“La vraie faute est celle qu’on ne corrige pas ...”

Confucius
An introduction to optical/IR interferometry

Brief summary of main results obtained during the last lecture:

\[ V = \left| \gamma_{12}(0,u,v) \right| = \left| \iint_{S} I'(\xi,\eta) \exp \left\{ -i2\Pi(u\xi + v\eta) \right\} d\xi \, d\eta \right| \]

\[ I'(\xi,\eta) = \iint \gamma_{12}(0,u,v) \exp \left\{ i2\Pi(\xi u + \eta v) \right\} d(u)d(v) \]

- For the case of a 1D uniformly brightening star whose angular diameter is \( \phi = b/z' \), we found that the visibility of the fringes is zero when \( \lambda/B = b/z' = \phi \) where \( B \) is the baseline of the interferometer.

- For the case of a double star with an angular separation \( \phi = b/z' \), we found that the visibility of the fringes is zero when \( \lambda/2B = b/z' = \phi \).
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- 5 Light coherence
- 5.5 Aperture synthesis

Exercises:
- the case of a gaussian-like source?
- let us assume that the observed visibility $|Y_{12}(0, u)|$ of a celestial object is $|\cos(\pi u \theta)|$, please retrieve the intensity distribution $I'$ of the source
Case of a double point-like source with a flux ratio $= 1$
Case of a double point-like source with a flux ratio 0.7/0.3
Variation of the fringe contrast as a function of the angular separation between the two stars:
If the source is characterized by a uniform disk light distribution, the corresponding visibility function is given by

$$\nu = \left( \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \right) = | \gamma_{12}(0) | = TF(I) = \frac{2J_1(\pi\theta_{UD}B / \lambda)}{\pi\theta_{UD}B / \lambda}$$
SW Virginis
M7.3 III semi-regular variable in 1996 & 1997

\[ V_{DU}(B) = \frac{2J_1\left(\pi \theta \frac{B}{\lambda}\right)}{\pi \theta \frac{B}{\lambda}} \]

\[ \phi_{UD} = 16,53 \pm 0,14 \text{ mas} \]
\[ \chi^2 = 0,81 \]

Fréquence spatiale (cycles/arcsec)
\[ I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2} |C| \cos \left( \frac{bx}{\lambda} + \phi_C \right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]
\[ I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2} |C| \cos \left( \frac{bx}{\lambda} + \phi_C \right) \quad \text{with} \quad |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]
For the case of the Sun:

\[ \theta_{UD} = 1.22 \lambda / B = 1.22 \times 0.55 / B(\mu) = 30' \times 60''' / 206265 \]

\[ B(\mu) = 206265 \times 1.22 \times 0.55 / (30 \times 60) = 76.9 \mu \]

\[ d(\mu) = 7.2 \text{ or } 14.4 \mu \quad \Rightarrow \quad \sigma = 2.44 \lambda / d = 7.8^\circ \text{ or } 3.9^\circ \]

See the masks!
First fringes on the Sun: 9/4/2010

$B = 29.4 \mu$

$d = 11.8 \mu$
Carlina PSF

↔ 50µ

. 14µ
KEOPS PSF

↔ 50µ

· 14µ
OVLA PSF

\[ \pm 50 \mu \]

\[ \pm 14 \mu \]
OVLA PSF

$\leftrightarrow 50\mu$

$14\mu$
ELSA_Sun_24
Interferometric observations on 10/4/2010 of Procyon, Mars and Saturn, using the 80cm telescope at Haute-Provence Observatory and adequate masks (coll. with Hervé le Coroller) …
Procyon
B = 12 mm
d = 2 mm
Mars
B = 12 mm
d = 2 mm
Saturn
\[ B = 4 \text{ mm} \]
\[ d = 2 \text{ mm} \]
Saturn
B = 12 mm
d = 2 mm
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- Some examples of optical interferometers
First fringes with I2T
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers
An introduction to optical/IR interferometry

- Some examples of optical interferometers

http://www.aeos.ulg.ac.be/HARI/
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

  Interferometry to-day is:

  Very Large Telescope Interferometer (VLTI)

  • 4 x 8.2m UTs
  • 4 x 1.8m ATs
  • Max. Base: 200m
An introduction to optical/IR interferometry

- Some examples of optical interferometers
VLTI (Chili)

VLTI delay lines
Note: $uv$ plane coverage for an object at zenith. More generally, the projected baselines must be used.
Examples of $uv$ plane coverage

Dec -15

Dec -65
How does the $uv$ plane coverage affect imagery?

Model

4 telescopes, 6 hrs

8 telescopes, 6 hrs
An introduction to optical/IR interferometry

- Some examples of optical interferometers

Interferometry to-day is also:

The CHARA interferometer

- 6 x 1m telescopes
- Max. Base: 330m
An introduction to optical/IR interferometry

- Some examples of optical interferometers
  Interferometry to-day is also:

Palomar Testbed Interferometer (PTI)

- 3 x 40cm telescopes
- Max. Base: 110m
An introduction to optical/IR interferometry

- Some examples of optical interferometers

Interferometry to-day is also:

Keck interferometer

- 2 x 10m telescopes
- Base: 85m
Closure phases – what are these?

- Measure visibility phase ($\Phi$) on three baselines simultaneously.

- Each is perturbed from the true phase ($\phi$) in a particular manner:
  \[ \Phi_{12} = \phi_{12} + \varepsilon_1 - \varepsilon_2 \]
  \[ \Phi_{23} = \phi_{23} + \varepsilon_2 - \varepsilon_3 \]
  \[ \Phi_{31} = \phi_{31} + \varepsilon_3 - \varepsilon_1 \]

- Construct the linear combination of these:
  \[ \Phi_{12} + \Phi_{23} + \Phi_{31} = \phi_{12} + \phi_{23} + \phi_{31} \]

The error terms are antenna dependent – they vanish in the sum. The source information is baseline dependent – it remains. We still have to figure out how to use it!
Closure phase is a peculiar linear combination of the true Fourier phases:

– In fact, it is the argument of the product of the visibilities on the baselines in question, hence the name triple product (or bispectrum):

\[ V_{12} V_{23} V_{31} = |V_{12}| |V_{23}| |V_{31}| \exp(i2\pi[\Phi_{12} + \Phi_{23} + \Phi_{31}]) = T_{123} \]

– So we have to use the closure phases as additional constraints in some nonlinear iterative inversion scheme.
6 Some examples of optical interferometers

Interferometry to-day is also:

Nulling interferometry

- Measurement of « stellar leakage »
- Allow to resolve stars with a small size interferometer
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6 Some examples of optical interferometers

Interferometry to-day is also:

![Graph 1](image1.png)
![Graph 2](image2.png)
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- 6 Other examples of interferometers: ALMA
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- 6 Other examples of interferometers: DARWIN
7 Some results

<table>
<thead>
<tr>
<th>Star</th>
<th>Spectral type</th>
<th>Luminosity class</th>
<th>Angular diameter $\times 10^{-3}$ seconds of arc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Boo</td>
<td>K2</td>
<td>Giant</td>
<td>20</td>
</tr>
<tr>
<td>$\alpha$ Tau</td>
<td>K5</td>
<td>Giant</td>
<td>20</td>
</tr>
<tr>
<td>$\alpha$ Sco</td>
<td>M1-M2</td>
<td>Super-giant</td>
<td>40</td>
</tr>
<tr>
<td>$\beta$ Peg</td>
<td>M2</td>
<td>Giant</td>
<td>21</td>
</tr>
<tr>
<td>$\sigma$ Cet</td>
<td>M6e</td>
<td>Giant</td>
<td>47</td>
</tr>
<tr>
<td>$\alpha$ Ori</td>
<td>M1-M2</td>
<td>Super-giant variable</td>
<td>34→47</td>
</tr>
</tbody>
</table>

Table 2.1. Stars measured with Michelson’s interferometer. From Pease (1931).
An introduction to optical/IR interferometry

- Some results

<table>
<thead>
<tr>
<th>NOM</th>
<th>SPECTRE</th>
<th>DIAMÈTRE $\lambda = 0.55 \mu m$ en ms. d'arc</th>
<th>MESURÉ $\lambda = 2.2 \mu m$ en ms. d'arc</th>
<th>R/R $\odot$</th>
<th>TEMPÉRATURE EFFECTIVE $\lambda = 0.55 \mu m$ en degrés Kelvin</th>
<th>TEMPÉRATURE EFFECTIVE $\lambda = 2.2 \mu m$ en degrés Kelvin</th>
<th>DISTANCE en parsecs (1 pc = 3.26 au)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α Cas</td>
<td>KOIII</td>
<td>5.4±0.6</td>
<td>14.4±0.6</td>
<td>25±6</td>
<td>4700±300</td>
<td>3711±64</td>
<td>45±9</td>
</tr>
<tr>
<td>β And</td>
<td>MOIII</td>
<td>13.2±1.7</td>
<td></td>
<td>33±9</td>
<td>3900±250</td>
<td>3900±250</td>
<td>23±3</td>
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<tr>
<td>γ And</td>
<td>K3III</td>
<td>6.6±0.8</td>
<td></td>
<td>50±14</td>
<td>4850±250</td>
<td>7000±500</td>
<td>75±15</td>
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<tr>
<td>α Per</td>
<td>F5IIb</td>
<td>2.9±0.4</td>
<td></td>
<td>55±9</td>
<td>8200±800</td>
<td>8200±800</td>
<td>176±6</td>
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<tr>
<td>α Cyg</td>
<td>A2IIa</td>
<td>2.7±0.3</td>
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<td>145±45</td>
<td>4300±350</td>
<td>4300±350</td>
<td>500±100</td>
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<tr>
<td>α Ari</td>
<td>K2III</td>
<td>7.8±1</td>
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<td>15±5</td>
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<tr>
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<td>8±2</td>
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<td>3960±270</td>
<td>11±1</td>
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<tr>
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<td>4220±300</td>
<td>3904±34</td>
<td>31±11</td>
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<td>4240±120</td>
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<td>μ Gem</td>
<td>M3III</td>
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<td>94±30</td>
<td>3960±95</td>
<td>3960±95</td>
<td>60±15</td>
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<tr>
<td>α Tau</td>
<td>K5III</td>
<td></td>
<td></td>
<td>47±7</td>
<td>3904±34</td>
<td>4240±120</td>
<td>21±3</td>
</tr>
<tr>
<td>α Boo</td>
<td>K2III</td>
<td></td>
<td></td>
<td>25±6</td>
<td>4240±120</td>
<td></td>
<td>11±2</td>
</tr>
<tr>
<td>α Aur</td>
<td>G5III</td>
<td>9.0±1.2</td>
<td></td>
<td>11.7±2</td>
<td>6400±200</td>
<td></td>
<td>13.7±0.6</td>
</tr>
<tr>
<td>α Aur</td>
<td>G0III</td>
<td>4.8±1.5</td>
<td></td>
<td>7.1±2</td>
<td>5950±200</td>
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<td>α Lyr</td>
<td>AOV</td>
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<td>2.6±0.2</td>
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<td></td>
<td>8.1±0.3</td>
</tr>
</tbody>
</table>
An introduction to optical/IR interferometry

8 Three important theorems … and some applications

8.1 The fundamental theorem

8.2 The convolution theorem

8.3 The Wiener-Khintchin theorem

Réf.: P. Léna; Astrophysique: méthodes physiques de l’observation (Savoirs Actuels / CNRS Editions)
8.1 The fundamental theorem

\[
a(p,q) = TF_\text{TF}(A(x,y))(p,q),
\]

\[
a(p, q) = \int_{R^2} A(x, y) \exp[-i2\pi(px + qy)]dxdy,
\]

with

\[
p = x' / (\lambda f)
\]

\[
q = y' / (\lambda f)
\]
8.1 The fundamental theorem

The distribution of the complex amplitude $a(p,q)$ in the focal plane is given by the Fourier transform of the distribution of the complex amplitude $A(x,y)$ in the entrance pupil plane.
An introduction to optical/IR interferometry

8.1 The fundamental theorem
8.1 The fundamental theorem

**Démonstration**

\[ A(x,y) \exp(i2\pi vt), \quad (8.1.3.1) \]

\[ A(x,y) = A(x,y) \exp(i\phi(x,y)) P_0(x,y). \quad (8.1.3.2) \]
8.1 The fundamental theorem

Démonstration

\[ A(x, y) \exp(i2\pi nt + i\psi), \quad (8.1.3.3) \]

\[ \delta = d(M \text{ I N}) - d(O \text{ J N}), \quad (8.1.3.4) \]

\[ \psi = 2\pi \delta / \lambda. \quad (8.1.3.5) \]
8.1 The fundamental theorem

Déémonstration

\[ \delta = -d(O, K) = -|(OM \ u)|, \quad (8.1.3.6) \]

\[ A(x,y) \exp(i2\pi(\nu t - xx'/\lambda f - yy'/\lambda f)). \quad (8.1.3.7) \]

\[ p = x'/\lambda f, \ q = y'/\lambda f, \quad (8.1.3.8) \]

\[ \exp(i2\pi \nu t) \ A(x,y) \exp(-i2\pi(xp + yq)). \quad (8.1.3.9) \]
8.1 The fundamental theorem

**Démonstration**

\[ a(p, q) = \int_{\mathbb{R}^2} A(x, y) \exp \left[ -i2\pi (px + qy) \right] \, dx \, dy, \]

(8.1.3.10)

\[ a(p, q) = TF \_\_ \left[ A(x, y) \right](p, q) \]

(8.1.3.11)
8.1 The fundamental theorem

Application: Point Spread Function determination

\[ A(x,y) = A_0 \, P_0(x,y), \quad (8.1.1) \]

\[ P_0(x,y) = \Pi(x/a) \, \Pi(y/a). \quad (8.1.2) \]
An introduction to optical/IR interferometry

8.1 The fundamental theorem

\[ a(p, q) = TF \int_{-a/2}^{a/2} \int_{-a/2}^{a/2} A_0 \exp[-i2\pi(px + qy)] dx dy \]  \hspace{1cm} (8.1.3)

\[ a(p, q) = A_0 \int_{-a/2}^{a/2} \exp[-i2\pi px] dx \int_{-a/2}^{a/2} \exp[-i2\pi qy] dy \]  \hspace{1cm} (8.1.4)

\[ a(p, q) = A_0 a^2 \left[ \sin(\pi pa) / (\pi pa) \right] \left[ \sin(\pi qa) / (\pi qa) \right] \]  \hspace{1cm} (8.1.5)

\[ i(p, q) = a(p, q) a^*(p, q) = |a(p, q)|^2 = |h(p, q)|^2 = i_0 a^4 \left[ \sin(\pi pa) / (\pi pa) \right]^2 \left[ \sin(\pi qa) / (\pi qa) \right]^2. \]  \hspace{1cm} (8.1.6)
An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

\[ \Delta p = \Delta x' / (\lambda f); \quad \Delta q = \Delta y' / (\lambda f) = 2/a \implies \Delta \phi_x' = \Delta \phi_y' = 2\lambda / a \] (8.1.7)
An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination when observing a star along another direction

\[ \psi = 2\pi \delta / \lambda = 2\pi (xb/f + yc/f) / \lambda, \quad (8.1.5.7) \]

\[ A(x,y) = P_0(x,y) A_0 \exp[2i\pi(xb/f + yc/f) / \lambda]. \quad (8.1.5.8) \]

\[ a(p,q) = A_0 \int_{-a/2}^{a/2} \exp[-2i\pi(p-b/f \lambda)x] dx \int_{-a/2}^{a/2} \exp[-2i\pi(q-c/f \lambda)y] dy \quad (8.1.5.9) \]

\[ a(p,q) = A_0 a^2 \left( \frac{\sin \left( \pi \left( p-b / f \lambda \right) a \right)}{\pi \left( p-b / f \lambda \right) a} \right) \left( \frac{\sin \left( \pi \left( q-c / f \lambda \right) a \right)}{\pi \left( q-c / f \lambda \right) a} \right) \quad (8.1.5.10) \]
8.1 The fundamental theorem

Application: Point Spread Function determination

$$h(p,q) = \text{TF}_P(P(x,y))(p,q)$$

$$i(\rho') = |a(\rho')|^2 = (A_0 \pi)^2 \left[ R_2^2 \frac{2 J_1(Z_2)}{Z_2} - R_1^2 \frac{2 J_1(Z_1)}{Z_1} \right]^2,$$

with $Z_2 = 2\pi R_2 \rho' / (\lambda f)$ and $Z_1 = 2\pi R_1 \rho' / (\lambda f).$
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**BESSEL FUNCTIONS (REMINDER)**

Integral representation of the Bessel functions

\[
J_0(x) = \frac{1}{\pi} \int_0^\pi \cos(x \sin(\vartheta)) \, d\vartheta \quad J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n \vartheta - x \sin(\vartheta)) \, d\vartheta
\]

Undefined integral

\[
\int x' J_0(x') \, dx' = x J_1(x)
\]

Series development (x \(\sim\) 0):

\[
J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \frac{x^6}{2^2 4^2 6^2} + \ldots
\]

\[
J_1(x) = \frac{x}{2} - \frac{x^3}{2^2 4} + \frac{x^5}{2^2 4^2 6} - \frac{x^7}{2^2 4^2 6^2 8} + \ldots
\]

\[
J_n(x) = \left(\frac{2}{\pi x}\right)^{1/2} \cos(x - n\pi/2 - \pi/4) \ldots \text{and when } x \text{ is large!}
\]

Graphs of the \(J_0(x)\) and \(J_1(x)\) functions
An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

\[ x = \rho \cos(\theta), \quad y = \rho \sin(\theta), \quad p = \rho' \cos(\theta') / (\lambda f), \quad q = \rho' \sin(\theta') / (\lambda f). \]

\[ a(\rho', \theta') = A_0 \int_{R_1}^{R_2} \int_0^{2\pi} \exp \left[ -2i\pi \rho \rho' \cos(\theta - \theta') / (\lambda f) \right] d(\theta - \theta') \rho d\rho \tag{8.1.5.13} \]

\[ a(\rho', \theta') = a(\rho') = A_0 \pi \left[ \frac{2R_2^2}{Z_2} J_1(Z_2) - \frac{2R_1^2}{Z_1} J_1(Z_1) \right] \tag{8.1.5.14} \]

\[ Z_2 = 2\pi R_2 \frac{\rho'}{\lambda f} \]

\[ Z_1 = 2\pi R_1 \frac{\rho'}{\lambda f} \tag{8.1.5.15} \]

Pour le cas \( R_1 = 0 \)

\[ i(\rho') = |a(\rho')|^2 = 4(A_0 \pi)^2 R_2^4 \left( \frac{J_1(Z_2)}{Z_2} \right)^2 \tag{8.1.5.16} \]
An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

\[ i(\rho') = |a(\rho')|^2 = (A_0 \pi)^2 \left[ R_2^2 2 J_1(Z_2) / Z_2 - R_1^2 2 J_1(Z_1) / Z_1 \right]^2, \]  
with \( Z_2 = 2\pi R_2 \rho' / (\lambda f) \) and \( Z_1 = 2\pi R_1 \rho' / (\lambda f) \).
8.1 The fundamental theorem

Application: Point Spread Function determination

\[ \rho' (=r) = 1.22 \frac{\lambda f}{D} \quad (D = 2 R_2, R_1 = 0). \]  

\[ \frac{2\pi \int_0^r i(\rho')\rho' d\rho'}{2\pi \int_0^\infty i(\rho')\rho' d\rho'} = 0.84 \]

\[ h(p,q) = TF_{\_}(P(x,y))(p,q). \]