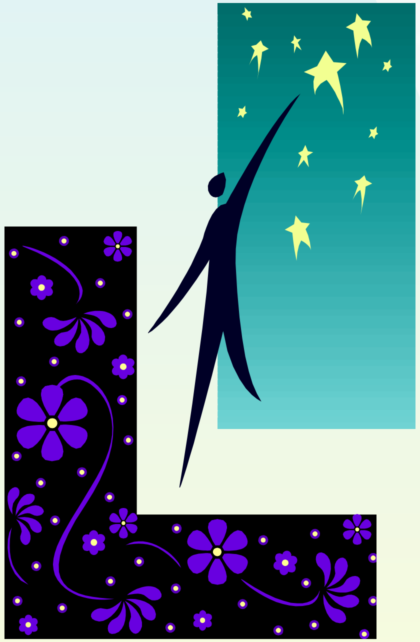
“Seuls les sages, même réduits à l’extrême mendicité, sont riches.“

***Cicéron*, 1er siècle avant J.-C.**

An introduction to optical/IR interferometry

Brief summary of main results obtained during the past lectures:



$$\rho = R / z$$

$$T_{eff} = (F/\sigma)^{1/4} = (f / \sigma \rho^2)^{1/4}$$

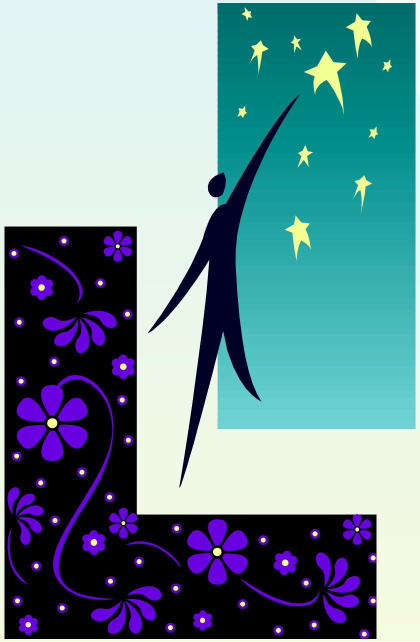
$$E = A(z) \exp[i2\pi\nu t]$$

$$E = A(z, t) \exp[i2\pi\nu t]$$

$$\tau = 1 / \Delta\nu \qquad \lambda_{eff} = \lambda^2 / \Delta\lambda$$

$$I = A A^* = |A|^2 = a^2 .$$

An introduction to optical/IR interferometry



If $\Delta \geq \lambda / (2B)$, fringe disappearance!

$$I_q = I + I + 2I \, |\gamma_{12}(0)| \cos\left(\beta_{12} - 2\Pi \nu \tau\right)$$

$$\gamma_{12}(\tau) = \langle V_1^*(t) V_2(t-\tau) \rangle / I$$

Fringe visibility: $$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right) = |\gamma_{12}(0)|$$

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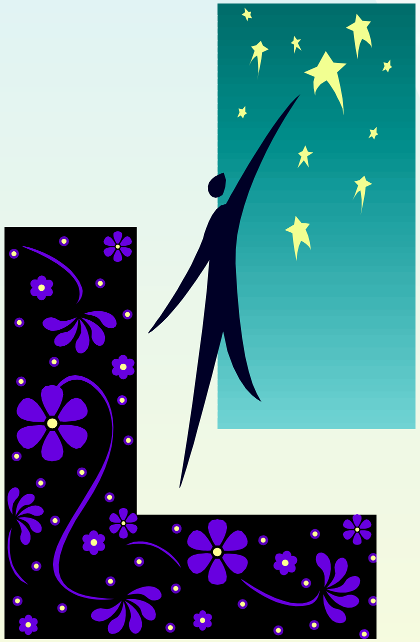


$$V = \left|\gamma_{12}(0,u,v)\right| = \left|\iint_S I'(\zeta,\eta)\exp\left\{-i2\Pi\left(u\zeta + v\eta\right)\right\}d\zeta\, d\eta\right|$$

$$I'(\zeta,\eta) = \iint \gamma_{12}(0,u,v)\exp\left\{i2\Pi\left(\zeta u + \eta v\right)\right\}d(u)d(v)$$

- For the case of a 1D uniformly brightening star whose angular diameter is ϕ = b/z’, we found that the visibility of the fringes is zero when λ/B = b/z’ = ϕ where B is the baseline of the interferometer

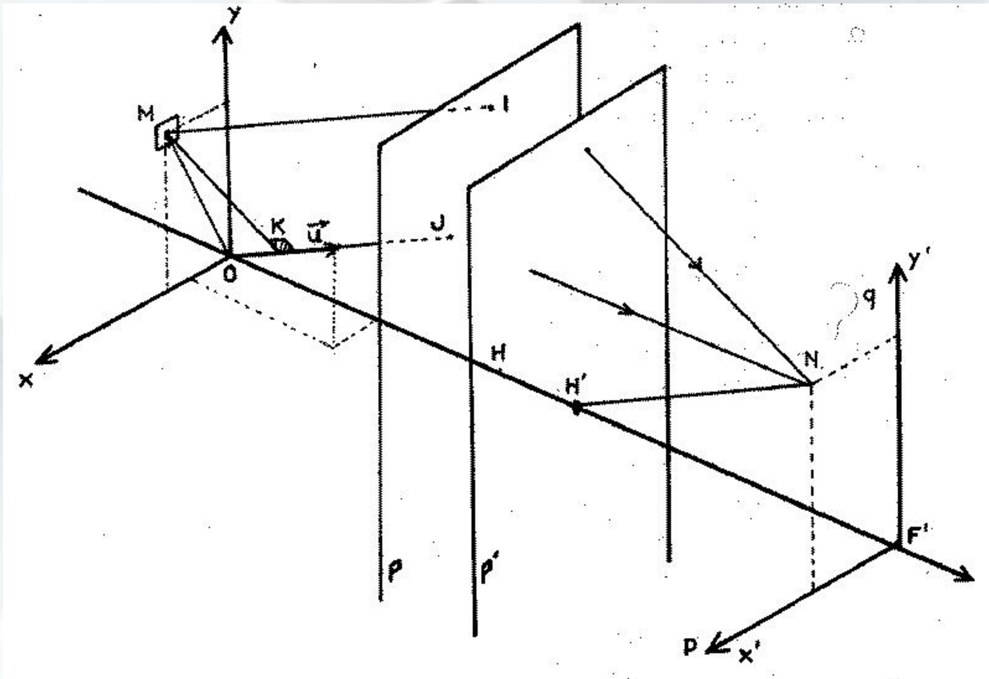
- For the case of a double star with an angular separation ϕ = b/z’, we found that the visibility of the fringes is zero when λ/2B = b/z’ = ϕ

An introduction to optical/IR interferometry

8.1 The fundamental theorem

a(p,q) = TF_(A(x,y))(p,q),

$$a(p,q) = \int_{R^2} A(x,y)\exp\left[-i2\pi\left(px+qy\right)\right]dxdy,$$



with

p = x’ /(λ f)

q = y’ /(λ f)

An introduction to optical/IR interferometry

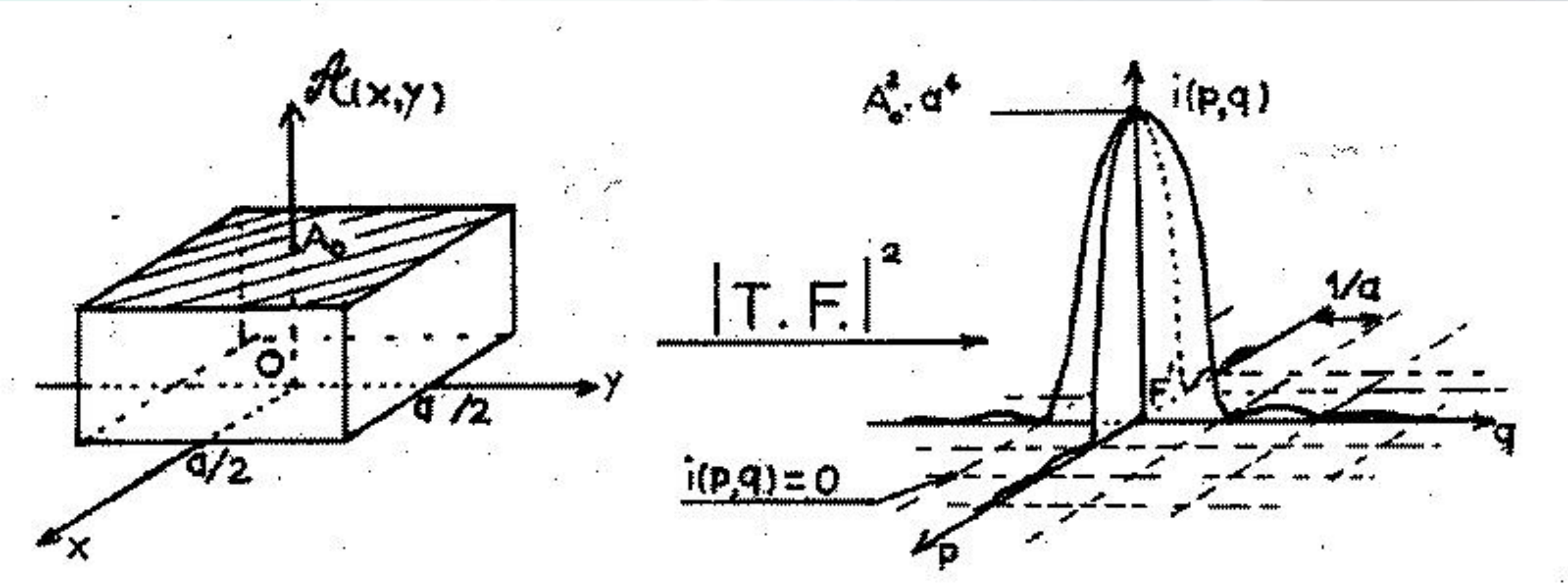
8.1 The fundamental theorem

The distribution of the complex amplitude a(p,q) in the focal plane is given by the Fourier transform of the distribution of the complex amplitude A(x,y) in the entrance pupil plane.

An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination



$\Delta p = \Delta x' /(\lambda f); \Delta q = \Delta y' /(\lambda f) = 2/a \Rightarrow \Delta\phi_{x'} = \Delta\phi_{y'} = 2\lambda/a$ (8.1.7)

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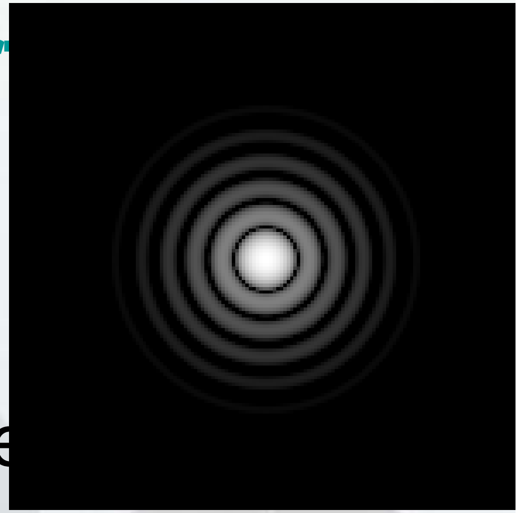
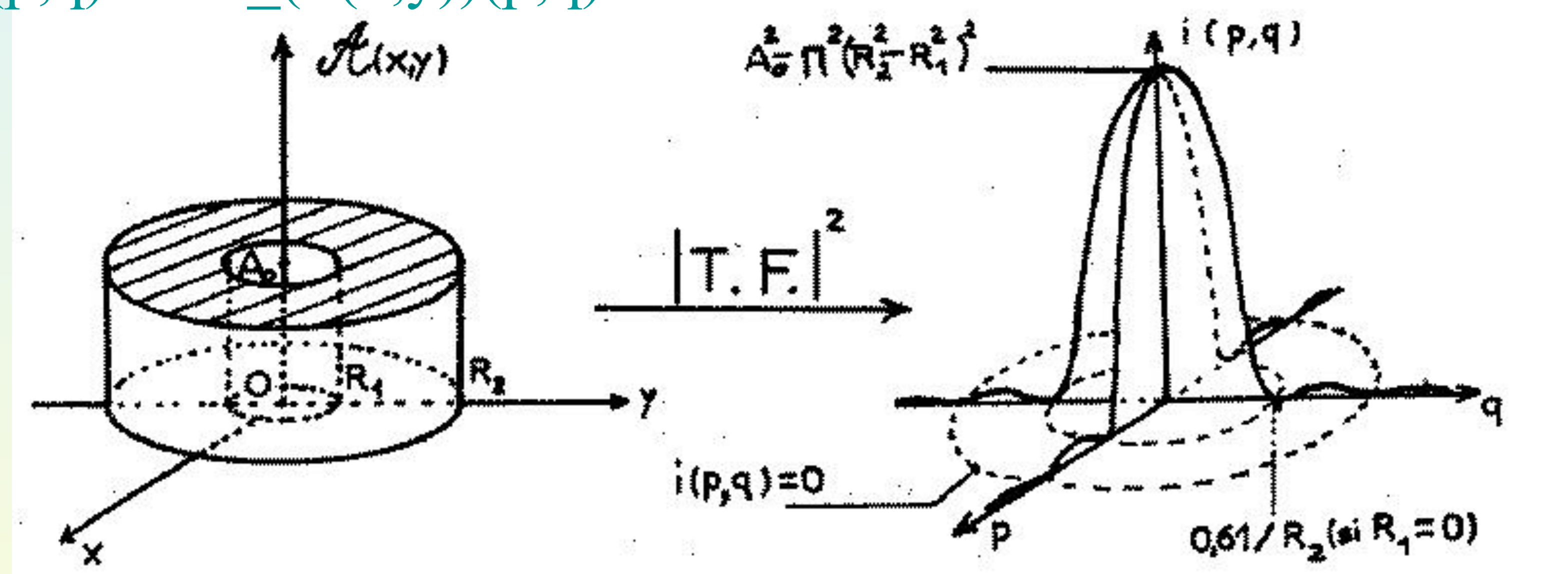


8.1 The fundamental theorem

Application: Point Spread Function dete

h(p,q) = TF_(P(x,y))(p,q)



$i(\rho') = |a(\rho')|^2 = (A_0\,\pi)^2\,[R_2^{\,2}\,2\,J_1(Z_2)\,/\,Z_2\; - R_1^{\,2}\,2\,J_1(Z_1)\,/\,Z_1]^2$, (8.1.8)

with $Z_2 = 2\pi\,R_2\,\rho'\;/\,(\lambda f)$ and $Z_1 = 2\pi\,R_1\,\rho'\;/\,(\lambda f)$. (8.1.9)

From the previous result, i.e. for the case of a circular aperture with a radius R, the distribution of the complex amplitude in the focal plane is given by the expression:

$a(\rho') = (A_0 \pi) [R^2 \, 2 \, J_1(Z) / Z]$,

where $Z = 2\pi \, R \, \rho' \, / \, (\lambda f)$

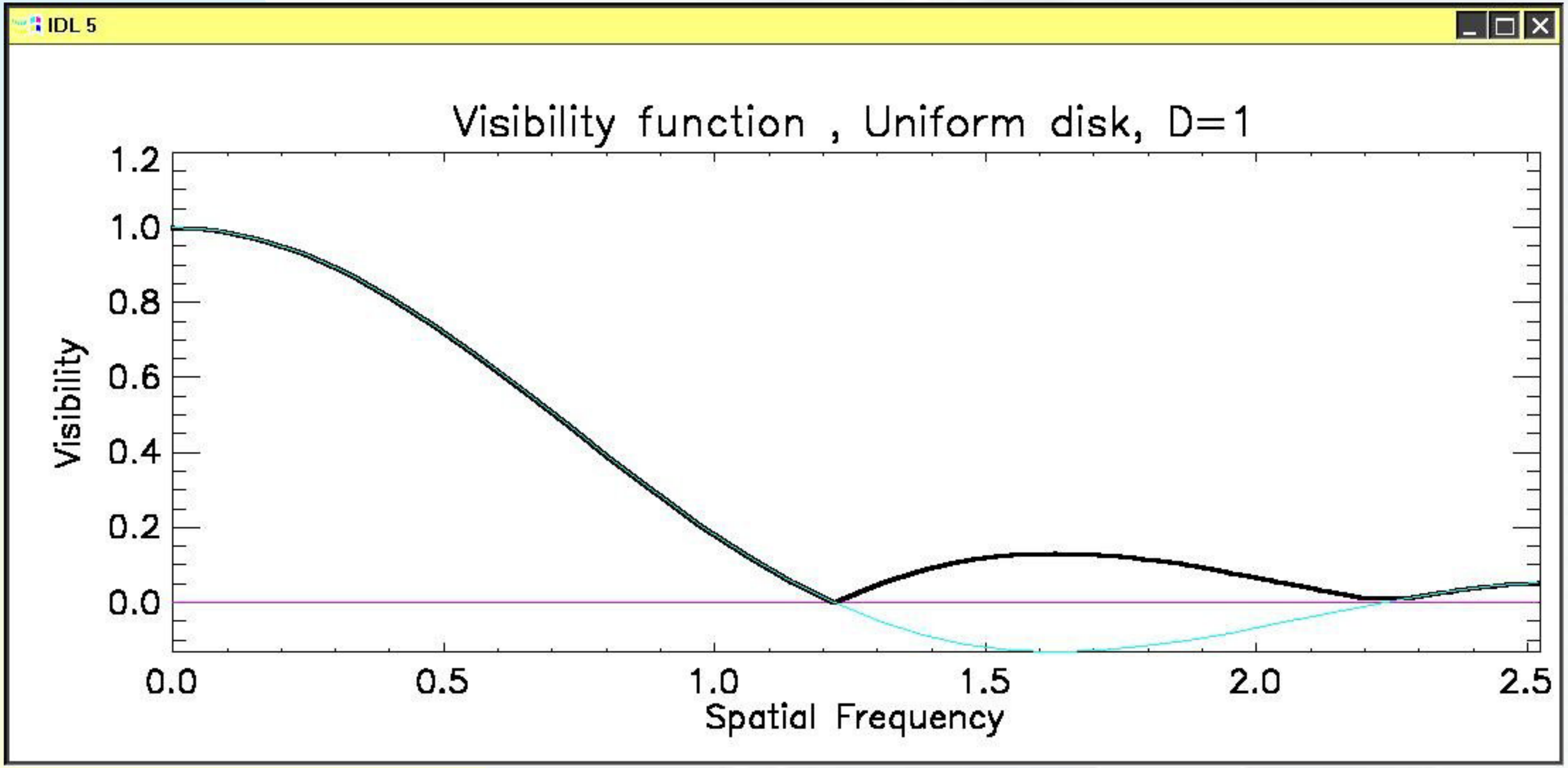
one should be able to demonstrate the next result, i.e., the visibility V of the fringes observed for the case of a uniformly bright circular disk source with an angular diameter θ_{UD} by means of an interferometer with a baseline B is given by:

$$\upsilon = \left(\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right) = \left| \gamma_{12}(0) \right| = TF(I') = \frac{2 J_1(\pi \theta_{UD} B / \lambda)}{\pi \theta_{UD} B / \lambda}$$

[redacted]

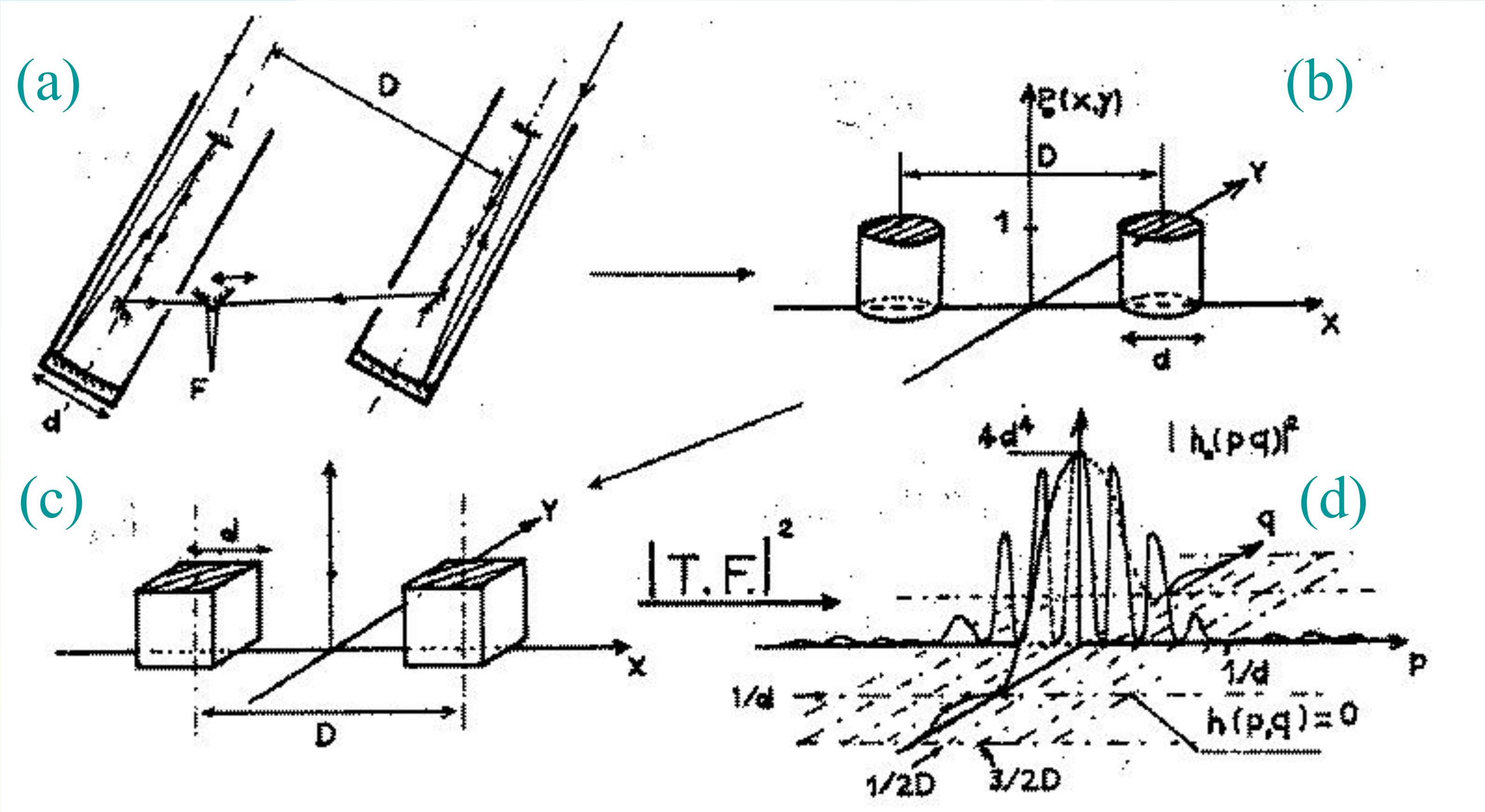
If the source is characterized by a uniform disk light distribution, the corresponding visibility function is given by

$$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right) = \left| \gamma_{12}(0) \right| = TF(I') = \frac{2J_1(\pi\theta_{UD}B/\lambda)}{\pi\theta_{UD}B/\lambda}$$



An introduction to optical/IR interferometry

8.1 The fundamental theorem: 2 telescope interferometer



Two coupled optical telescopes: simplified optical scheme (a). Distribution of the complex amplitude for the case of two circular (b) or square (c) apertures and corresponding impulse response (d).

An introduction to optical/IR interferometry

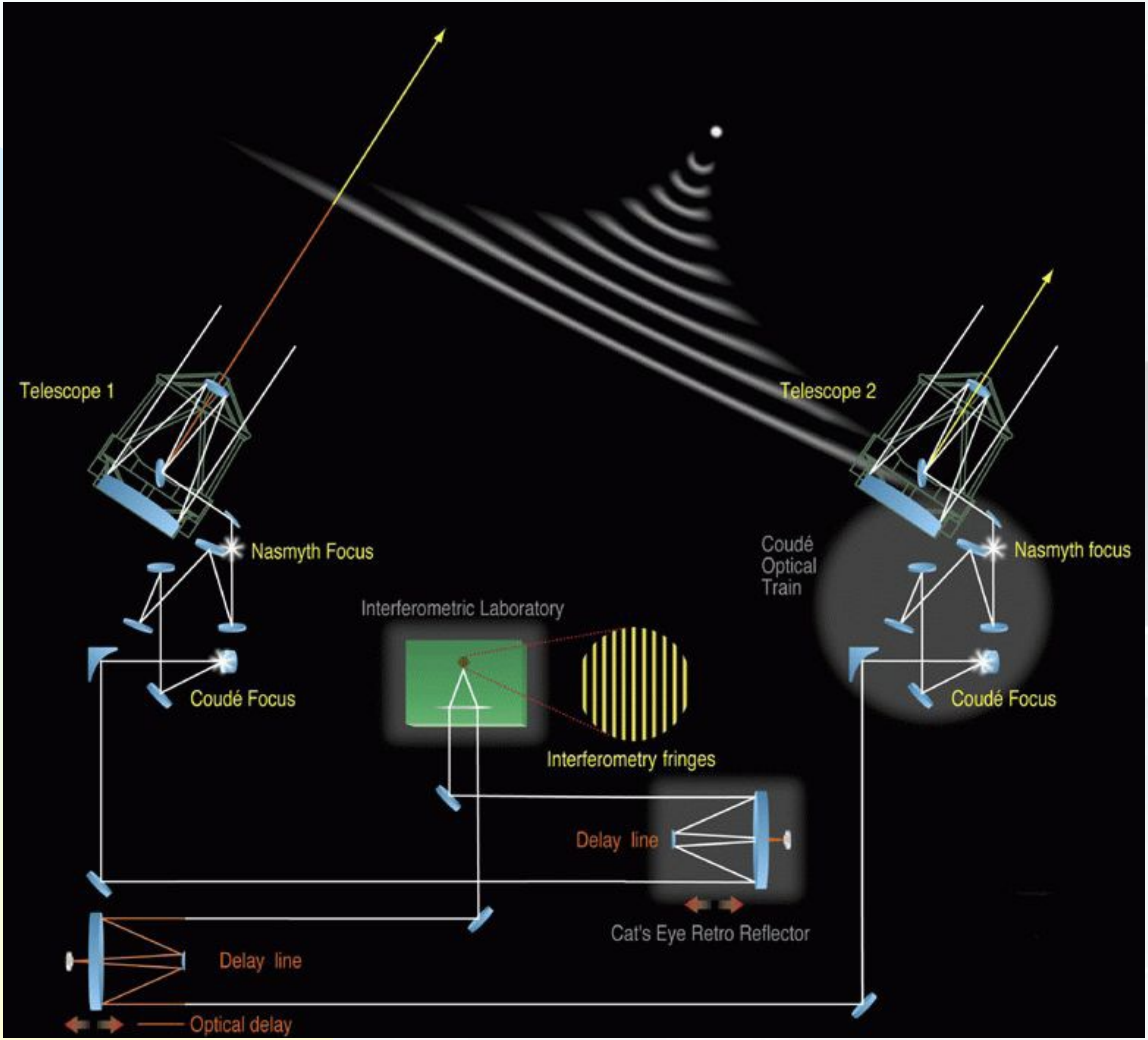
8.1 The fundamental theorem: 2 telescope interferometer

$$h(p,q) = TF(P(x,y))(p,q) = \int_{R^2} P(x,y)\exp\left[-i2\pi(px+qy)\right]dxdy \qquad (8.1.10)$$

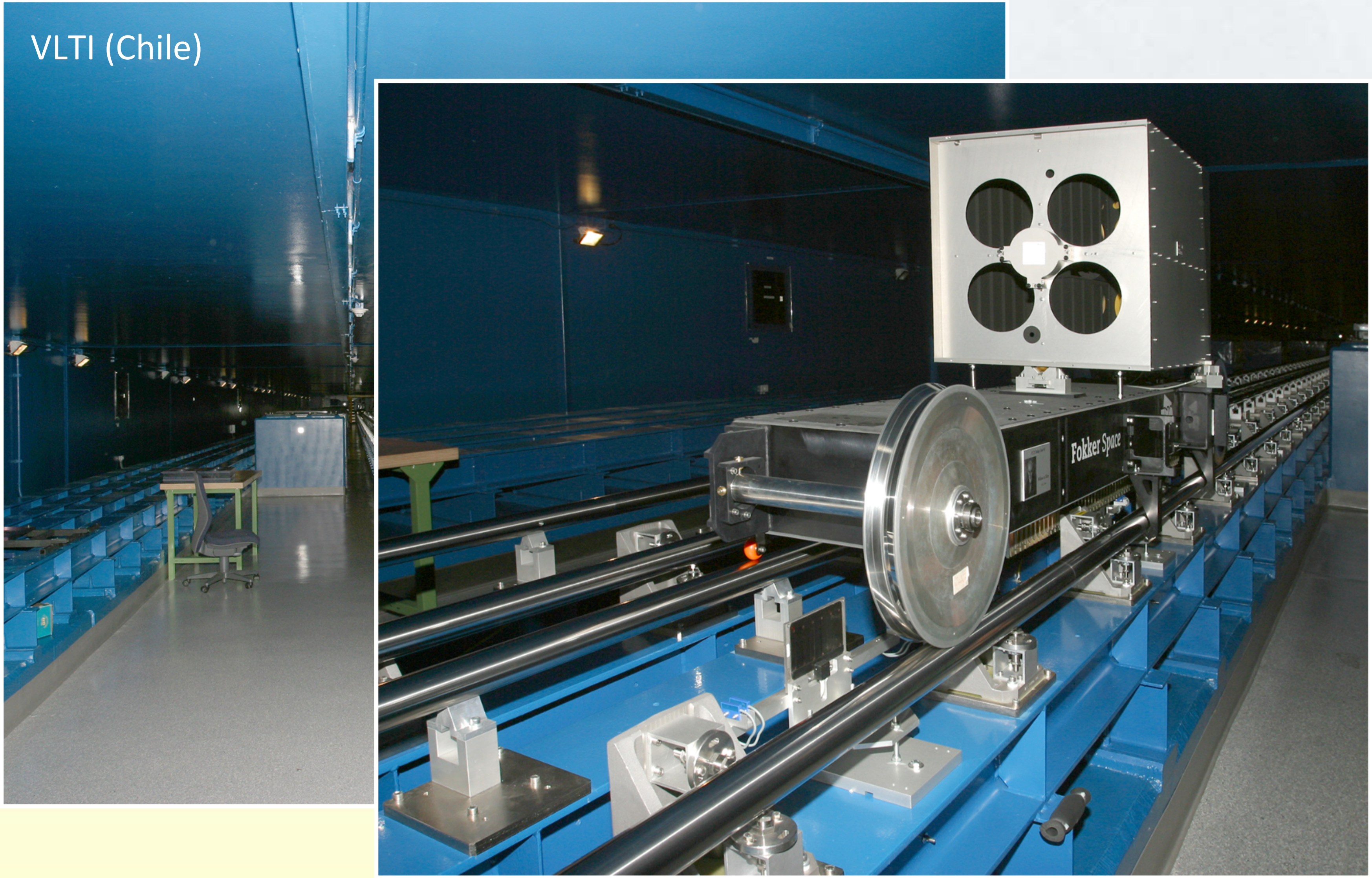
$$\begin{aligned}
h(p,q) &= TF(P_0(x+D/2)+P_0(x-D/2))(p,q) = \\
&TF(P_0(x+D/2))(p,q) + TF(P_0(x-D/2))(p,q) = \\
&\exp(i\pi D)\,TF(P_0(x))(p,q) + \exp(-i\pi D)\,TF(P_0(x))(p,q) = \\
&(\exp(i\pi D)+\exp(-i\pi D))\,TF(P_0(x))(p,q) = \\
&2\cos(\pi D)\,TF(P_0(x))(p,q)
\end{aligned} \qquad (8.1.11)$$

For the particular case of two square apertures:

$$i(p,q) = \left|h(p,q)\right|^2 = 4\cos^2(\pi pD)\,d^4\left(\frac{\sin(\pi qd)}{\pi qd}\right)^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \qquad (8.1.12)$$


Telescope 1
Telescope 2
Nasmyth Focus
Nasmyth focus
Coudé
Optical
Train
Interferometric Laboratory
Coudé Focus
Coudé Focus
Interferometry fringes
Delay line
Cat's Eye Retro Reflector
Delay line
Optical delay

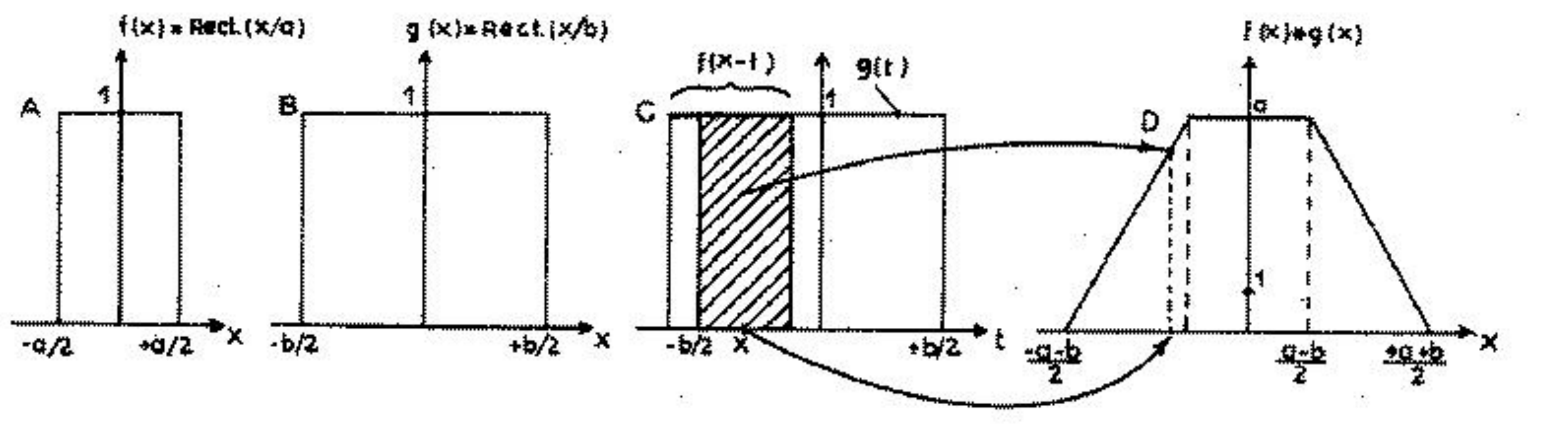
Delay lines at the VLTI



An introduction to optical/IR interferometry

8.2 The convolution theorem

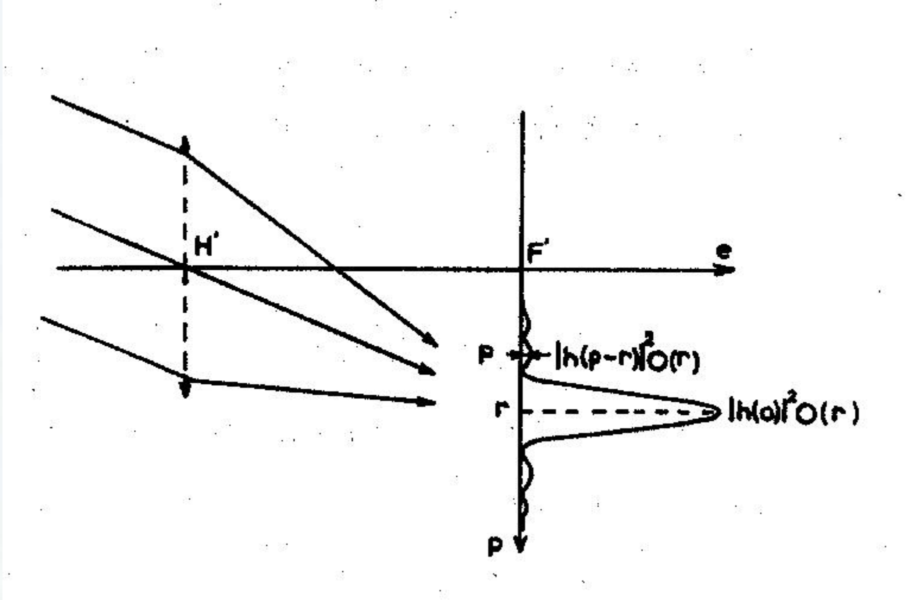
$$f(x) * g(x) = (f * g)(x) = \int_{R^n} f(x-t)g(t)dt$$



Convolution product of two 1D rectangle functions. A) f(x), B) g(x), C) g(t) and f(x-t); the dashed area represents the integral of the product of f(x-t) and g(t) for the given x offset, D) f(x)*g(x) = (f*g)(x) represents the previous integral as a function of x.

An introduction to optical/IR interferometry

8.2 The convolution theorem



$e(p,q) = O(p,q) * |h(p,q)|^2,$

$$e(p,q) = \int_{R^2} O(r,s) \left| h(p-r, q-s) \right|^2 drds$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

For the case of a point-like source:

$O(p,q) = E\,\delta(p,q)$, (8.2.1)

[$\delta(x) = 0$ if $x \neq 0$, $\delta(x) = \infty$ if $x = 0$] and (8.2.2)

$e(p,q) = O(p,q) * |h(p,q)|^2 = E\,\delta(p,q) * |h(p,q)|^2 = E\,|h(p,q)|^2$ (8.2.3)

An introduction to optical/IR interferometry

8.2 The convolution theorem

More generally, since
TF_(f * g) = TF_(f) TF_(g). (8.2.4)

We find, because
$e(p,q) = O(p,q) * |h(p,q)|^2$ (8.2.5)

that:
$TF\_(e(p,q)) = TF\_(O(p,q))\, TF\_(|h(p,q)|^2)$, (8.2.6)

and, finally,
$O(p,q) = TF^{-1}\,[TF\_(e(p,q)) / TF\_(|h(p,q)|^2)]$. (8.2.7)

An introduction to optical/IR interferometry

8.2 The convolution theorem

$O(p,q) = (\lambda^2 E / \phi^2)\, \Pi(p \lambda / \phi)\, \Pi(q \lambda / \phi).$ (8.2.8)

$e(p,q) = O(p,q) * |h_0(p,q)|^2.$

$e(p) = O(p) * |h_0(p)|^2,$ (8.2.9)

$$e(p) = 2d^2(\lambda/\phi)\sqrt{E}\int_{p-\phi/2\lambda}^{p+\phi/2\lambda}\left(\frac{\sin(\pi rd)}{\pi rd}\right)^2 \cos^2(\pi rD)dr \quad (8.2.10)$$

$\left(\frac{\sin(\pi rd)}{\pi rd}\right)^2 \approx$ Cte sur $[p-\phi/2\lambda, p+\phi/2\lambda]$, and (8.2.11)

$$e(p) = 2d^2(\lambda/\phi)\sqrt{E}\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2\int_{p-\phi/2\lambda}^{p+\phi/2\lambda} \cos^2(\pi rD)dr. \quad (8.2.12)$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

$$e(p) = 2d^2 \left(\frac{\sin(\pi p d)}{\pi p d} \right)^2 \left[O(p) * \cos^2(\pi p D) \right], \qquad (8.2.13)$$

$$e(p) = 2d^2 \left(\frac{\sin(\pi p d)}{\pi p d} \right)^2 \left[\frac{1}{2} \int_R O(p)\,dp + \frac{1}{2} O(p) * \cos(2\pi p D) \right] \qquad (8.2.14)$$

$$e(p) = A \left[B + \frac{1}{2} \mathrm{Re}\left(O(p) * \exp(i 2\pi p D) \right) \right], \qquad (8.2.15)$$

$$A = 2d^2 \left(\frac{\sin(\pi p d)}{\pi p d} \right)^2 \quad et \quad B = \frac{1}{2} \int_R O(p)\,dp, \qquad (8.2.16)$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

$$e(p) = A\left[B + \frac{1}{2}\mathrm{Re}\left(\int_R O(r)\exp\left(i2\pi(p-r)D\right)dr\right)\right], \qquad (8.2.17)$$

$$e(p) = A\left[B + \frac{1}{2}\cos(2\pi pD)\,TF\_(O(r))(D)\right], \qquad (8.2.18)$$

$$\gamma(D) = (e_{max} - e_{min}) / (e_{max} + e_{min}), \qquad (8.2.19)$$

$$\gamma(D) = TF\_(O(r))(D) / (2B) = TF\_(O(r))(D) / \int O(p)\,dp. \qquad (8.2.20)$$

An introduction to optical/IR interferometry

8.3 The Wiener-Khintchin theorem

In our case, this theorem merely states that the Fourier tranform of the PSF (see Eq. (8.2.7)) is the auto-correlation function of the distribution of the complex amplitude in the pupil plane:

$$TF(\left|h(a,b)\right|^2) = \iint P^*(x,y)P(x-a,y-b)dx\,dy$$

Démonstration (1/2):

Let us evaluate: $$\iint P^*(x,y)P(x-a,y-b)\,dx\,dy$$

Let us also remind:

$$h(p,q)=\iint P(x,y)\exp(-i2\pi(px+qy))\,dx\,dy$$

And thus (Fourier inverse transform),

$$P(x,y)=\iint h(p,q)\exp(i2\pi(px+qy))\,dp\,dq$$, and also

$$P^*(x,y)=\iint h^*(p',q')\exp(-i2\pi(p'x+q'y))\,dp'\,dq'$$

We then find that

$$\iint P^*(x,y)P(x-a,y-b)\,dx\,dy=$$

$$\iint\iint h^*(p',q')h(p,q)\exp(-i2\pi(ap+bq))\,dp\,dq$$

$$\iint \exp(-i2\pi((p'-p)x+(q'-q)y))\,dx\,dy\,dp'\,dq'$$

Démonstration (2/2):

Since

$$\iint \exp(-i2\pi((p'-p)x+(q'-q)y))dxdy =$$

$\delta(p-p')\delta(q-q')$, we then find

$$\iint P^{*}(x,y)P(x-a,y-b)dx\,dy =$$

$$\iint \iint h^{*}(p',q')h(p,q)\delta(p-p')\delta(q-q')$$

$$\exp(-i2\pi(ap+bq))dpdqdp'dq'$$

and finally

$$\iint P^{*}(x,y)P(x-a,y-b)dx\,dy =$$

$$\iint \left|h(p,q)\right|^{2}\exp(-i2\pi(ap+bq))dpdq =$$

$$= TF(\left|h(p,q)\right|^{2})(a,b).$$

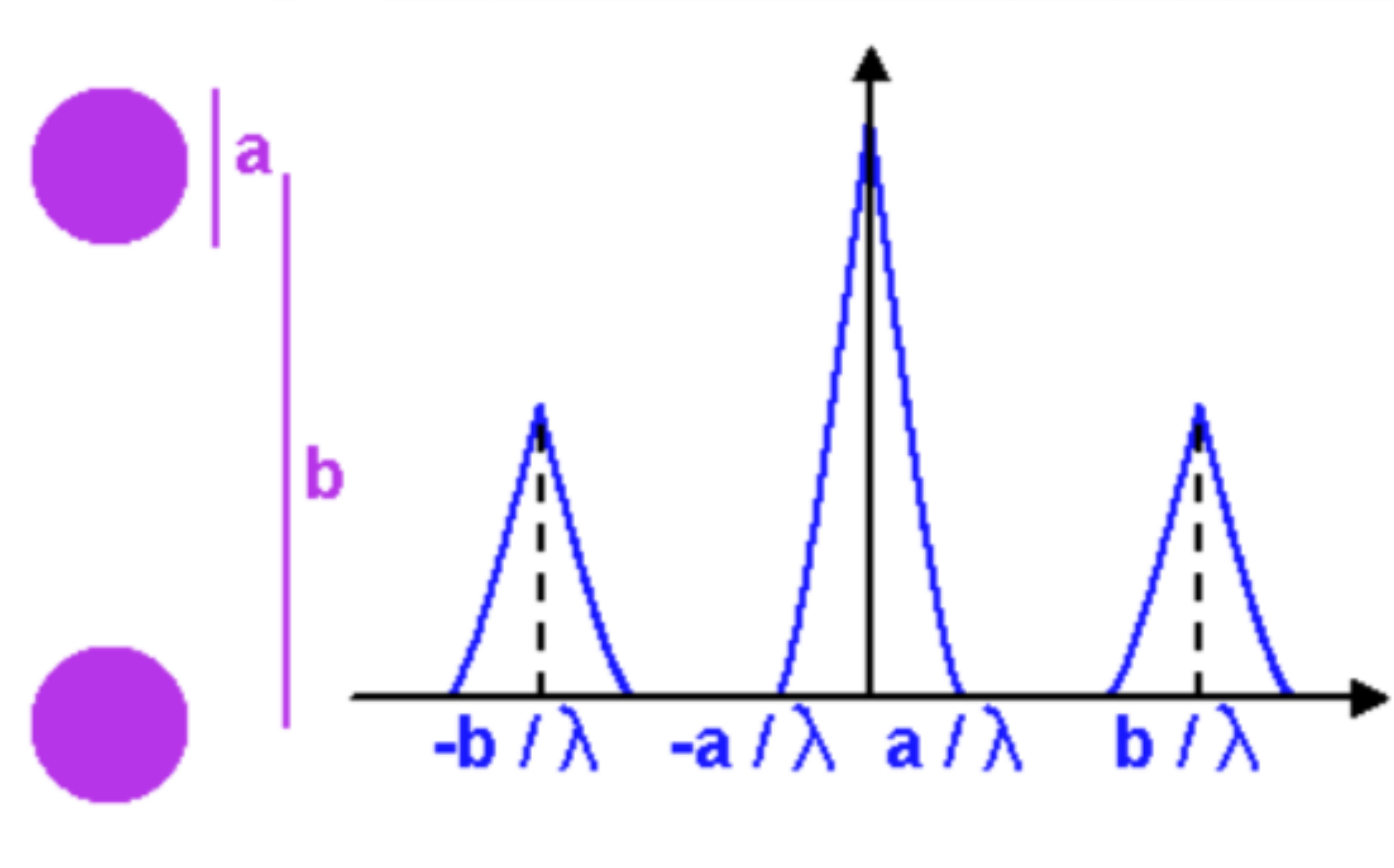
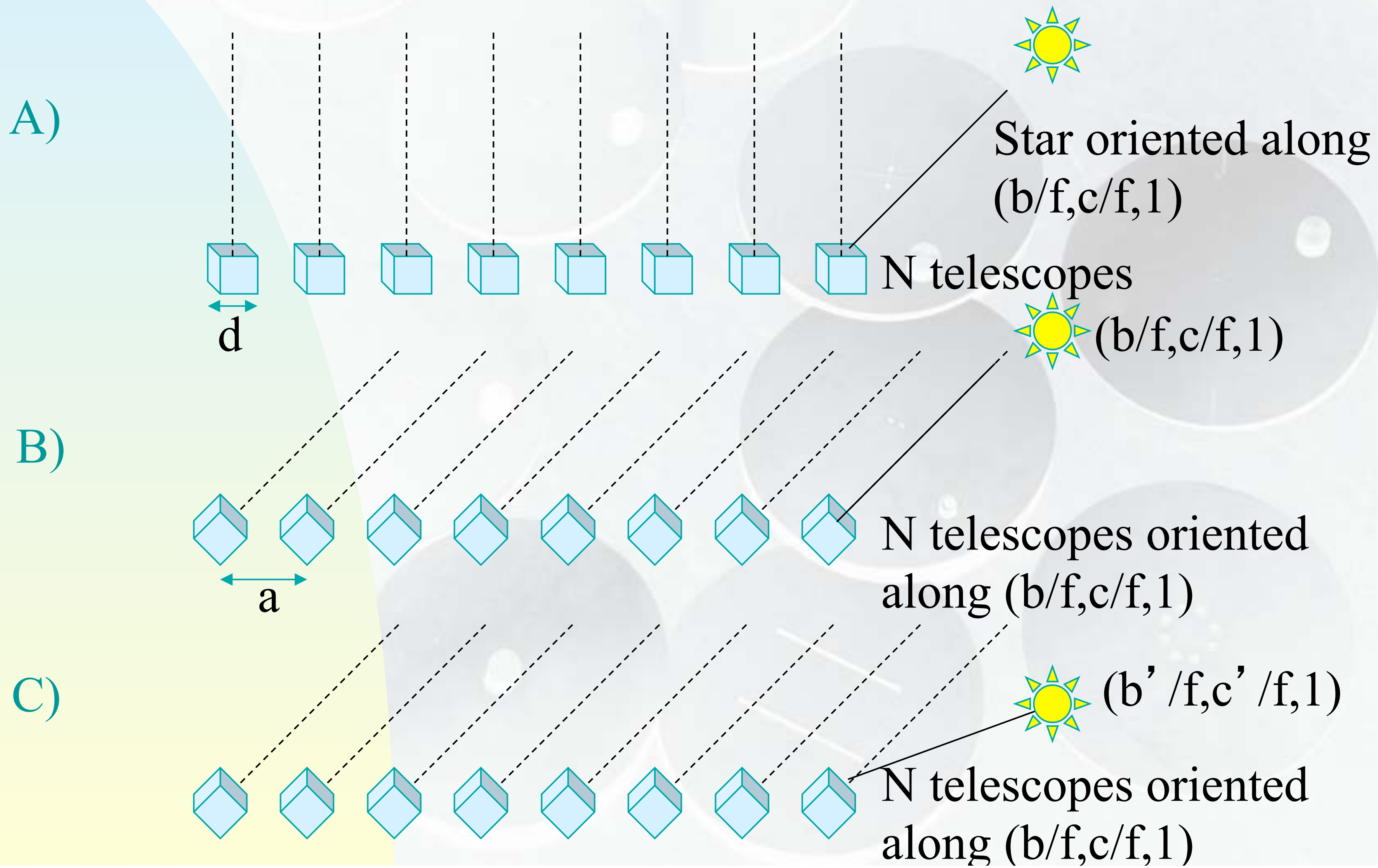


Diagram illustrating the autocorrelation as a function of the space frequency, for the case of an interferometer composed of 2 telescopes, with a diameter a, separated by the baseline b. The autocorrelation of the pupil gives access to high space frequencies.

• Exercices: Calculate the response functions (PSFs) for the cases described in the figures below!



• Exercices: Calculate the response functions (PSFs) for the cases described in the figures below!