

**“Choisir, c’est se priver du reste ...”**

**André Gide**

<http://www.aeos.ulg.ac.be/teaching.php>

Astrophysics and Space Techniques (2) [ASTR0004-2](#) Surdej Jean

Slides of the 4<sup>th</sup> lecture “Astrophysics and Space Techniques” (2015-16)

And 1<sup>st</sup> lecture “Observing the sky”. Un ancien vidéo de ce cours en français est accessible via le lien <http://orbi.ulg.ac.be/handle/2268/74101>  
fichier pdf taille (octets) : 1698109

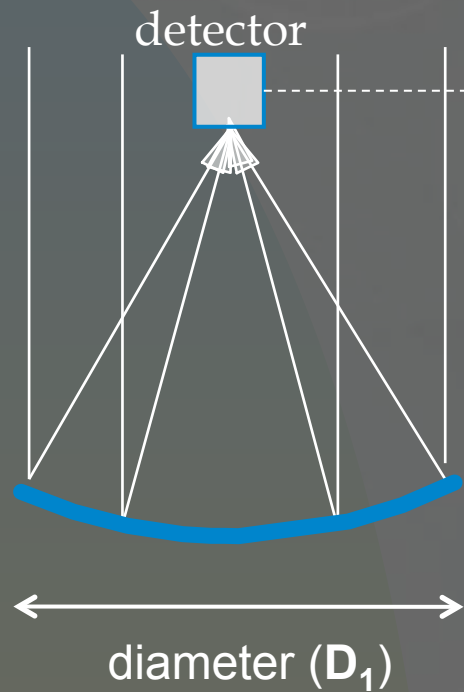


# *Introduction to optical/IR interferometry*

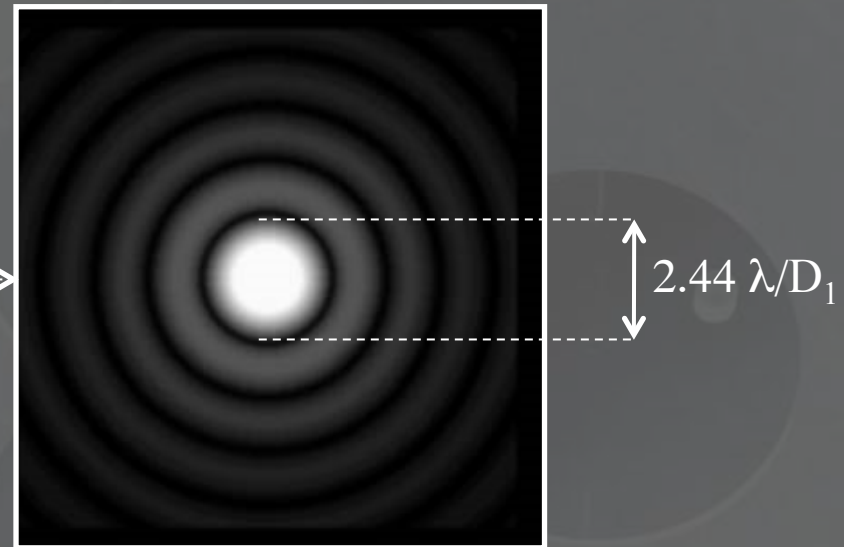
**Jean Surdej**  
([JSurdej@ulg.ac.be](mailto:JSurdej@ulg.ac.be))

<http://hdl.handle.net/2268/155589>

## Telescope

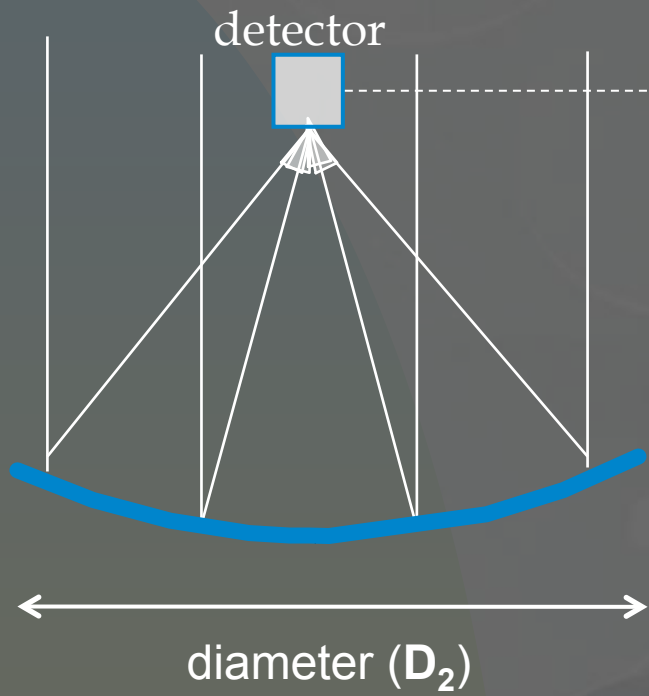


## Airy disk

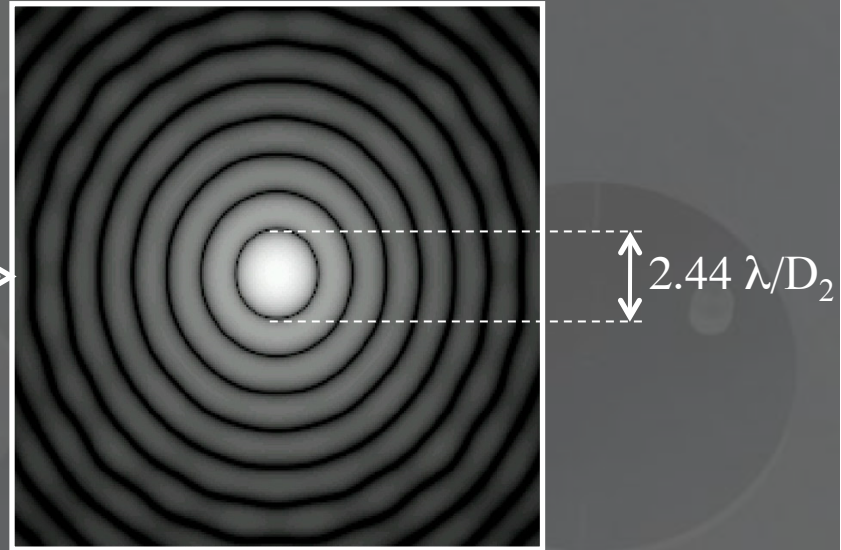


- The image of a star is like a dot!

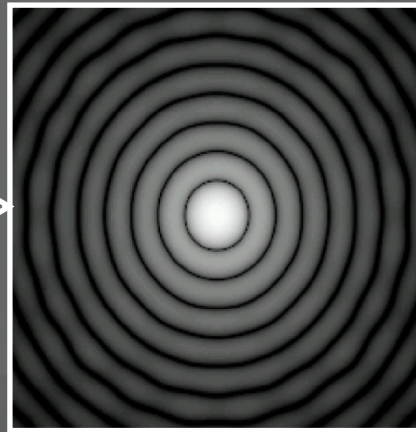
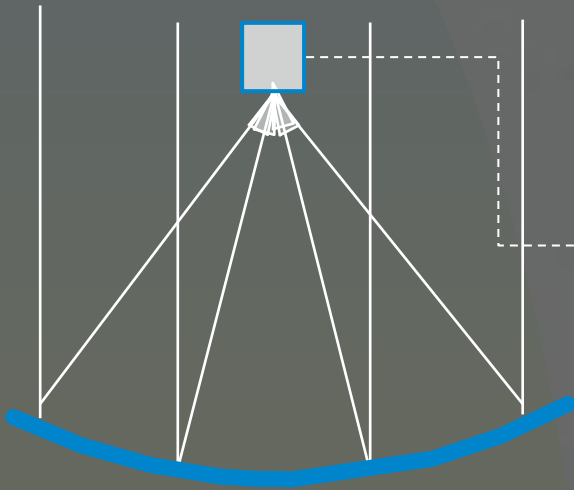
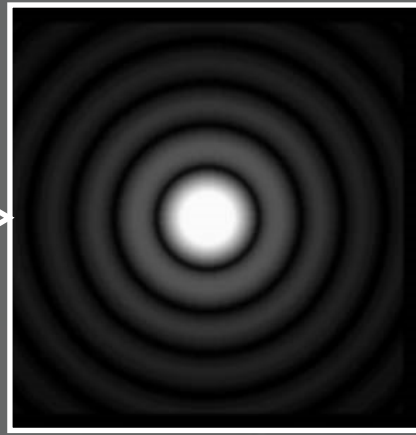
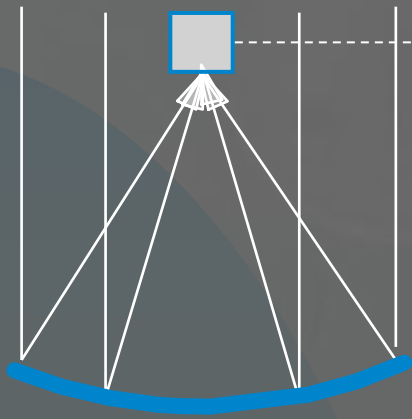
# Telescope



# Airy disk

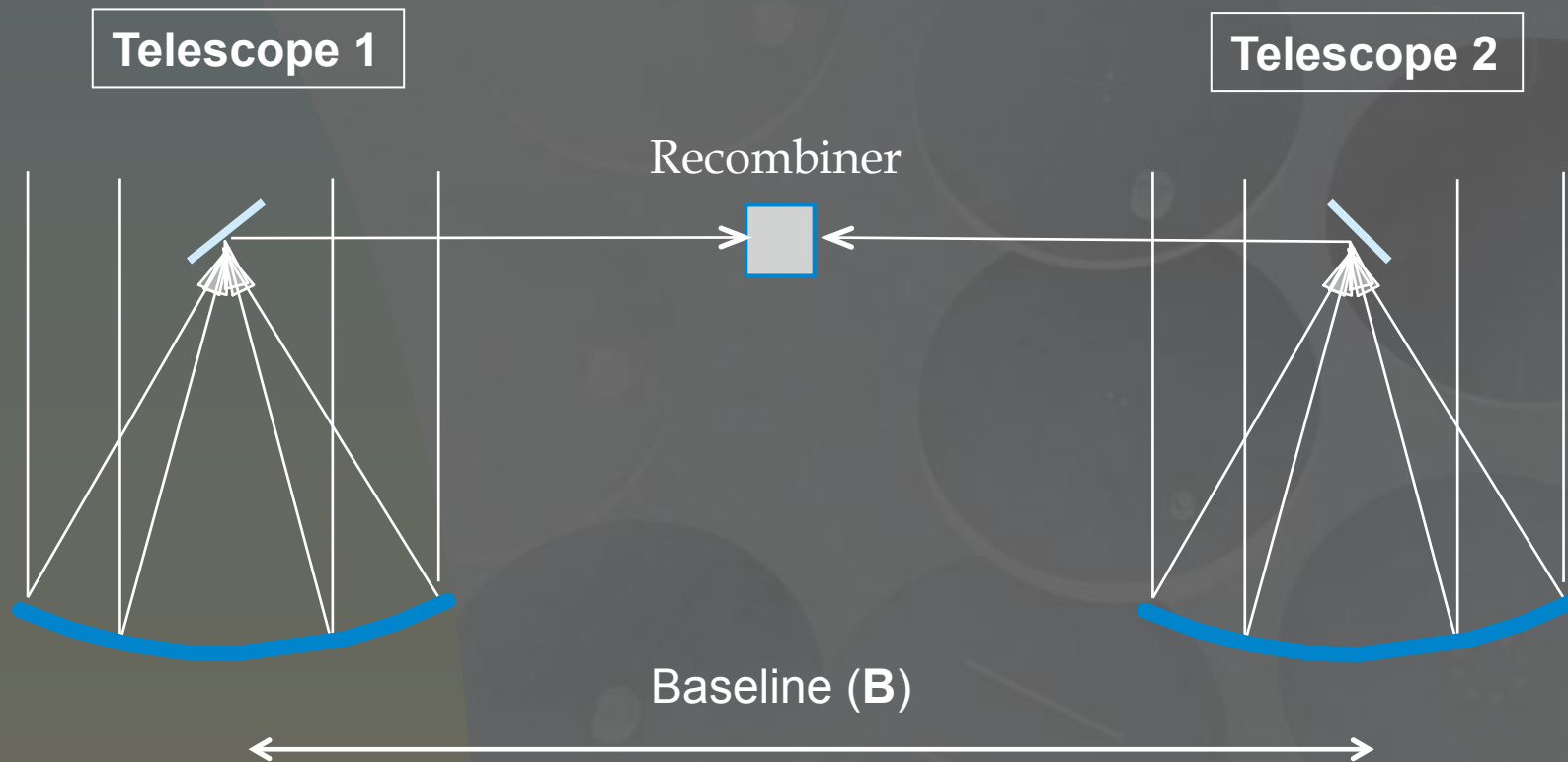


- The image of a star is still like a dot!



- Need for very large telescopes !!!

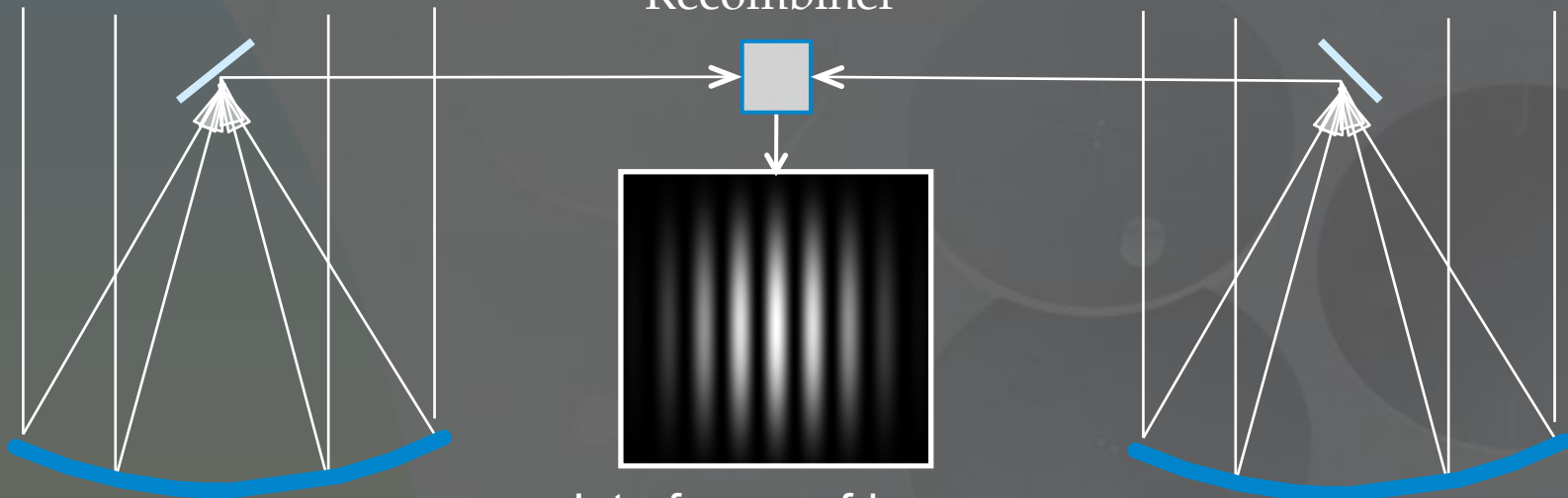
- H. Fizeau and E. Stephan (1868-1870):  
“In terms of angular resolution, two small apertures distant of  $B$  are equivalent to a single large aperture of diameter  $B$ ”



Telescope 1

Telescope 2

Recombiner



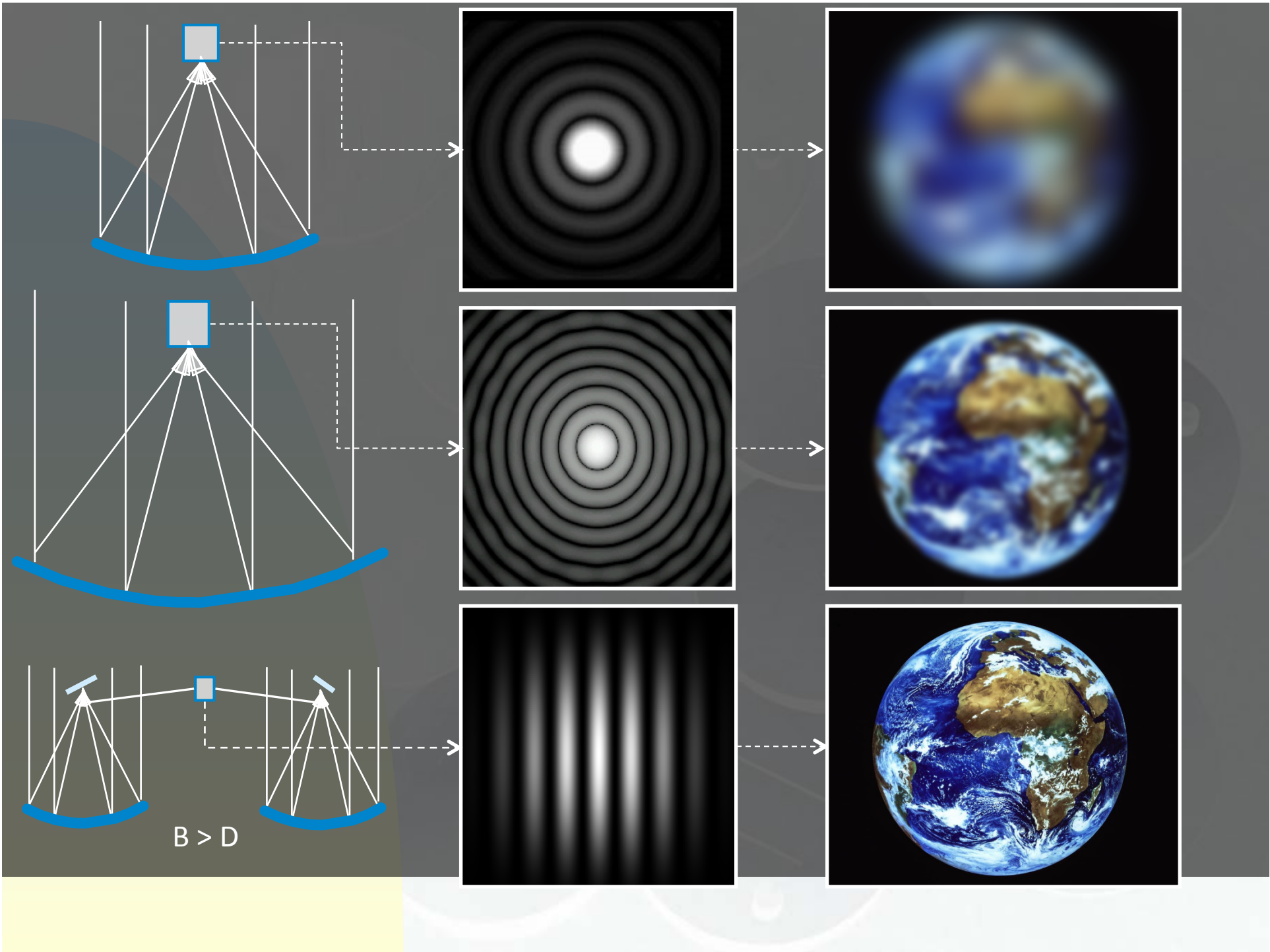
Interference fringes

Inter-fringe =  $\lambda/B$

Baseline (**B**)



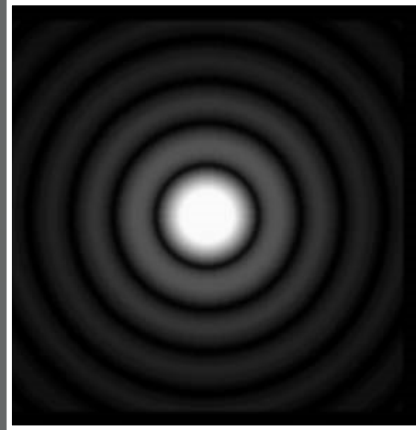








=



⊗



### Convolution theorem

$$I(\zeta, \eta) = \iint PSF(\zeta - \zeta', \eta - \eta') O(\zeta', \eta') d\zeta' d\eta' = PSF(\zeta, \eta) \otimes O(\zeta, \eta)$$

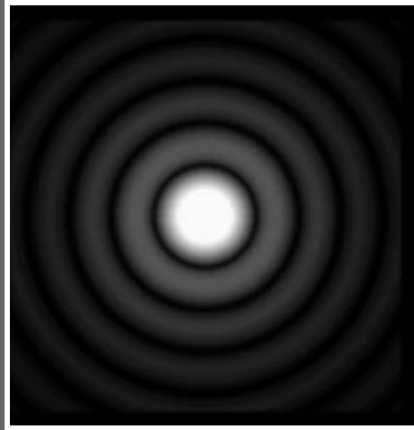
$$TF(I(\zeta, \eta))(u, v) = TF(PSF(\zeta, \eta))(u, v) \cdot TF(O(\zeta, \eta))(u, v)$$

$$\mathbf{u} = \mathbf{B}_u / \lambda, \mathbf{v} = \mathbf{B}_v / \lambda$$

$$O(\zeta, \eta) = TF(-1)TF(O(\zeta, \eta)) = TF(-1)(TF(I(\zeta, \eta)) / TF(PSF(\zeta, \eta)))$$



=



⊗



## Wiener Kitchen theorem

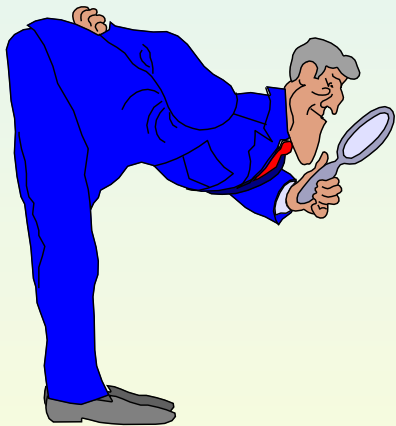
$$TF(PSF(\zeta, \eta))(u, v) = \iint A^*(x, y) A(x+u, y+v) dx dy$$

# An introduction to optical/IR interferometry

- 1 Introduction
- 2 Reminders
- 3 Brief history of stellar diameter measurements
- 4 Interferometry with two independent telescopes
- 5 Light coherence (**Zernicke-van Cittert theorem**)
- 6 Examples of optical interferometers
- 7 Results
- 8 Three important theorems (**Fundamental theorem, Convolution theorem and Wiener-Khintchin theorem**)!

# An introduction to optical/IR interferometry

- 1 Introduction



Z

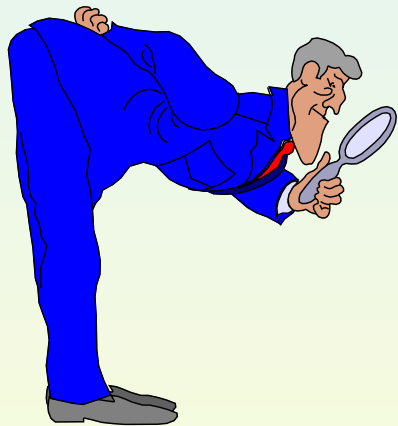
$\Delta$



# An introduction to optical/IR interferometry

- 1 Introduction

$$\rho = R / z \quad (1.1)$$



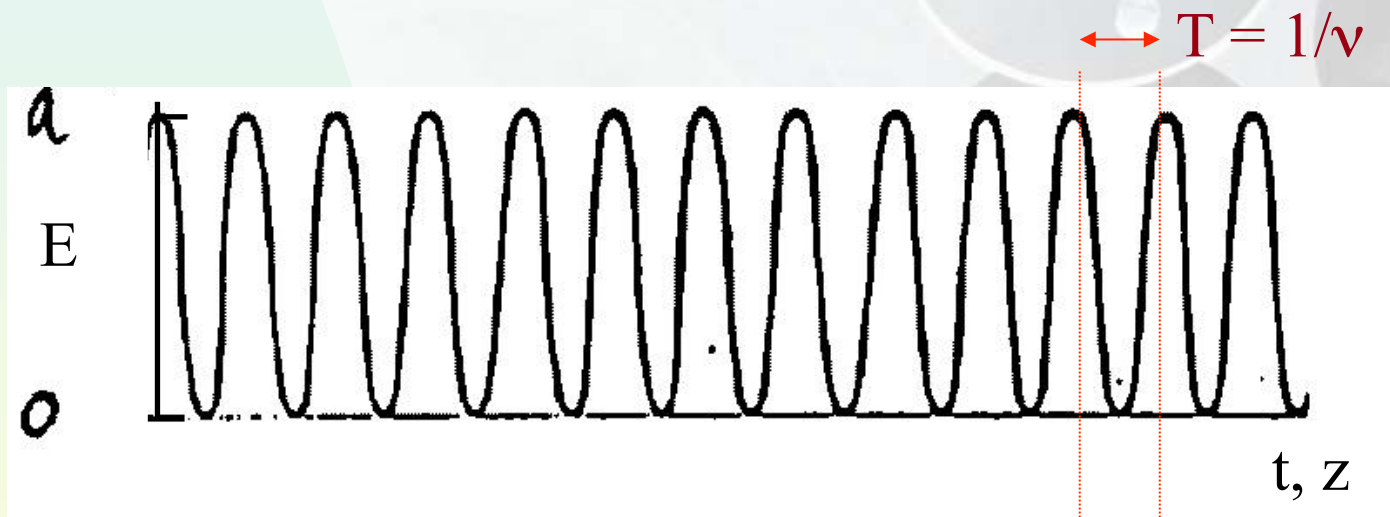
$$\Delta = 2\rho$$
A diagram of a lens with radius  $R$  and focal length  $z$ . A blue line labeled  $z$  extends from the lens to the magnifying glass. A yellow double-headed arrow indicates a distance of  $2R$ .

$$F = f / \rho^2 \quad (1.2)$$

$$T_{\text{eff}} = (F/\sigma)^{1/4} = (f / \sigma \rho^2)^{1/4} \quad (1.3)$$

# An introduction to optical/IR interferometry

- 2 Reminders
- 2.1. Representation of an electromagnetic wave



$$E = a \cos[2\pi (\nu t - z / \lambda)] \quad (2.1.1)$$

$$\text{where } \lambda = c T = c / \nu. \quad (2.1.2)$$

# An introduction to optical/IR interferometry

## ■ 2.1. Representation of an electromagnetic wave

$$E = \text{Re}\{ a \exp[i2\pi(\nu t - z / \lambda)] \} \quad (2.1.3)$$

$$E = \text{Re}\{ a \exp[-i \phi] \exp[i2\pi\nu t] \} \quad (2.1.4)$$

where  $\phi = 2\pi z / \lambda.$  (2.1.5)

$$E = a \exp[-i \phi] \exp[i2\pi\nu t] \quad (2.1.6)$$



# An introduction to optical/IR interferometry

## ■ 2.1. Representation of an electromagnetic wave

$$E = A \exp[i2\pi\nu t] \quad (2.1.7)$$

with  $A = a \exp[-i\phi] \quad (2.1.8)$

$$\nu \sim 6 \cdot 10^{14} \text{ Hz for } \lambda = 5000 \text{ \AA}$$

# An introduction to optical/IR interferometry

- 2.1. Representation of an electromagnetic wave

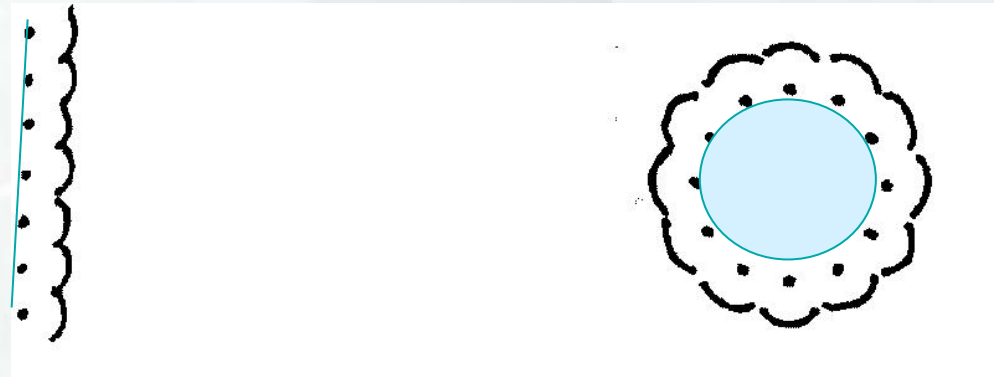
$$\langle E^2 \rangle = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} E^2 dt \quad (2.1.9)$$

$$\langle E^2 \rangle = a^2 / 2 \quad (2.1.10)$$

$$I = A A^* = |A|^2 = a^2. \quad (2.1.11)$$

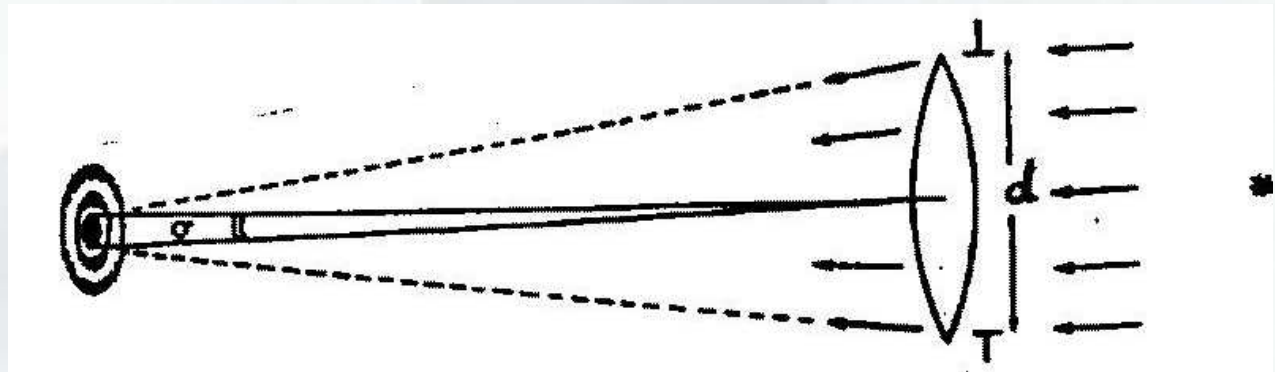
# An introduction to optical/IR interferometry

- 2.2. The Huygens-Fresnel principle



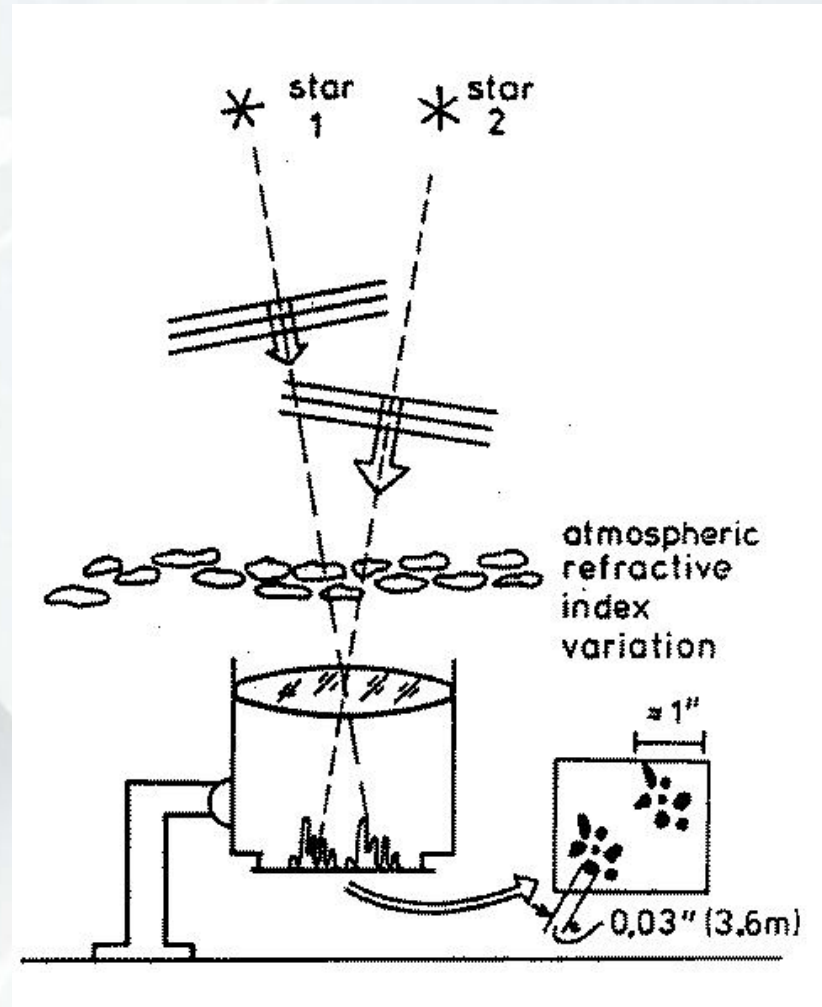
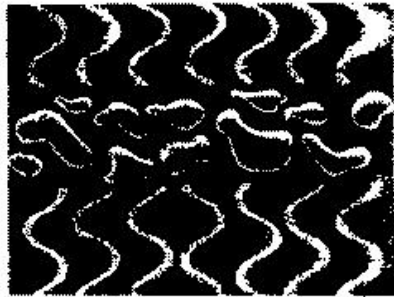
$$\sigma = 2.44 \lambda / d$$

(2.2.1)



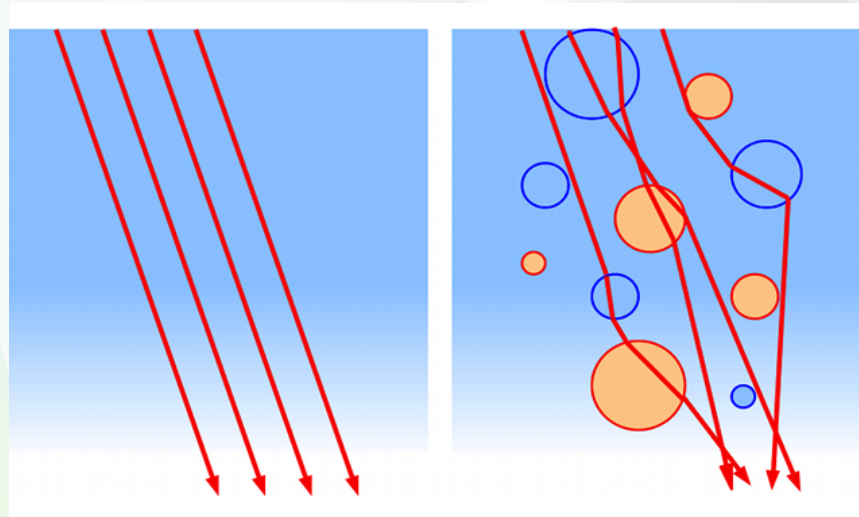
# An introduction to optical/IR interferometry

- 2.3. Atmospheric turbulences



# An introduction to optical/IR interferometry

## ■ 2.3. Atmospheric turbulences



Fried parameter:  $r_0$

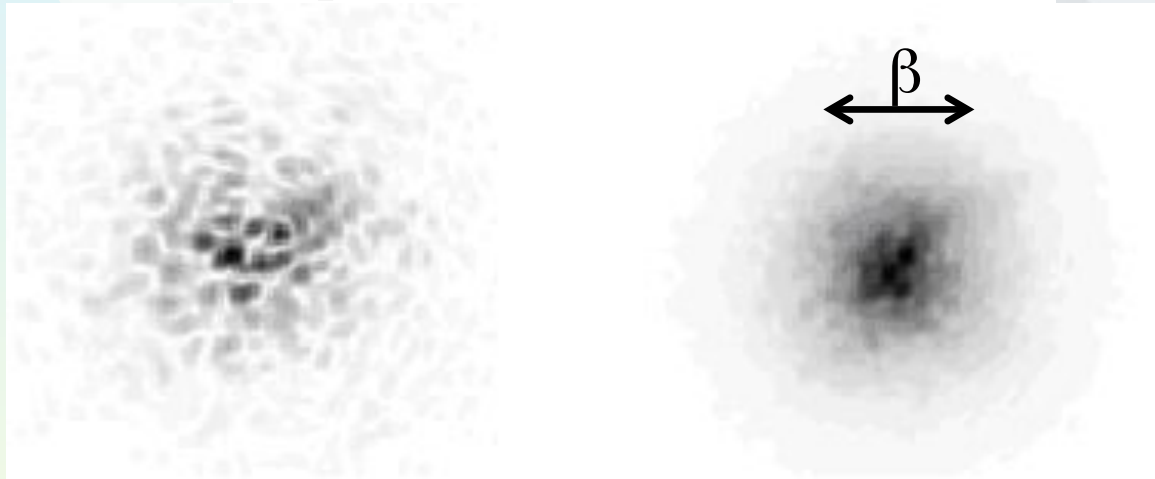
Coherence time (Greenwood time) :  $\tau_0 = 0,314 r_0/v$

Seeing :  $\beta = 0,98 \lambda / r_0$

# An introduction to optical/IR interferometry

- 2.3. Atmospheric turbulences

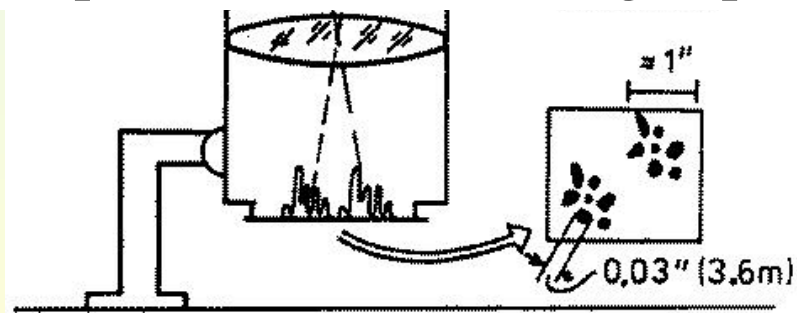
Speckles



Short exposure

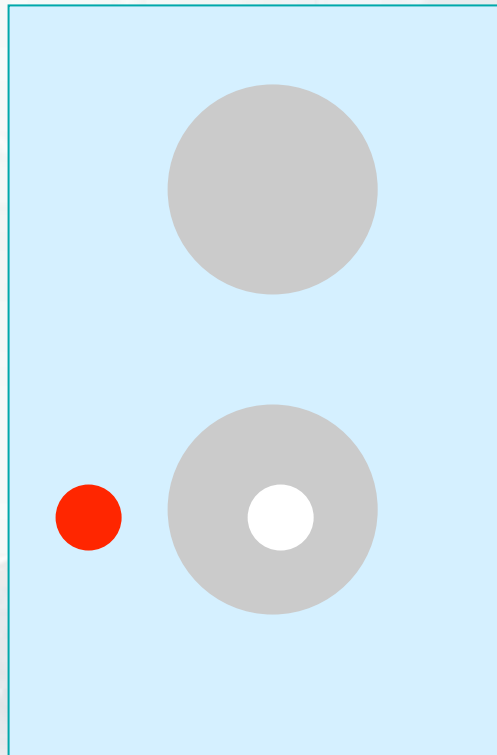
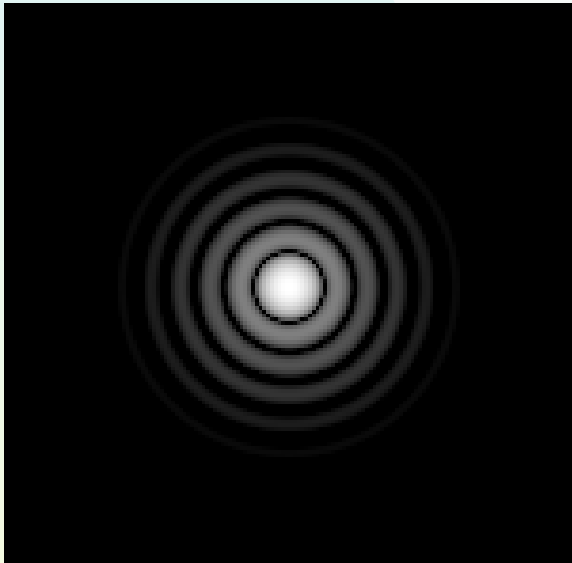
vs

Long exposure



# An introduction to optical/IR interferometry

- 2.2. The Huygens-Fresnel principle



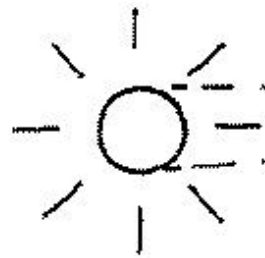
1st experiment!



# An introduction to optical/IR interferometry

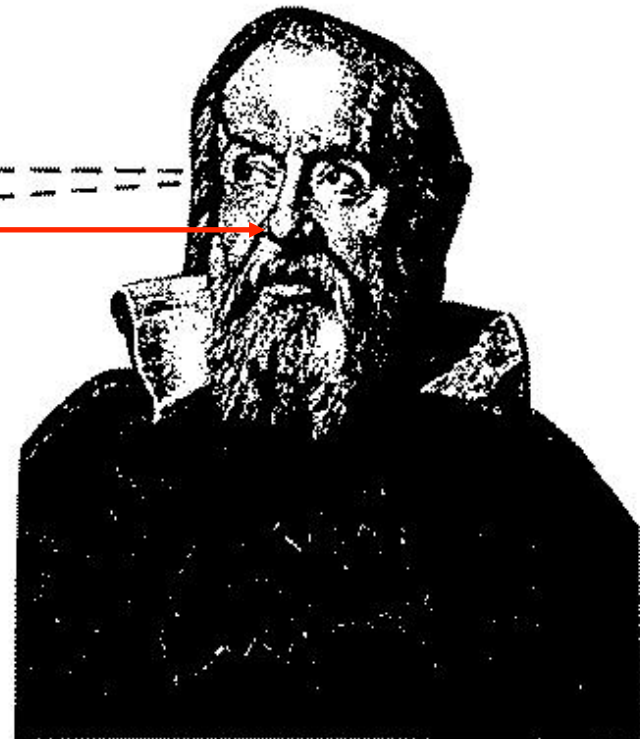
- 3 Brief history of stellar diameter measurements

a) Galileo (1632)



$2e$

$z$



$$\Delta = 2\rho = D / z$$

# An introduction to optical/IR interferometry

## ■ 3 Brief history of stellar diameter measurements

### b) Newton:

$$V_{\odot} - V = -5 \log (z / z_{\odot}), \quad (3.1)$$

$$\Delta = 2 R_{\odot} / z, \quad (3.2)$$

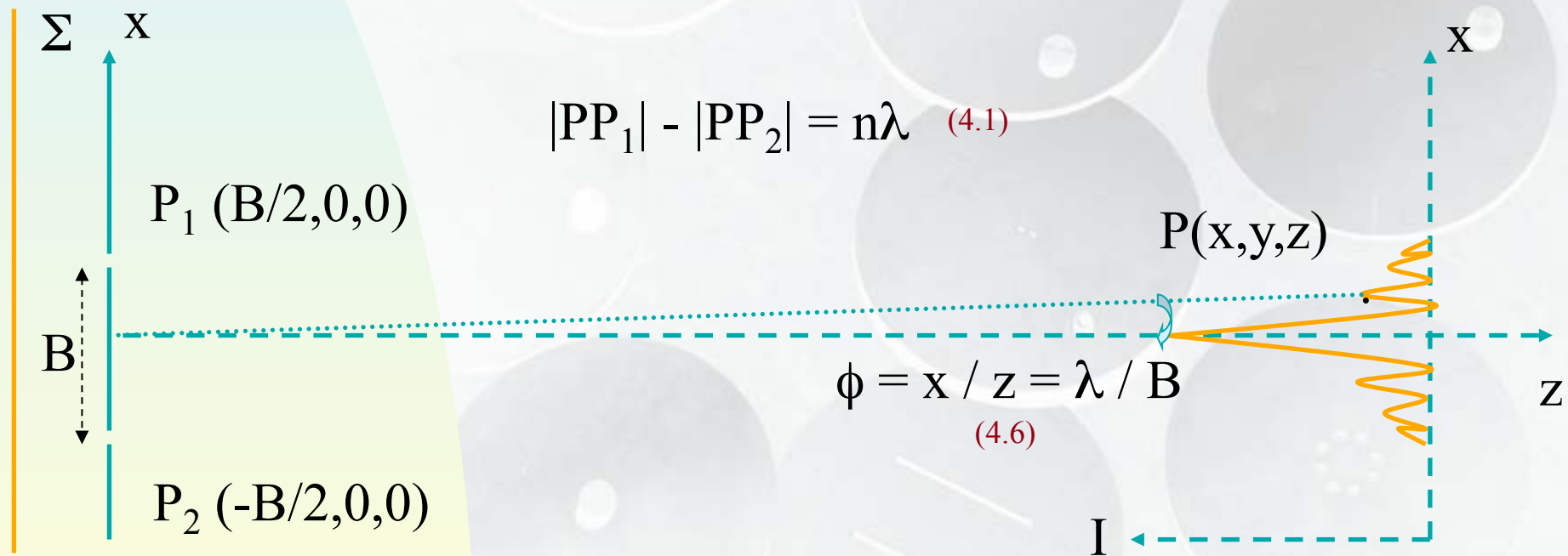
$$\Delta \sim 2 \cdot 10^{-3}'' \quad (8 \cdot 10^{-3}'' ). \quad (3.3)$$

### c) Fizeau-type interferometry

# An introduction to optical/IR interferometry

- 4 Interferometry with two independent telescopes

## a) Young's double hole experiment (24-11-1803)



# An introduction to optical/IR interferometry

## ■ 4 Interferometry with two independent telescopes

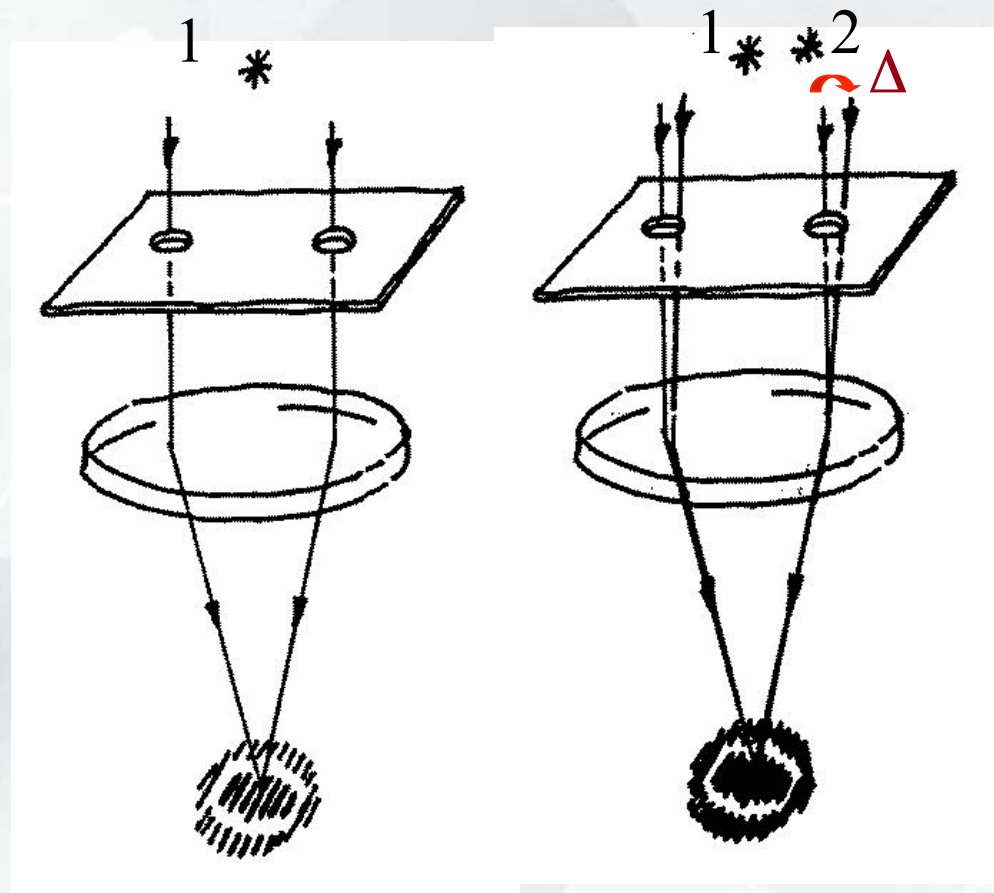
### b) Fizeau ... the father of stellar interferometry (1868)

If  $\Delta \geq \phi/2 = \lambda / (2B)$ , (4.7)

fringe disappearance!

Fringe visibility:

$$v = \left( \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right)$$

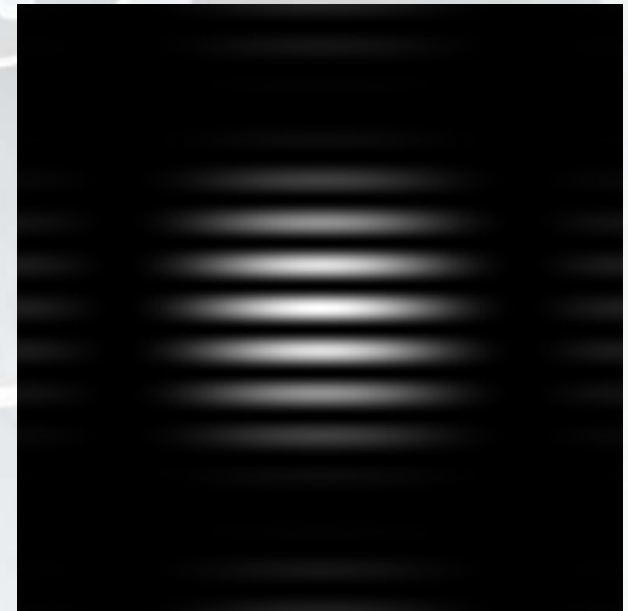
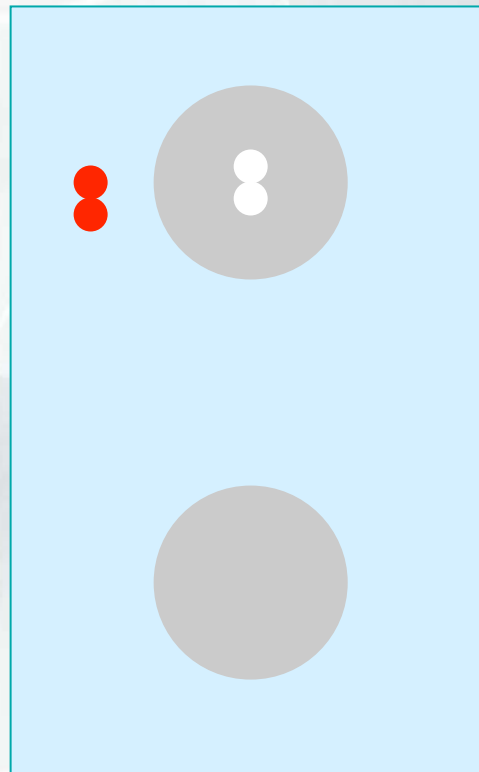


# An introduction to optical/IR interferometry

- 4 Interferometry with two independent telescopes

**b) Fizeau ... the father of stellar interferometry (1868)**

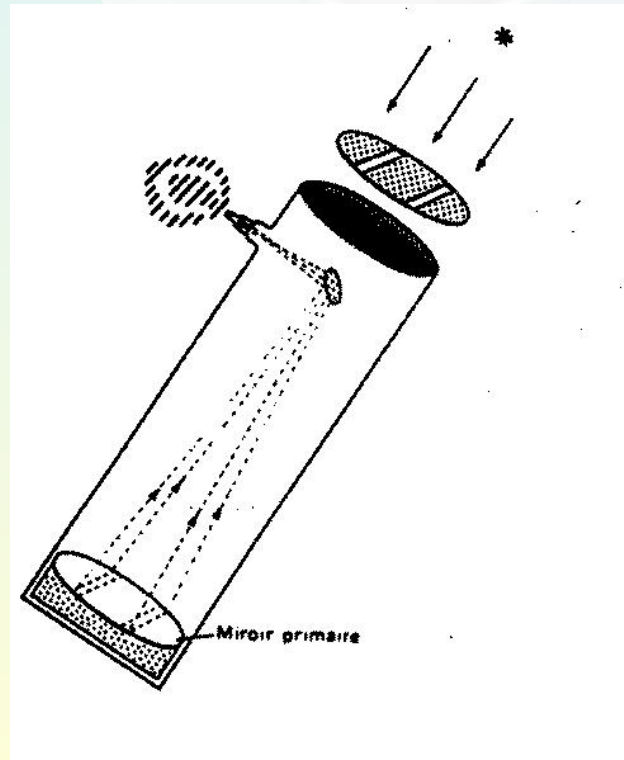
2nd experiment!



# An introduction to optical/IR interferometry

- 4 Interferometry with two independent telescopes

## b) Fizeau ... the father of stellar interferometry (1868)



Stéphan, 1873  
 $\Delta \ll 0,16''$

# An introduction to optical/IR interferometry

- Marseille 80 cm telescope





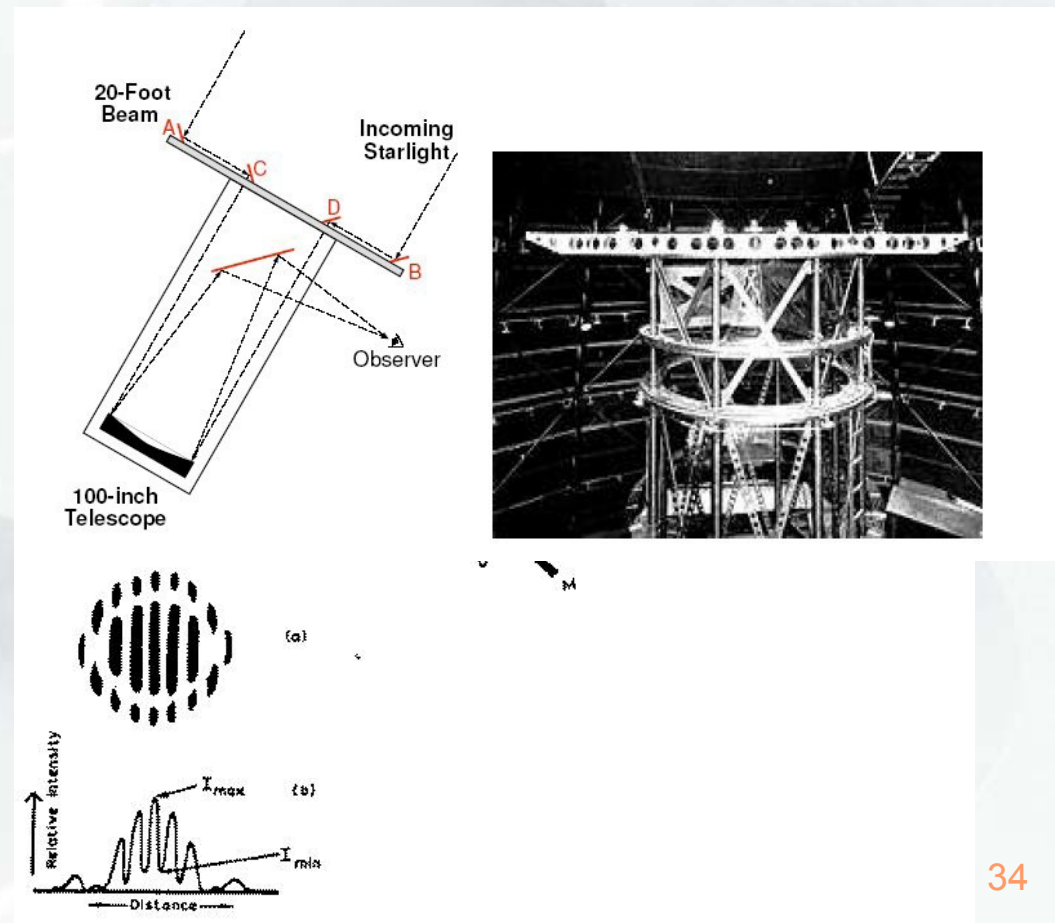


<http://www-obs.cnrs-mrs.fr/dynamique/pap/compact.html>

# An introduction to optical/IR interferometry

- 4 Interferometry with two independent telescopes
- b) Fizeau ... the father of stellar interferometry (1868)**

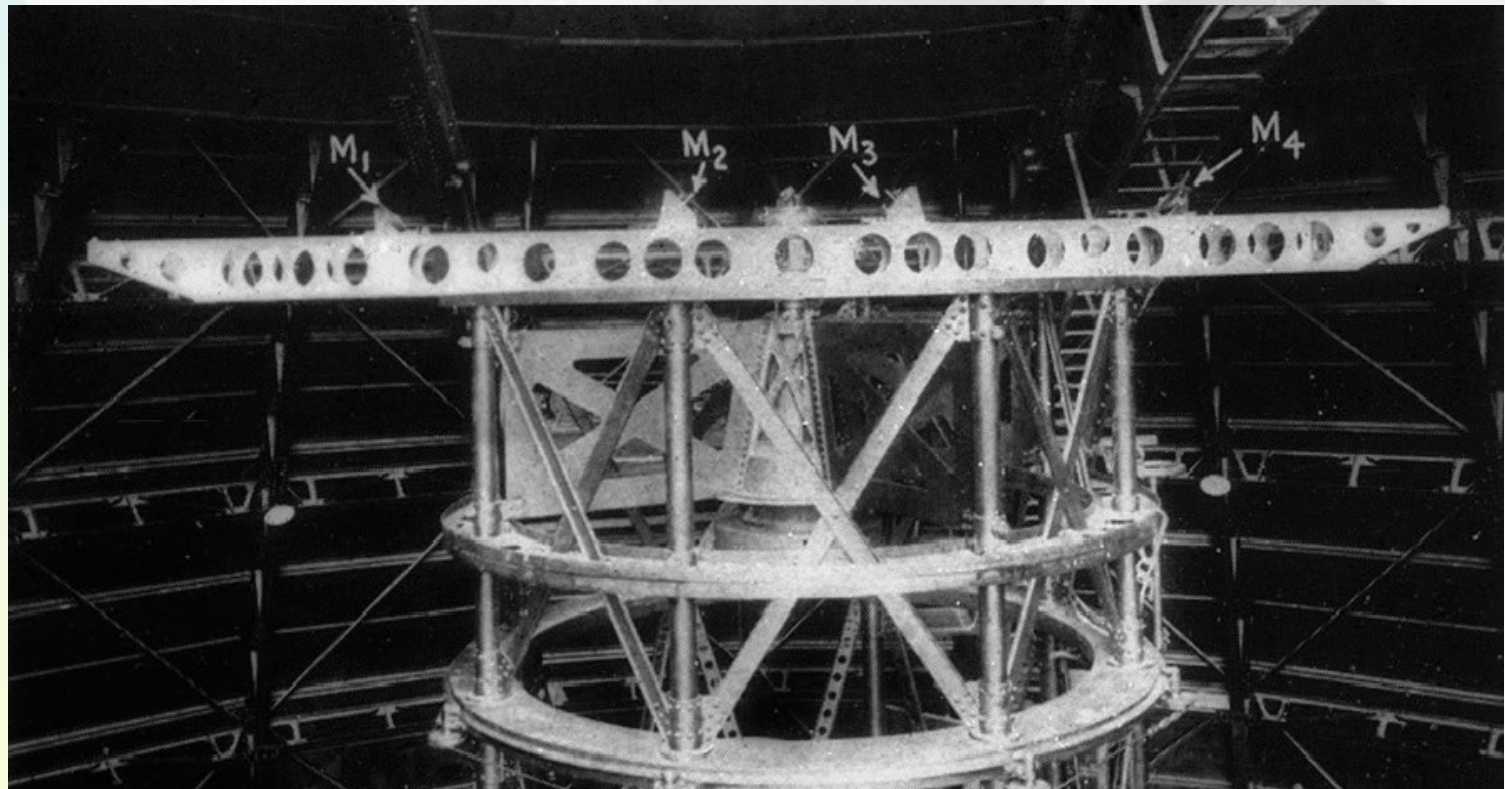
- Michelson, 1890 (satellites of Jupiter)
- Michelson and Pease (1920)



# An introduction to optical/IR interferometry

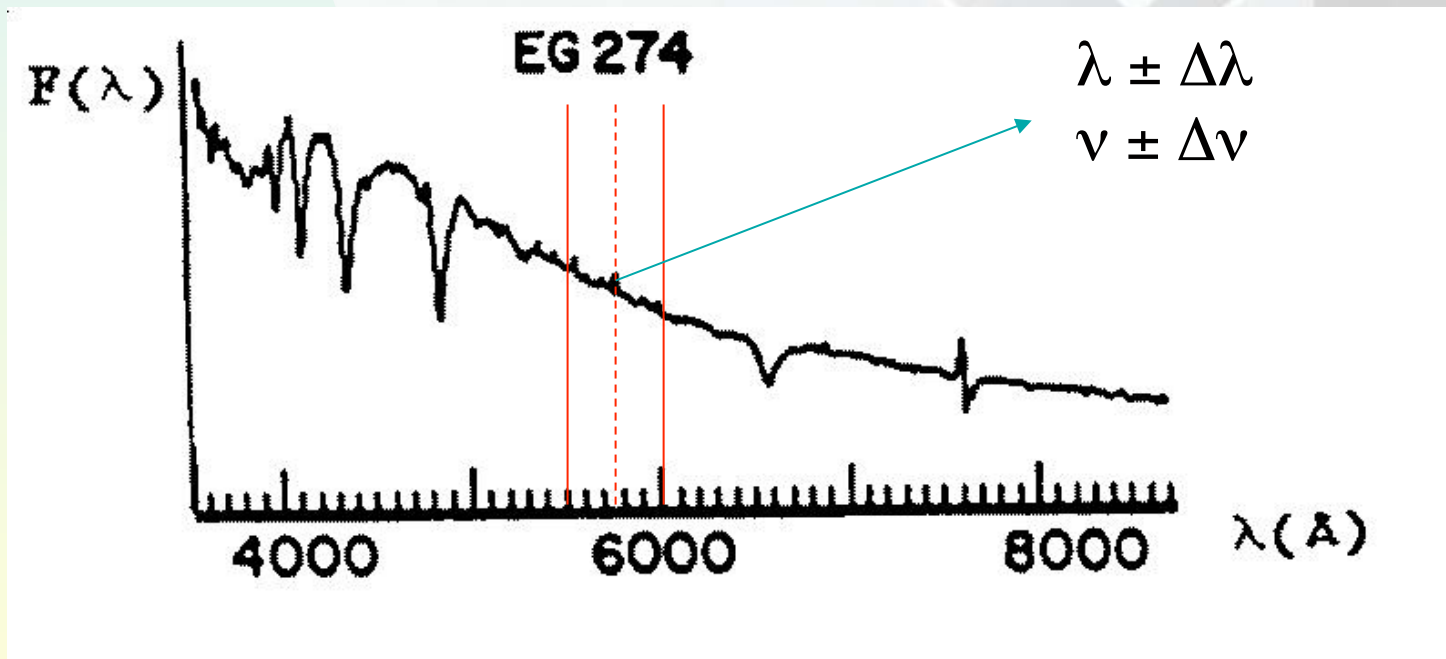
- 4 Interferometry with two independent telescopes
- b) Fizeau ... the father of stellar interferometry (1868)**

- Anderson
- Brown and Twiss (1956)
- Radio Interferometry (1950)



# An introduction to optical/IR interferometry

- 5 Light coherence
- 5.1 Quasi monochromatic light (waves and wave groups)





# An introduction to optical/IR interferometry

- 5 Light coherence
- 5.1 Quasi monochromatic light (waves and wave groups)

$$I = \langle V(z, t) V^*(z, t) \rangle \quad (5.1.1)$$

$$V(z, t) = \int_{\nu-\Delta\nu}^{\nu+\Delta\nu} a(\nu') \exp(i2\Pi(\nu' t - z/\lambda')) d\nu' \quad (5.1.2)$$

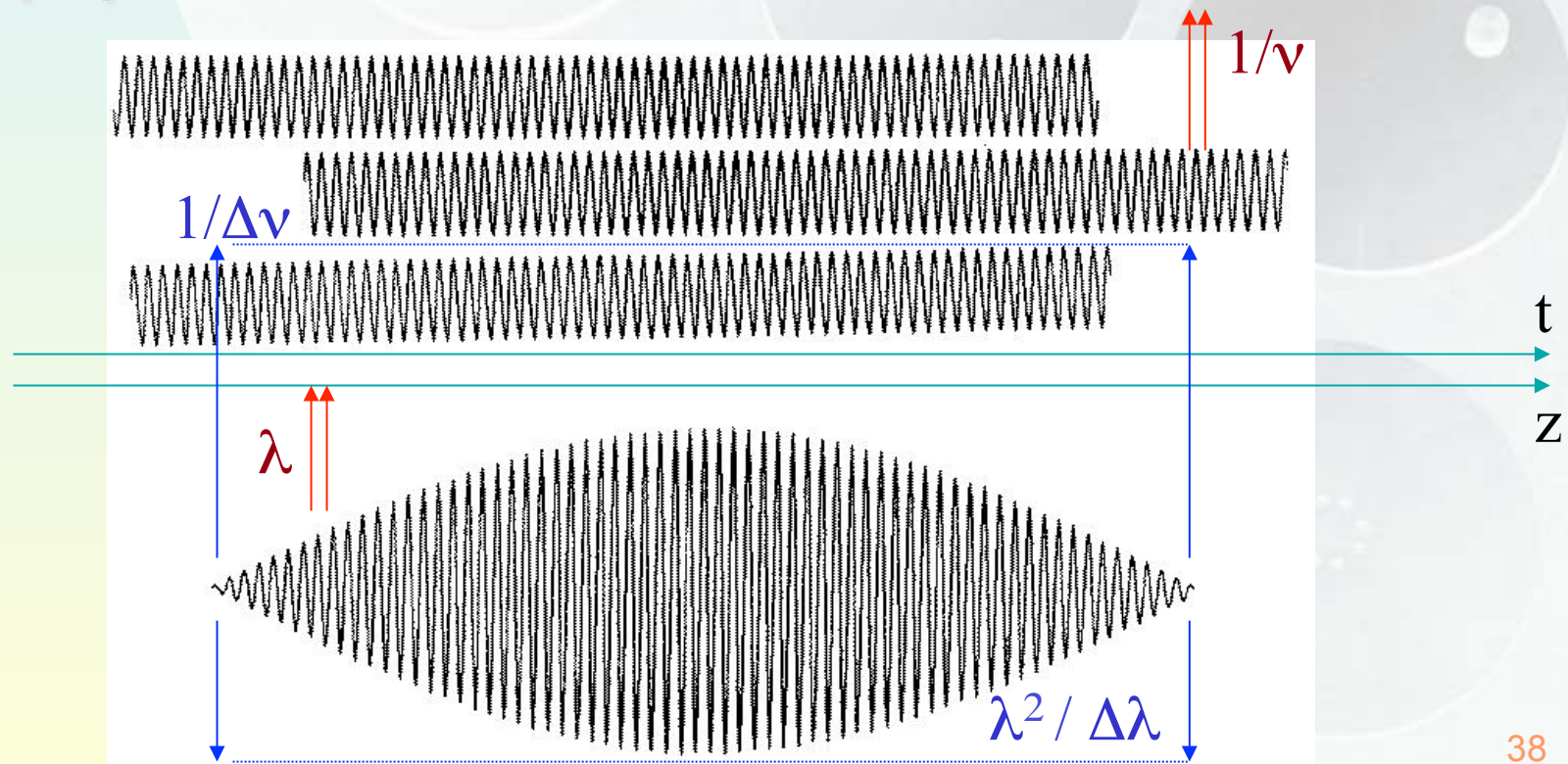
$$\exp(-i2\Pi(\nu t - z/\lambda)) \exp(i2\Pi(\nu t - z/\lambda))$$

$$V(z, t) = A(z, t) \exp(i2\Pi(\nu t - z/\lambda)) \quad (5.1.3)$$

$$A(z, t) = \int_{\nu-\Delta\nu}^{\nu+\Delta\nu} a(\nu') \exp(i2\Pi((\nu' - \nu)t - z(1/\lambda' - 1/\lambda))) d\nu' \quad (5.1.4)$$

# An introduction to optical/IR interferometry

- 5 Light coherence
- 5.1 Quasi monochromatic light (waves and wave groups)



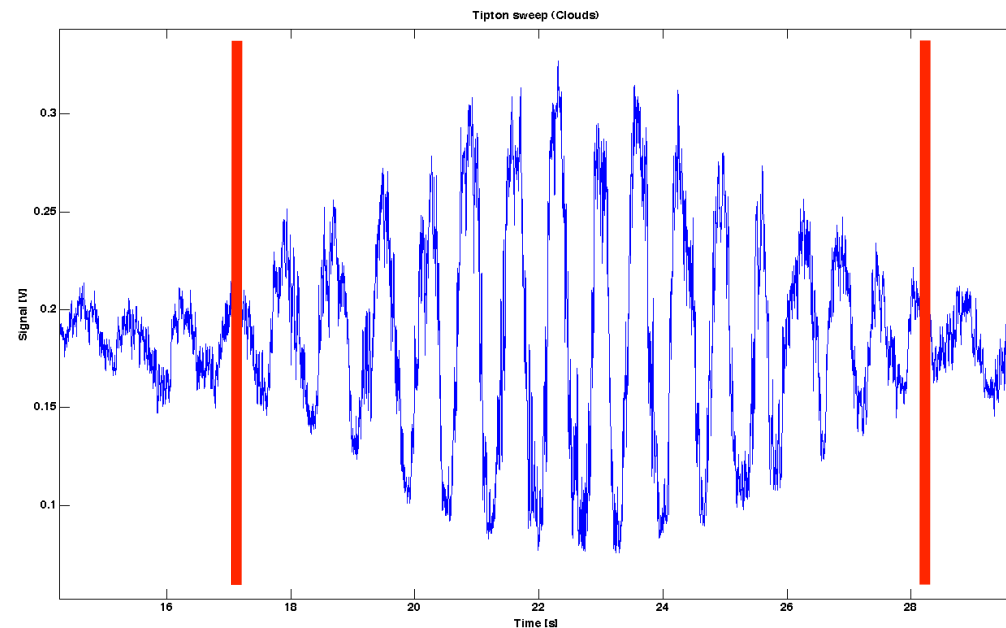
# An introduction to optical/IR interferometry

- 5 Light coherence
- 5.1 Quasi monochromatic light (waves and wave groups)

$$\lambda_0 = 2.2\mu\text{m}$$

$$\lambda \in [2.07 ; 2.33]\mu\text{m}$$

$$\Delta\lambda = 0.13\mu\text{m}$$

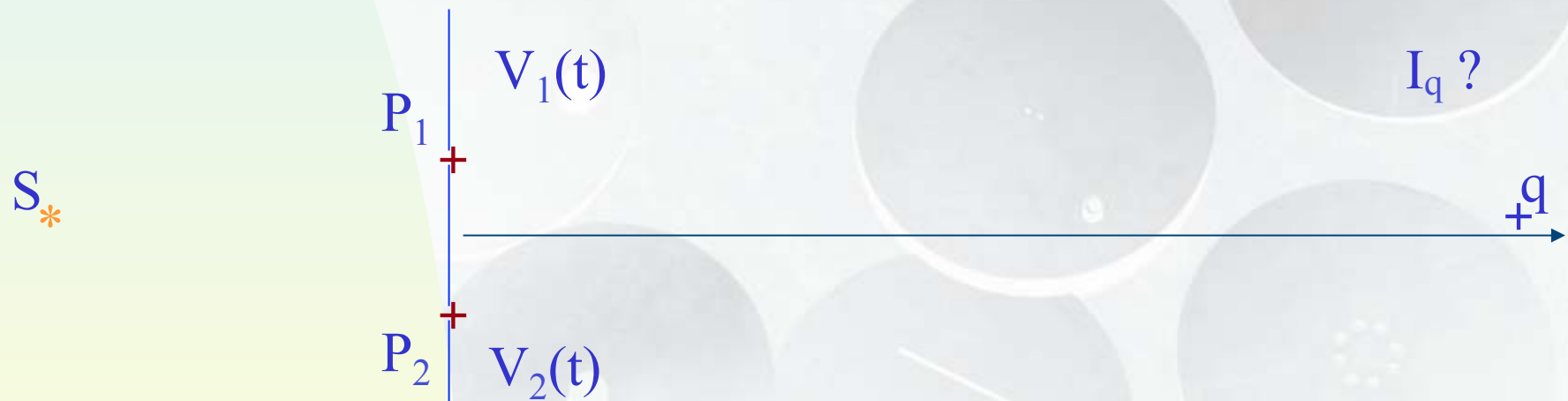


# An introduction to optical/IR interferometry

- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = \langle V_q^*(t) V_q(t) \rangle \quad (5.2.1)$$

$$V_q(t) = V_1(t - t_{1q}) + V_2(t - t_{2q}) \quad (5.2.2)$$



$$V_q(t) = V_1(t) + V_2(t - \tau) \quad (5.2.3)$$

$$\tau = t_{2q} - t_{1q} \quad (5.2.4) \quad 40$$



# An introduction to optical/IR interferometry

- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = I + I + 2I \operatorname{Re}\{\gamma_{12}(\tau)\} \quad (5.2.5)$$

$$\gamma_{12}(\tau) = \langle V_1^*(t) V_2(t - \tau) \rangle / I \quad (5.2.6)$$

$$\gamma_{12}(\tau) = \langle A_1^*(z, t) A_2(z, t - \tau) \rangle \exp(-i2\pi\nu\tau) / I \quad (5.2.7)$$

If  $\tau \ll 1/\Delta\nu$

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau=0)| \exp(i\beta_{12} - i2\pi\nu\tau) \quad (5.2.8)$$

# An introduction to optical/IR interferometry

- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = I + I + 2I |\gamma_{12}(0)| \cos(\beta_{12} - 2\pi\nu\tau) \quad (5.2.9)$$

$$v = \left( \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right) = |\gamma_{12}(0)| \quad (5.2.10)$$