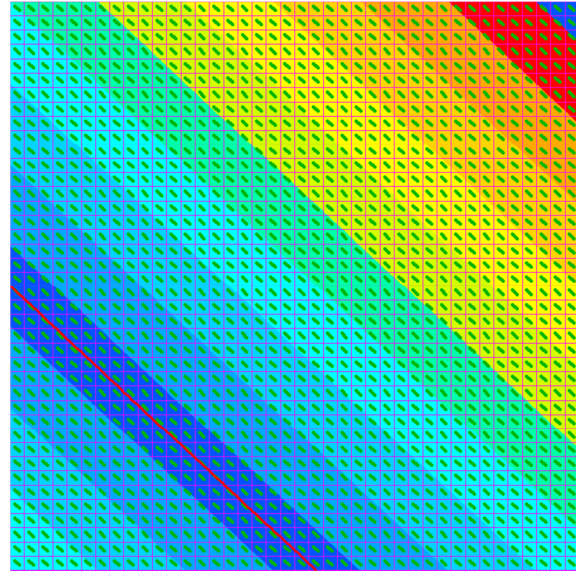


Figure 4. Solution of the optimization problem with a straight course.

Since the optimization problem considered in this section is very simple the validity of the optimal solution presented in Fig. 4 can be verified by performing a parametric study over the design domain.

Therefore, the value of the objective function has been computed for every integer value of θ between -90° and 90° . The result of the parametric study is presented in Fig. 5. The minimal value of the objective function is 8719 J obtained for $\theta=-43^{\circ}$ which confirms the result of the optimization procedure. Moreover, the parametric study shows that the present optimization problem does not possess local optima except the one generated by the upper bound $\theta < 90^{\circ}$. However, as the objective function is periodic, the solution $\theta=90^{\circ}$ would disappear if the

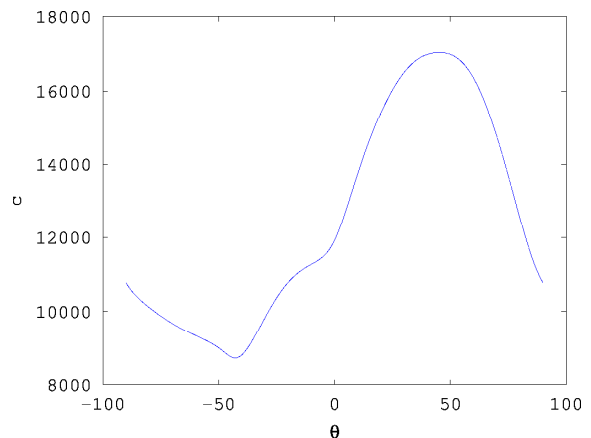


Figure 5. Deformation energy against the angle design variable.

upper bound was larger and lead to the same solution as $\theta = -43^\circ$.

4. CURVILINEAR COURSE OPTIMIZATION

4.1. Optimization problem

In order to make advantage of fiber placement capabilities, we can consider a more general optimization problem with a curvilinear fiber course presented in Fig. 6. In this new optimization problem, the geometry of the structure and the boundary conditions remain identical to the one considered in previous section while the definition of the initial course is modified. Firstly, the optimization process is now able to move the seed point along the left side of the plate by adjusting the design variable y_s . Secondly, the initial course corresponds now to a quadratic spline defined by three points:

- The seed point,
- A final point located on the right edge at a vertical distance from the seed point equal to y_p (the second design variable),
- A middle point placed at 25mm from the left edge and at a vertical distance from the seed point equal to $y_p/5$.

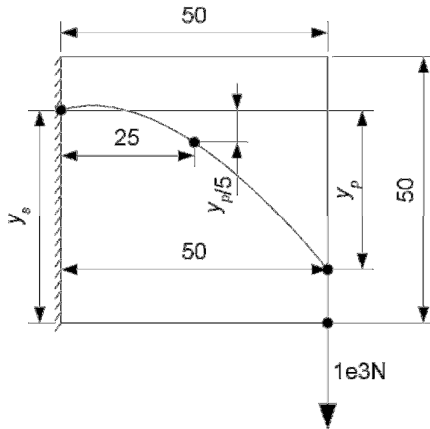


Figure 6. Curved fiber path optimization problem.

The mathematical formulation of the optimization problem is given in Eq. (3). Again, the objective of the optimization problem is to minimize the deformation energy.

$$\begin{aligned} & \min_{y_p, y_s} c(y_p, y_s), \\ & \text{s. t. } \begin{cases} 1 \leq y_s \leq 49, \\ -100 \leq y_p \leq 100. \end{cases} \end{aligned} \quad (3)$$

The position of the seed point y_s is restricted to the range [1,49] in order to avoid discontinuities of the objective function that may arise when the seed point is close to the lower or upper edge of the plate. Indeed, when the seed point is close to one of those edges, the initial course may suddenly go out of the domain for a small modification of y_s , which leads to very different layups.

4.2. Solution

The optimization problem is solved with the same

procedure as previously based on finite difference sensitivities. The initial design is chosen as in previous application with $y_s = 25$ and $y_p = 0$ which result in horizontal rectilinear fibers. In the final design is presented in Fig. 7, the position of the seed point has reached its upper bound ($y_s = 49$) while the end point of the initial course is defined by $y_p = -48.9$.

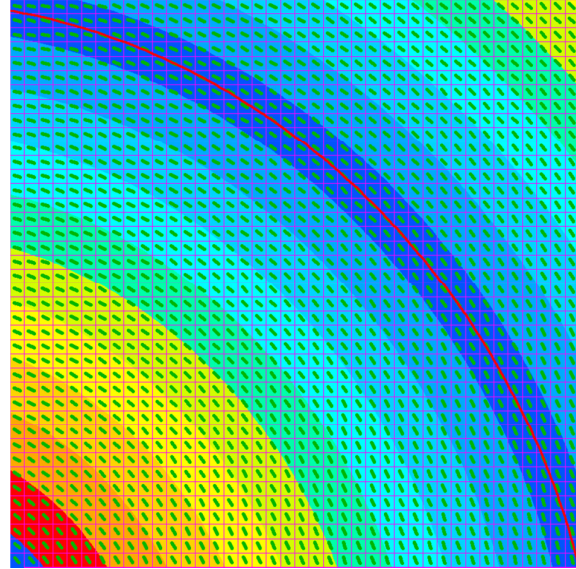


Figure 7. Solution of the optimization problem with a curvilinear course.

Under the design load case, the final design give a deformation energy equal to 4676 J, which is 61% less than the initial design and 46% less than the rectilinear design obtained previously. This shows that the capability of AFP to follow curved courses leads to a strong improvement of the mechanical performance and confirms the conclusions of Hyer and Charette [14]. Nevertheless, we can observe that curvature of the courses increases in the lower left corner such that the introduction of an optimization constraint on curvature would be helpful to ensure the manufacturability of the part.

In order to check the optimality of the design obtained by the optimization procedure and to investigate the

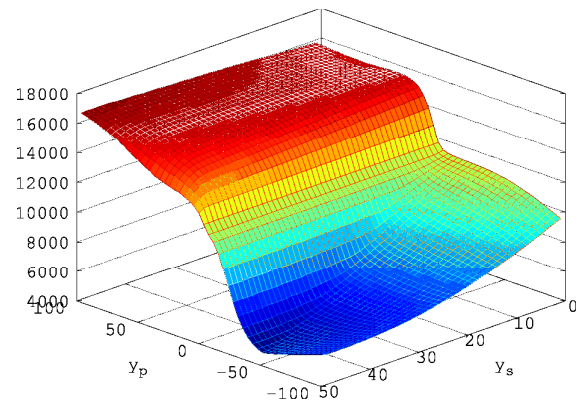


Figure 8. Objective function for the curvilinear course optimization problem.

existence of local optima, a parametric study has been performed. The result of this parametric study is presented in Fig. 8. The plotted surface represents the objective function value over the optimization domain. We can check that the presented design actually corresponds to the optimal solutions. However, a closer analysis of Fig. 8 shows that there is also a local optimum for $y_s=1$ and $y_p=62$ into which the optimization procedure could get trapped if a different initial design is selected.

5. CONICAL SURFACE

5.1. Optimization problem

In order to illustrate the ability of the method to handle 3D surfaces, the last presented numerical application is the optimal design of a conical shell depicted in Fig. 9. The cone axis coincides with Z axis. The large base of the cone is clamped while a force and a torsion torque are applied on the small base. The shell is composed of two plies which can be oriented independently. For each ply, the reference curve is a straight line in the 'artificial' 2D plane and its angle with respect to the x axis of the 'artificial' 2D plane is a design variable. We have therefore two design variables θ_1 and θ_2 . The mathematical formulation of the optimization problem is:

$$\begin{aligned} & \min_{\theta_1, \theta_2} c(\theta_1, \theta_2), \\ & \text{s. t. } \begin{cases} -90^\circ \leq \theta_1 \leq 90^\circ, \\ -90^\circ \leq \theta_2 \leq 90^\circ. \end{cases} \end{aligned} \quad (4)$$

Moreover, because the surface is conical, if the reference course makes more than one revolution around the cone, it becomes non equidistant to itself. As a consequence it is of course impossible to generate equidistant courses. To circumvent this problem, we assume that the shell is manufactured in two parts which are draped with the same parameters. That's why in Fig. 9, one seed point is placed on each side of the (YZ) plane and the shell is cut in two by this plane.

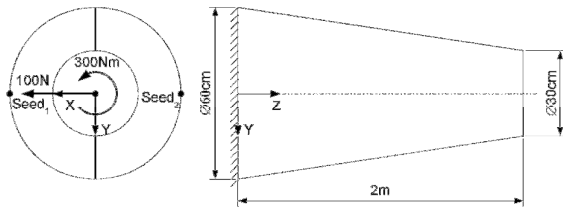


Figure 9. Conical surface dimensions and load case.

5.2. Solution

Starting with $\theta_1 = \theta_2 = 0$, the optimization procedure converges to the solution presented in Fig. 10. Ply 2 is the symmetrical of ply 1 with respect to plane (X-Z) as at the end of the optimization process $\theta_1 = 37.7^\circ$ and $\theta_2 = -37.7^\circ$. These opposed orientations shows that in the present load case, torsion dominates. The deformation energy is cut by 83% between the initial design and the final design.

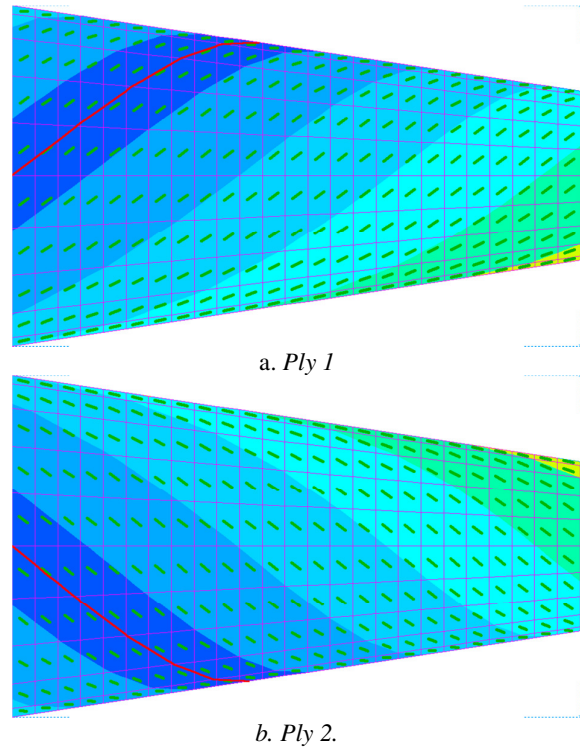


Figure 10. Solution of the conical shell optimization problem.

The results of a parametric study are presented in Fig. 11 which present the isovalue lines of the objective function over the design space. One can notice that the objective function has several local extrema but fortunately, only two of them are minima. The solution obtained by the optimization procedure is indicated by a star label and actually corresponds to a minimum of the objective function. The second minimum corresponds in fact to the first one but with changed sign for each design variable. As a consequence, we can conclude that the stacking sequence has no significant importance for the present application.

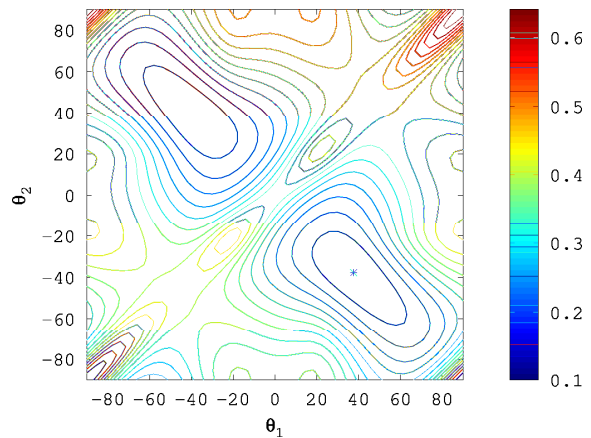


Figure 11. Objective function isolines over the design space.

6. CONCLUSION

In this paper, a new approach for computing fiber orientations resulting from ATL/AFP draping is used as the basis for composite shell optimization. The main advantages of this approach over previous research is that it allows determining fiber orientations and deposition courses that correspond to a layup free from overlaps and gaps between successive courses. Moreover, the method does not require a geometrical representation of the layup surface but can simply be applied on a 2D mesh of this surface.

In order to show the generality of the proposed approach, several numerical applications were proposed, firstly on 2D surfaces, next on a 3D surface. Additionally, the results of these applications confirms the benefits of using curved fiber paths on mechanical performance as previously observed [14] with the great advantage in the present case that optimal fiber orientation correspond to a manufacturable part.

Future works should focus on the introduction of other manufacturing constraints into the optimization problem such as a minimum curvature radius for the layup courses. Moreover, the improvement of the sensitivity analysis can also be investigated. The development of a semi-analytical sensitivity analysis would strongly improve the efficiency of the optimization process and would allow increasing the number of design variables in order to consider more complex courses definition.

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