“La vraie faute est celle qu’on ne corrige pas …”

Confucius
An introduction to optical/IR interferometry

Brief summary of main results obtained during the last lecture:

\[ V = \left| \gamma_{12}(0,u,v) \right| = \left| \iint_{S} I'(\xi,\eta) \exp\{-i2\Pi(u\xi + v\eta)\} d\xi d\eta \right| \]

\[ I'(\xi,\eta) = \iint \gamma_{12}(0,u,v) \exp\{i2\Pi(\xi u + \eta v)\} d(u)d(v) \]

- For the case of a 1D uniformly brightening star whose angular diameter is \( \phi = b/z' \), we found that the visibility of the fringes is zero when \( \lambda/B = b/z' = \phi \) where B is the baseline of the interferometer.

- For the case of a double star with an angular separation \( \phi = b/z' \), we found that the visibility of the fringes is zero when \( \lambda/2B = b/z' = \phi \).
An introduction to optical/IR interferometry

- 5 Light coherence
- 5.5 Aperture synthesis

Exercises:
- the case of a gaussian-like source?
- let us assume that the observed visibility $|Y_{12}(0,u)|$ of a celestial object is $|\cos(\pi u \theta)|$, please retrieve the intensity distribution $I'$ of the source
Case of a double point-like source with a flux ratio = 1
Case of a double point-like source with a flux ratio 0.7/0.3
Variation of the fringe contrast as a function of the angular separation between the two stars:
If the source is characterized by a uniform disk light distribution, the corresponding visibility function is given by

\[ \nu = \left(\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \right) = \left(\gamma^{12}(0) \right) = TF(I) = \frac{2J_1(\pi \theta_{UD} B / \lambda)}{\pi \theta_{UD} B / \lambda} \]
SW Virginis
M7.3 III semi-regular variable in 1996 & 1997

\[ V_{DU}(B) = \frac{2J_1 \left( \frac{\pi \theta B}{\lambda} \right)}{\pi \theta \frac{B}{\lambda}} \]

\[ \phi_{UD} = 16.53 \pm 0.14 \text{ mas} \]

\[ \chi^2 = 0.81 \]

1.22/θ
\[ I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos \left( \frac{bx}{\lambda} + \phi C \right) \text{ with } |C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \]
\[ I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2} |C| \cos \left( \frac{b x}{\lambda} + \phi_C \right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]
For the case of the Sun:

$\theta_{UD} = 1.22 \lambda / B = 1.22 \times 0.55 / B(\mu) = 30' \times 60'' / 206265$

$B(\mu) = 206265 \times 1.22 \times 0.55 / (30 \times 60) = 76.9 \mu$

d(\mu) = 7.2 or 14.4 \mu \quad \Rightarrow \quad \sigma = 2.44 \lambda / d = 7.8^\circ \text{or } 3.9^\circ$

See the masks!
First fringes on the Sun:
9/4/2010

\[ B = 29.4 \mu \]
\[ d = 11.8 \mu \]
OVLA PSF

- 50\(\mu\)
- 14\(\mu\)
ELSA PSF

↔ 50µ

• 14µ
Interferometric observations on 10/4/2010 of Procyon, Mars and Saturn, using the 80cm telescope at Haute-Provence Observatory and adequate masks (coll. with Hervé le Coroller) …
Procyon
B = 12 mm
d = 2 mm
Mars
$B = 12 \, \text{mm}$
$d = 2 \, \text{mm}$
Saturn
B = 4 mm
d = 2 mm
Saturn
B = 12 mm
d = 2 mm
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- Some examples of optical interferometers
First fringes with I2T
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

http://www.aeos.ulg.ac.be/HARI/
An introduction to optical/IR interferometry

6 Some examples of optical interferometers

Interferometry to-day is:

Very Large Telescope Interferometer (VLTI)

• 4 x 8.2m UTs
• 4 x 1.8m ATs
• Max. Base: 200m
Cerro Paranal
An introduction to optical/IR interferometry

- Some examples of optical interferometers
VLTI delay lines
Note: uv plane coverage for an object at zenith. More generally, the projected baselines must be used.
Examples of $uv$ plane coverage

Dec -15

Dec -65
How does the $uv$ plane coverage affect imagery?

Model

4 telescopes, 6 hrs

8 telescopes, 6 hrs
An introduction to optical/IR interferometry

- Some examples of optical interferometers

Interferometry to-day is also:

The CHARA interferometer

- 6 x 1m telescopes
- Max. Base: 330m
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

  Interferometry to-day is also:

Palomar Testbed Interferometer (PTI)

- 3 x 40cm telescopes
- Max. Base: 110m
An introduction to optical/IR interferometry

6 Some examples of optical interferometers

Interferometry to-day

is also:

Keck interferometer

- 2 x 10m telescopes
- Base: 85m
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- 6 Some examples of optical interferometers

Interferometry to-day is also:

Nullin interferometry

- Measurement of « stellar leakage »
- Allow to resolve stars with a small size interferometer
6 Some examples of optical interferometers

Interferometry to-day is also:
An introduction to optical/IR interferometry

- 6 Other examples of interferometers: ALMA
An introduction to optical/IR interferometry

- 6 Other examples of interferometers: DARWIN
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7 Some results

<table>
<thead>
<tr>
<th>Star</th>
<th>Spectral type</th>
<th>Luminosity class</th>
<th>Angular diameter $\times 10^{-3}$ seconds of arc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Boo</td>
<td>K2</td>
<td>Giant</td>
<td>20</td>
</tr>
<tr>
<td>$\alpha$ Tau</td>
<td>K5</td>
<td>Giant</td>
<td>20</td>
</tr>
<tr>
<td>$\alpha$ Sco</td>
<td>M1–M2</td>
<td>Super-giant</td>
<td>40</td>
</tr>
<tr>
<td>$\beta$ Peg</td>
<td>M2</td>
<td>Giant</td>
<td>21</td>
</tr>
<tr>
<td>$\sigma$ Cet</td>
<td>M6e</td>
<td>Giant</td>
<td>47</td>
</tr>
<tr>
<td>$\alpha$ Ori</td>
<td>M1–M2</td>
<td>Super-giant variable</td>
<td>34–47</td>
</tr>
</tbody>
</table>

Table 2.1. Stars measured with Michelson’s interferometer. From Pease (1931).
Table 2. Diamètres stellaires mesurés à l'PIXT

<table>
<thead>
<tr>
<th>NOM</th>
<th>SPECTRE</th>
<th>DIAMÈTRE $\lambda = 0.55 \mu m$ en mas. d'arc</th>
<th>DIAMÈTRE $\lambda = 2.2 \mu m$ en mas. d'arc</th>
<th>RRVG</th>
<th>TEMPERATURE EFFECTIVE $\lambda = 0.55 \mu m$ en degrés Kelvin</th>
<th>TEMPERATURE EFFECTIVE $\lambda = 2.2 \mu m$ en degrés Kelvin</th>
<th>DISTANCE en parsecs (1 pc = 3.26 cl)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α Cas</td>
<td>K0III</td>
<td>5.4 ± 0.6</td>
<td>14.4 ± 0.6</td>
<td>36 ± 6</td>
<td>4700 ± 300</td>
<td>3711 ± 64</td>
<td>45 ± 1</td>
</tr>
<tr>
<td>β And</td>
<td>M0III</td>
<td>13.2 ± 1.2</td>
<td>33 ± 9</td>
<td>3800 ± 260</td>
<td>23 ± 3</td>
<td>70 ± 15</td>
<td>179 ± 8</td>
</tr>
<tr>
<td>γ And</td>
<td>K3III</td>
<td>6.8 ± 0.8</td>
<td>86 ± 14</td>
<td>4650 ± 250</td>
<td>7000 ± 800</td>
<td>5000 ± 100</td>
<td>23 ± 4</td>
</tr>
<tr>
<td>α Per</td>
<td>B1b</td>
<td>2.9 ± 0.4</td>
<td>65 ± 9</td>
<td>8200 ± 800</td>
<td>175 ± 8</td>
<td>11 ± 1</td>
<td>500 ± 100</td>
</tr>
<tr>
<td>ζ Cyg</td>
<td>A3Ia</td>
<td>2.7 ± 0.3</td>
<td>145 ± 45</td>
<td>4300 ± 350</td>
<td>4300 ± 350</td>
<td>4300 ± 350</td>
<td>23 ± 4</td>
</tr>
<tr>
<td>α Ari</td>
<td>K3III</td>
<td>7.6 ± 1</td>
<td>15 ± 6</td>
<td>4800 ± 220</td>
<td>4220 ± 300</td>
<td>4220 ± 300</td>
<td>23 ± 4</td>
</tr>
<tr>
<td>β Gem</td>
<td>K0III</td>
<td>7.8 ± 0.8</td>
<td>9 ± 2</td>
<td>4900 ± 220</td>
<td>3960 ± 170</td>
<td>3960 ± 170</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>β UmI</td>
<td>K4III</td>
<td>9.3 ± 1</td>
<td>30 ± 9</td>
<td>4220 ± 200</td>
<td>3960 ± 170</td>
<td>3960 ± 170</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>γ Oph</td>
<td>K5III</td>
<td>8.7 ± 0.8</td>
<td>10.2 ± 1.4</td>
<td>46 ± 10</td>
<td>3960 ± 170</td>
<td>3960 ± 170</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>δ Ori</td>
<td>G8III</td>
<td>3.8 ± 0.3</td>
<td>14.6 ± 0.8</td>
<td>94 ± 30</td>
<td>4530 ± 220</td>
<td>4530 ± 220</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>μ Gem</td>
<td>M3III</td>
<td>20.7 ± 0.4</td>
<td>47 ± 7</td>
<td>3960 ± 170</td>
<td>3960 ± 170</td>
<td>3960 ± 170</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>ο Tau</td>
<td>K5III</td>
<td>21.5 ± 1.2</td>
<td>25 ± 6</td>
<td>4240 ± 120</td>
<td>4240 ± 120</td>
<td>4240 ± 120</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>α Boo</td>
<td>K2II</td>
<td>5.6 ± 1.2</td>
<td>11.7 ± 2</td>
<td>5420 ± 200</td>
<td>5420 ± 200</td>
<td>5420 ± 200</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>η Aur</td>
<td>G8III</td>
<td>4.8 ± 1.5</td>
<td>7.1 ± 2</td>
<td>5950 ± 200</td>
<td>5950 ± 200</td>
<td>5950 ± 200</td>
<td>11 ± 2</td>
</tr>
<tr>
<td>ζ Aur</td>
<td>AOIV</td>
<td>3.0 ± 0.2</td>
<td>2.6 ± 0.2</td>
<td>8.1 ± 0.3</td>
<td>8.1 ± 0.3</td>
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<td>8.1 ± 0.3</td>
</tr>
</tbody>
</table>
An introduction to optical/IR interferometry

8 Three important theorems … and some applications

8.1 The fundamental theorem

8.2 The convolution theorem

8.3 The Wiener-Khintchin theorem

Réf.: P. Léna; Astrophysique: méthodes physiques de l’observation (Savoirs Actuels / CNRS Editions)
8.1 The fundamental theorem

\[ a(p,q) = \text{TF}_{(A(x,y))}(p,q), \]

\[ a(p,q) = \int_{R^2} A(x,y) \exp[-i2\pi(px + qy)]\,dx\,dy, \]

with

\[ p = \frac{x'}{\lambda f} \]
\[ q = \frac{y'}{\lambda f} \]
8.1 The fundamental theorem

The distribution of the complex amplitude $a(p,q)$ in the focal plane is given by the Fourier transform of the distribution of the complex amplitude $A(x,y)$ in the entrance pupil plane.
An introduction to optical/IR interferometry

8.1 The fundamental theorem
8.1 The fundamental theorem

Démonstration

\[ A(x,y) \exp(i2\pi vt), \]  
\[ A(x,y) = A(x,y) \exp(i\phi(x,y)) P_0(x,y). \]  

(8.1.3.1)  

(8.1.3.2)
8.1 The fundamental theorem

\[ A(x, y) \exp(i2\pi vt + i\psi), \quad (8.1.3.3) \]

\[ \delta = d(M \ I \ N) - d(O \ J \ N), \quad (8.1.3.4) \]

\[ \psi = 2\pi \delta / \lambda. \quad (8.1.3.5) \]
8.1 The fundamental theorem

**Démonstration**

\[ \delta = -d(O, K) = -\left|\langle OM u \rangle\right|, \]  

\[ A(x,y) \exp(i2\pi(vt - xx'/\lambda f - yy'/\lambda f)). \]  

\[ p = x'/\lambda f, \quad q = y'/\lambda f, \]  

\[ \exp(i2\pi vt) A(x,y) \exp(-i2\pi(xp + yq)). \]
8.1 The fundamental theorem

Démonstration

\[ a(p, q) = \int_{\mathbb{R}^2} A(x, y) \exp \left[ -i2\pi (px + qy) \right] \, dx \, dy, \]  

(8.1.3.10)

\[ a(p, q) = TF^{-1} \left[ A(x, y) \right] (p, q) \]  

(8.1.3.11)

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An introduction to optical/IR interferometry

8.1 The fundamental theorem

Application: Point Spread Function determination

\[ A(x,y) = A_0 \, P_0(x,y), \quad (8.1.1) \]

\[ P_0(x,y) = \Pi(x/a) \, \Pi(y/a). \quad (8.1.2) \]
8.1 The fundamental theorem

\[ a(p, q) = TF \left[ A(x, y) \right](p, q) = \int_{-a/2}^{a/2} \int_{-a/2}^{a/2} A_0 \exp\left[-i2\pi(px + qy)\right] dx dy \]  \hspace{1cm} (8.1.3)

\[ a(p, q) = A_0 \int_{-a/2}^{a/2} \exp\left[-i2\pi px\right] dx \int_{-a/2}^{a/2} \exp\left[-i2\pi qy\right] dy \]  \hspace{1cm} (8.1.4)

\[ a(p, q) = A_0 a^2 \left[ \sin(\pi pa) / (\pi pa) \right] \left[ \sin(\pi qa) / (\pi qa) \right]. \]  \hspace{1cm} (8.1.5)

\[ i(p, q) = a(p, q) a^*(p, q) = |a(p, q)|^2 = |h(p, q)|^2 = i_0 a^4 \left[ \sin(\pi pa) / (\pi pa) \right]^2 \left[ \sin(\pi qa) / (\pi qa) \right]^2. \]  \hspace{1cm} (8.1.6)
8.1 The fundamental theorem

Application: Point Spread Function determination

\[ \Delta p = \Delta x' / (\lambda f); \quad \Delta q = \Delta y' / (\lambda f) = 2/a \Rightarrow \Delta \phi_x = \Delta \phi_y = 2\lambda/a \] 

(8.1.7)
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8.1 The fundamental theorem

Application: Point Spread Function determination when observing a star along another direction

\[ \psi = 2\pi \frac{\delta}{\lambda} = 2\pi \frac{xb/f + yc/f}{\lambda}, \]  
(8.1.5.7)

\[ A(x,y) = P_0(x,y) A_0 \exp[2i\pi(\frac{xb}{f} + \frac{yc}{f})/\lambda]. \]  
(8.1.5.8)

\[ a(p,q) = A_0 \int_{-a/2}^{a/2} \exp[-2i\pi(\frac{p-b}{f\lambda})x] dx \int_{-a/2}^{a/2} \exp[-2i\pi(\frac{q-c}{f\lambda})y] dy \]  
(8.1.5.9)

\[ a(p,q) = A_0 a^2 \left( \frac{\sin\left(\pi\left(\frac{p-b}{f\lambda}\right)a\right)}{\pi\left(\frac{p-b}{f\lambda}\right)a} \right) \left( \frac{\sin\left(\pi\left(\frac{q-c}{f\lambda}\right)a\right)}{\pi\left(\frac{q-c}{f\lambda}\right)a} \right) \]  
(8.1.5.10)
8.1 The fundamental theorem

Application: Point Spread Function determination

\[ h(p,q) = \text{TF}_\text{P}(x,y)(p,q) \]

\[ i(\rho') = |a(\rho')|^2 = (A_0 \pi)^2 \left[ R_2^2 2 J_1(Z_2) / Z_2 - R_1^2 2 J_1(Z_1) / Z_1 \right]^2, \]

with \( Z_2 = 2\pi R_2 \rho' / (\lambda f) \) and \( Z_1 = 2\pi R_1 \rho' / (\lambda f) \).
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**BESSEL FUNCTIONS (REMEMBER)**

Integral representation of the Bessel functions

\[ J_0(x) = \frac{1}{\pi} \int_0^\pi \cos[x \sin(\theta)] d\theta \]

\[ J_n(x) = \frac{1}{\pi} \int_0^\pi \cos[n \theta - x \sin(\theta)] d\theta \]

Undefined integral

\[ \int x' J_0(x') dx' = x J_1(x) \]

Series development \((x \sim 0)\):

\[ J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{(2^4)2^2} - \frac{x^6}{(2^4 2^2)2^2} + \ldots \]

\[ J_1(x) = \frac{x}{2} - \frac{x^3}{(2^2)4} + \frac{x^5}{(2^2 4^2)6} - \frac{x^7}{(2^2 4^2 6^2)8} + \ldots \]

\[ J_n(x) = \frac{2}{(\pi x)^{1/2}} \cos(x - n\pi/2 - \pi/4) \ldots \text{ and when } x \text{ is large}! \]
8.1 The fundamental theorem

Application: Point Spread Function determination

\[ x = \rho \cos(\theta), \ y = \rho \sin(\theta), \ p = \rho' \cos(\theta') / (\lambda f), \ q = \rho' \sin(\theta') / (\lambda f). \]

\[ a(\rho', \theta') = A_0 \int_{R_1}^{R_2} \int_0^{2\pi} \exp\left[-2i\pi \rho' \cos(\theta - \theta') / (\lambda f)\right] d(\theta - \theta') \rho d\rho \]

\[ a(\rho', \theta') = a(\rho') = A_0 \pi \left[ \frac{2R_2^2}{Z_2} J_1(Z_2) - \frac{2R_1^2}{Z_1} J_1(Z_1) \right] \]

\[ Z_2 = 2\pi R_2 \frac{\rho'}{\lambda f} \]

et

\[ Z_1 = 2\pi R_1 \frac{\rho'}{\lambda f} \]

Pour le cas \( R_1 = 0 \)

\[ i(\rho') = |a(\rho')|^2 = 4(A_0\pi)^2 R_2^4 \left( \frac{J_1(Z_2)}{Z_2} \right)^2 \]
8.1 The fundamental theorem

Application: Point Spread Function determination

\[ h(p,q) = \text{TF}_P(P(x,y))(p,q) \]

\[
i(\rho') = |a(\rho')|^2 = (A_0 \pi)^2 \left[ R_2^2 2 J_1(Z_2) / Z_2 - R_1^2 2 J_1(Z_1) / Z_1 \right]^2,
\]

with \( Z_2 = 2\pi R_2 \rho' / (\lambda f) \) and \( Z_1 = 2\pi R_1 \rho' / (\lambda f) \).

(8.1.8)

(8.1.9)
8.1 The fundamental theorem

Application: Point Spread Function determination

\[ \rho' (r) = 1.22 \frac{\lambda f}{D} \] \hspace{1cm} (D = 2 R_2, R_1 = 0). \hspace{1cm} (8.1.5.18)

\[ h(p,q) = \text{TF}_\text{(P(x,y))}(p,q). \] \hspace{1cm} (8.1.5.20)