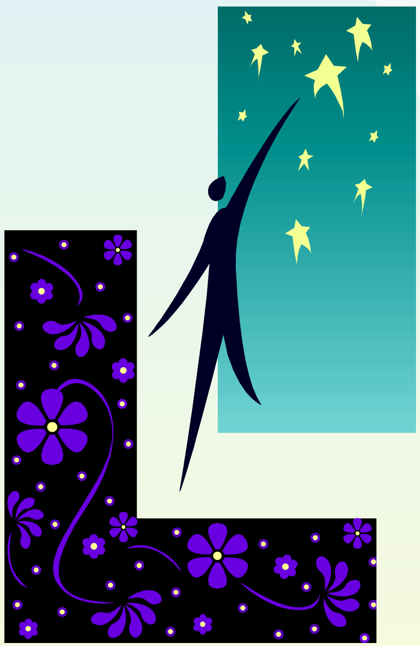
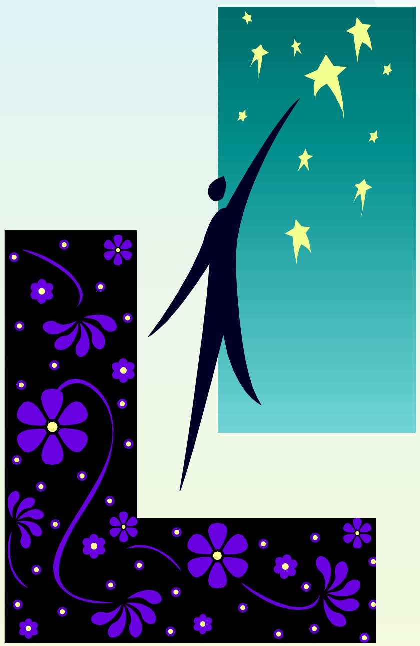


"Or les différents systèmes philosophiques, économiques et politiques qui régissent les hommes sont tous d'accord sur un point: bernons-les …"

Roger Waters
(Pink Floyd, 1974)

An introduction to optical/IR interferometry

Brief summary of main results obtained during the last lecture:



$$\rho = R / z$$

$$T_{eff} = (F/\sigma)^{1/4} = (f / \sigma\, \rho^2)^{1/4}$$

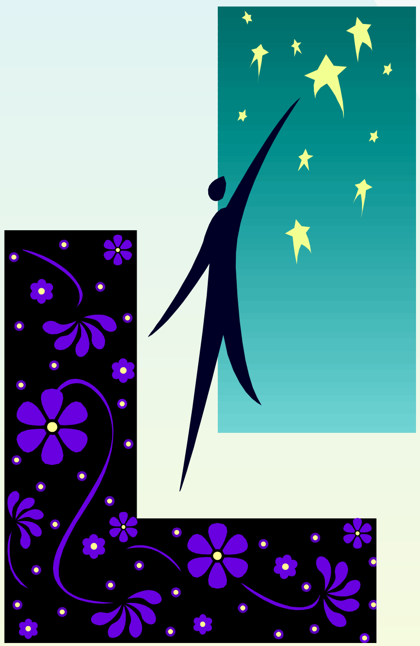
$$E = A(z) \exp[i2\pi\nu t]$$

$$E = A(z, t) \exp[i2\pi\nu t]$$

$$\tau = 1 / \Delta\nu \qquad \lambda_{eff} = \lambda^2 / \Delta\lambda$$

$$I = A A^* = |A|^2 = a^2 .$$

An introduction to optical/IR interferometry



If Δ ≥ λ / (2B), fringe disappearance!

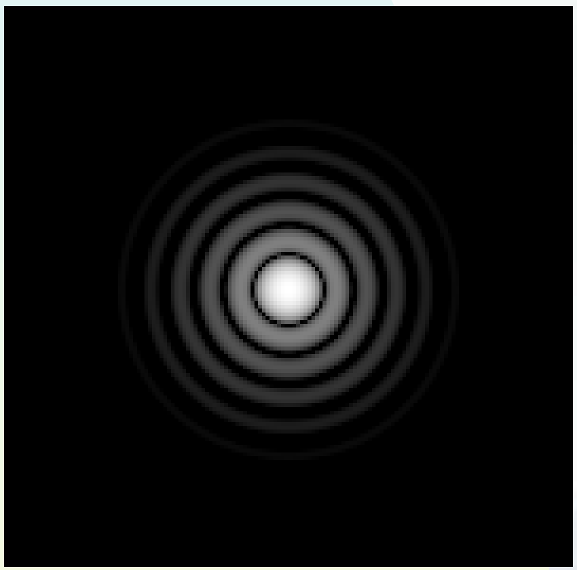
$$I_q = I + I + 2I \, |\gamma_{12}(0)| \cos\left(\beta_{12} - 2\Pi\nu\tau\right)$$

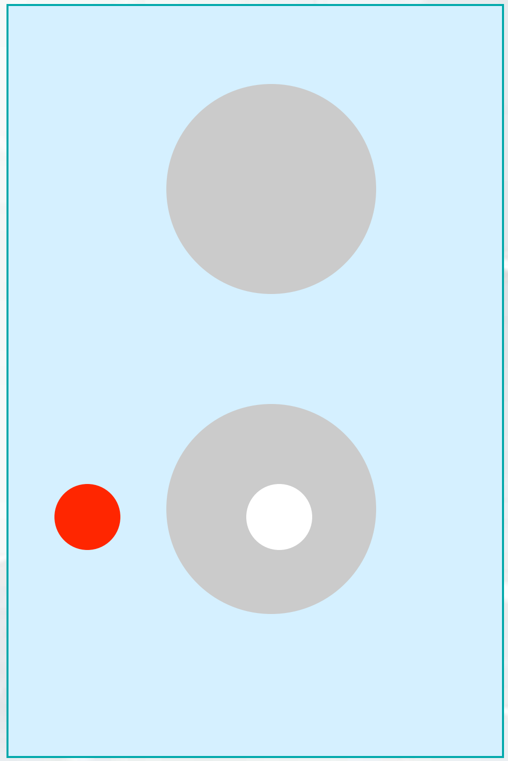
$$\gamma_{12}(\tau) = \langle V_1^*(t) V_2(t-\tau) \rangle / I$$

Fringe visibility: $$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right) = \left| \gamma_{12}(0) \right|$$

An introduction to optical/IR interferometry

- 2.2. The Huygens-Fresnel principle





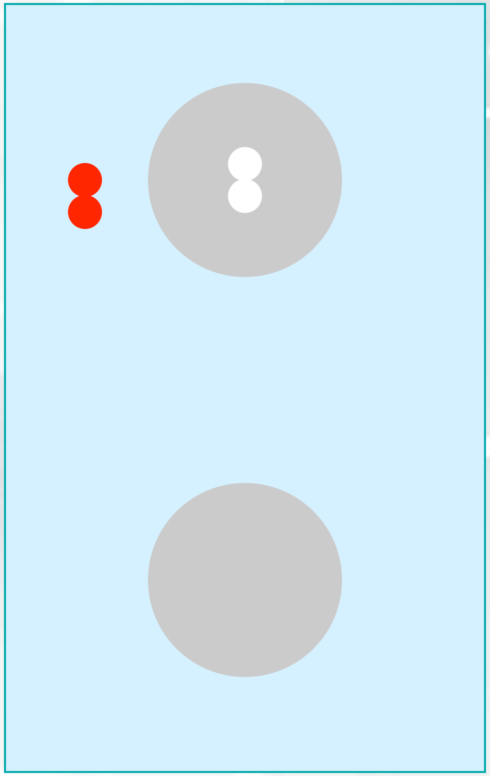
1st experiment!

An introduction to optical/IR interferometry

- 4 Interferometry with two independent telescopes

b) Fizeau … the father of stellar interferometry (1868)

2nd experiment!

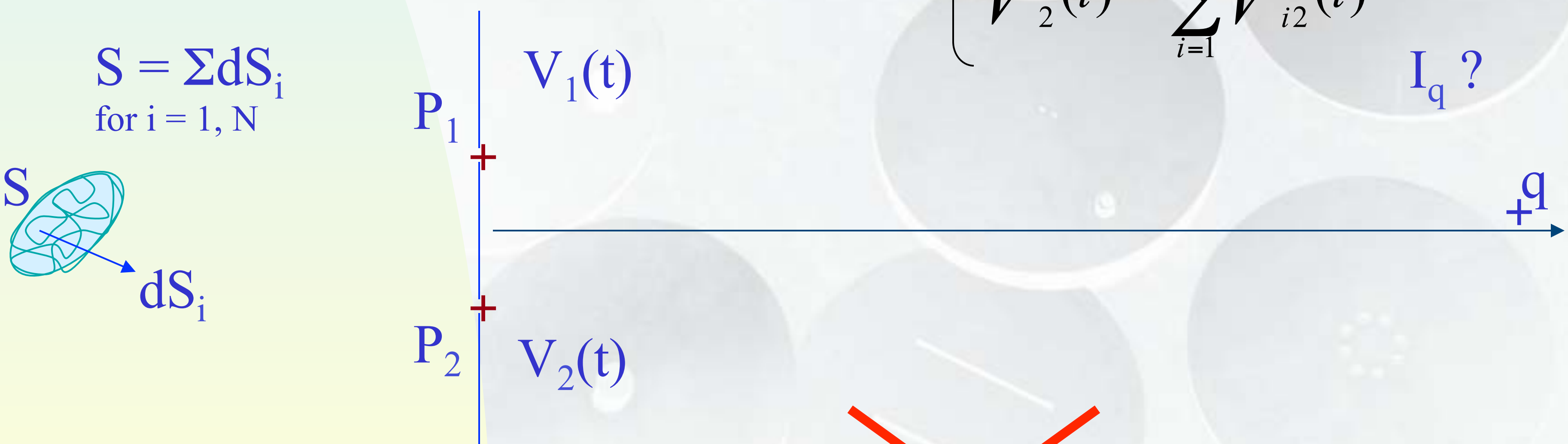


An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$\begin{cases} V_1(t) = \sum_{i=1}^{N} V_{i1}(t) \\ V_2(t) = \sum_{i=1}^{N} V_{i2}(t) \end{cases} \quad (5.3.2)$$

?? $$\gamma_{12}(\tau = 0) = \langle V_1^*(t) V_2(t) \rangle / I \quad (5.3.1)$$



$$\gamma_{12}(0) = \left[\sum_{i=1}^{N} \langle V_{i1}^* V_{i2} \rangle + \sum_{i \neq j}^{N} \langle V_{i1}^* V_{j2} \rangle \right] / I \quad (5.3.3)$$

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$\gamma_{12}(0) = \left[\sum_{i=1}^{N} \langle V_{i1}^{*} V_{i2} \rangle + \sum_{i \neq j}^{N} \langle V_{i1}^{*} V_{j2} \rangle \right] / I \quad (5.3.3)$$

$$\begin{cases} V_{i1}(t) = \left(a_i (t - r_{i1}/c) / r_{i1} \right) \exp\left\{ i 2 \Pi \nu (t - r_{i1}/c) \right\} \quad (5.3.4) \\ V_{i2}(t) = \left(a_i (t - r_{i2}/c) / r_{i2} \right) \exp\left\{ i 2 \Pi \nu (t - r_{i2}/c) \right\} \end{cases}$$

$$V_{i1}^{*}(t) V_{i2}(t) = \left| a_i (t - r_{i1}/c) \right|^2 / (r_{i1} r_{i2}) \exp\left\{ -i 2 \Pi \nu (r_{i2} - r_{i1}) / c \right\} \quad (5.3.5)$$

as long as:

$$\left| r_{i1} - r_{i2} \right| \leq c / \Delta\nu = \lambda^2 / \Delta\lambda = \ell \quad (5.3.6)$$

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$I(s)ds = \left| a_i(t - r/c) \right|^2 \quad (5.3.7)$$

$$\gamma_{12}(0) = \int_S \frac{I(s)}{r_1 r_2} \exp\left\{ -i2\Pi \left(r_2 - r_1 \right) / \lambda \right\} ds / I \quad (5.3.8)$$

!!! Theorem of Zernicke-van Cittert !!!

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$|r_2 - r_1| = |P_2P_i - P_1P_i| = |-(X^2 + Y^2) / 2\,Z' + (X\,\zeta + Y\,\eta)| \qquad (5.3.9)$$

$$\text{where} \quad \zeta = X' / Z' \quad \text{and} \quad \eta = Y' / Z' \qquad (5.3.10)$$

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$\gamma_{12}(0, X/\lambda, Y/\lambda) = \exp(-i\phi_{X,Y}) \frac{\iint_S I(\zeta,\eta)\exp\{-i2\Pi(X\zeta + Y\eta)/\lambda\}d\zeta d\eta}{\iint_S I(\zeta',\eta')d\zeta' d\eta'} \quad (5.3.11)$$

$$I'(\zeta,\eta) = I(\zeta,\eta) / \iint_S I(\zeta',\eta')d\zeta' d\eta' \quad (5.3.12)$$

Setting u = X/λ, v = Y/λ:

$$\gamma_{12}(0,u,v) = \exp(-i\phi_{u,v})\iint_S I'(\zeta,\eta)\exp\{-i2\Pi(u\zeta + v\eta)\}d\zeta d\eta \quad (5.3.13)$$

$$I'(\zeta,\eta) = \iint \gamma_{12}(0,u,v)\exp(i\phi_{u,v})\exp\{i2\Pi(\zeta u + \eta v)\}d(u)d(v) \quad (5.3.14)$$

An introduction to optical/IR interferometry

- **5.4 Fourier transform (cf. Léna 1996)**

5.4.1 Definitions:

$$TF\_f(s) = \int_{-\infty}^{\infty} f(x)\, e^{-2i\pi sx}\, dx, \qquad (5.4.1)$$

$$f(x) = \int_{-\infty}^{\infty} TF\_f(s)\, e^{2i\pi sx}\, ds, \qquad (5.4.2)$$

$$\int_{-\infty}^{\infty} \left| f(x) \right|^2 dx. \qquad (5.4.3)$$

An introduction to optical/IR interferometry

- **5.4 Fourier transform (cf. Léna 1996)**

5.4.1 Definitions: Generalisation:

$$TF\_f(\vec{w}) = \int_{-\infty}^{\infty} f(\vec{r})\, e^{-2i\pi \vec{r}\vec{w}}\, d\vec{r} \quad . \qquad (5.4.4)$$

5.4.2 Some properties:

a) Linearity:

TF_(af) = a TF_f, $a \in \Re$, a being a constant, (5.4.5)

TF_(f+g) = TF_f + TF_g. (5.4.6)

An introduction to optical/IR interferometry

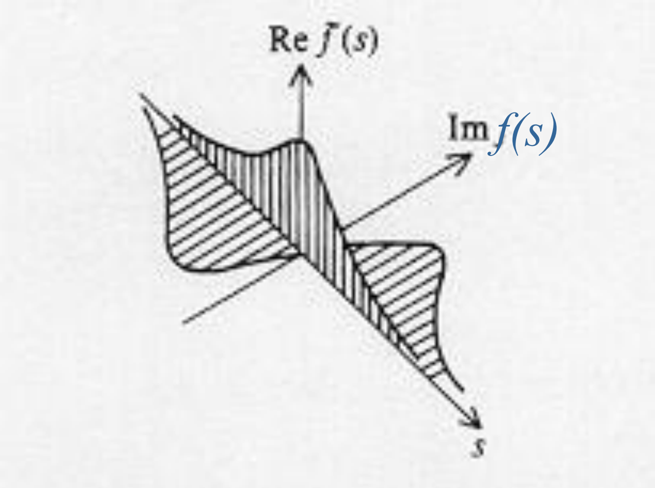
- **5.4 Fourier transform (cf. Léna 1996)**

5.4.2 Some properties: b) Symmetry & parity:

f(x) = P(x) + I(x), (5.4.7)

$$TF\_f(s) = 2\int_0^{\infty} P(x)\cos(2\pi xs)dx - 2i\int_0^{\infty} I(x)\sin(2\pi xs)dx \,. \quad (5.4.8)$$

Illustration of TF_f(s): f(x) is a real fonction. The real and imaginary parts of TF_f(s) are shown.



An introduction to optical/IR interferometry

5.4 Fourier transform (cf. Léna 1996)

c) Similitude:

$$TF\_(f(x/a))(s) = |a|\, TF\_(f(x))(sa), \qquad (5.4.9)$$

where $a \in \Re$, is a constant.

d) Translation:

$$TF\_(f(x - a))(s) = e^{-2i\pi as}\, TF\_(f(x))(s) \qquad (5.4.10)$$

An introduction to optical/IR interferometry

- **5.4 Fourier transform (cf. Léna 1996)**

e) Derivation:

$TF\_(df/dx)(s) = 2i\pi s\ TF\_f(s)$, $TF\_(d^nf/dx^n)(s) = (2i\pi s)^n\ TF\_f(s)$. (5.4.11)

5.4.3 Some important cases (one dimension):

a) Door function:

$\Pi(x) = 1$ if $x \in]-1/2, 1/2[$, (5.4.12)

$= 0$ if $x \in]-\infty, -1/2]$ or $x \in [1/2, \infty[$.

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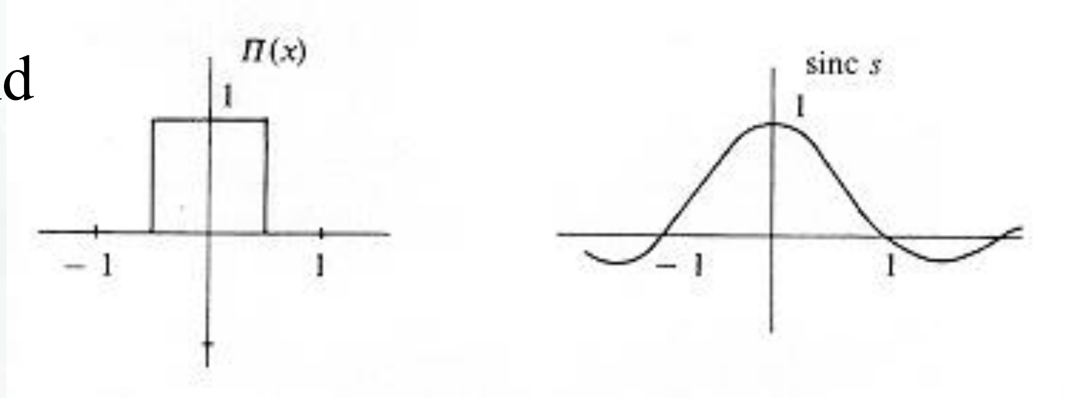
- **5.4 Fourier transform (cf. Léna 1996)**

The door function and its Fourier transform (cardinal sine)



TF_ (Π(x))(s) = sinc(s) = sin(πs) / πs. (5.4.13)

TF_ (Π(x/a))(s) = |a| sinc(as) = |a| sin(πas) / πas. (5.4.14)

An introduction to optical/IR interferometry

- **5.4 Fourier transform (cf. Léna 1996)**

b) Dirac distribution:

$$\delta(x) = \int_{-\infty}^{\infty} e^{2i\pi sx} ds \,. \qquad (5.4.15)$$

its Fourier transform is thus unity (= 1) in the interval]-∞, ∞[.

An introduction to optical/IR interferometry

- **5.4 Fourier transform (cf. Léna 1996)**

c) Dirac comb:

$\coprod(x) = \sum \delta(x-n)$, for n in the interval $]-\infty, \infty[$. (5.4.16)

$TF\_ \coprod(s) = \coprod(s)$. (5.4.17)

$\coprod(x)\, f(x) = \sum f(x)\, \delta(x-n)$, (5.4.18)

for n in the interval $]-\infty, \infty[$,

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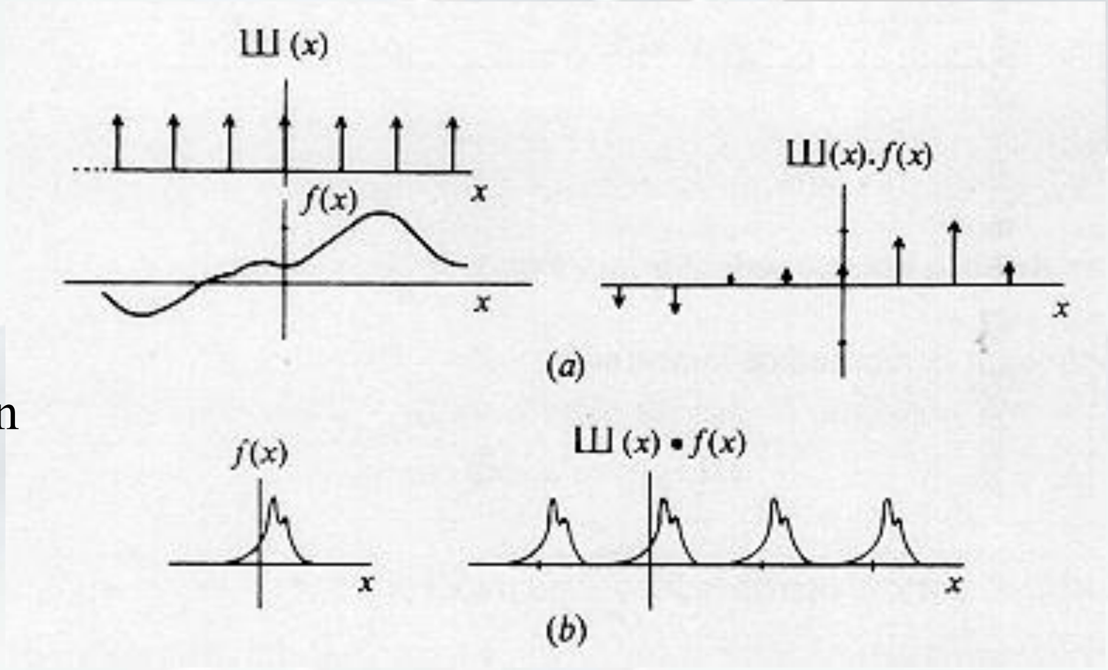
- **5.4 Fourier transform (cf. Léna 1996)**

c) Dirac comb:

$$Ш(x) * f(x) = \sum f(x-n), \quad \text{for n in the interval }]-\infty, \infty[, \qquad (5.4.19)$$



(a) Sampling of the function f(x) by means of the Dirac comb; (b) Replication of the function f(x) after convolution by the Dirac comb.

An introduction to optical/IR interferometry

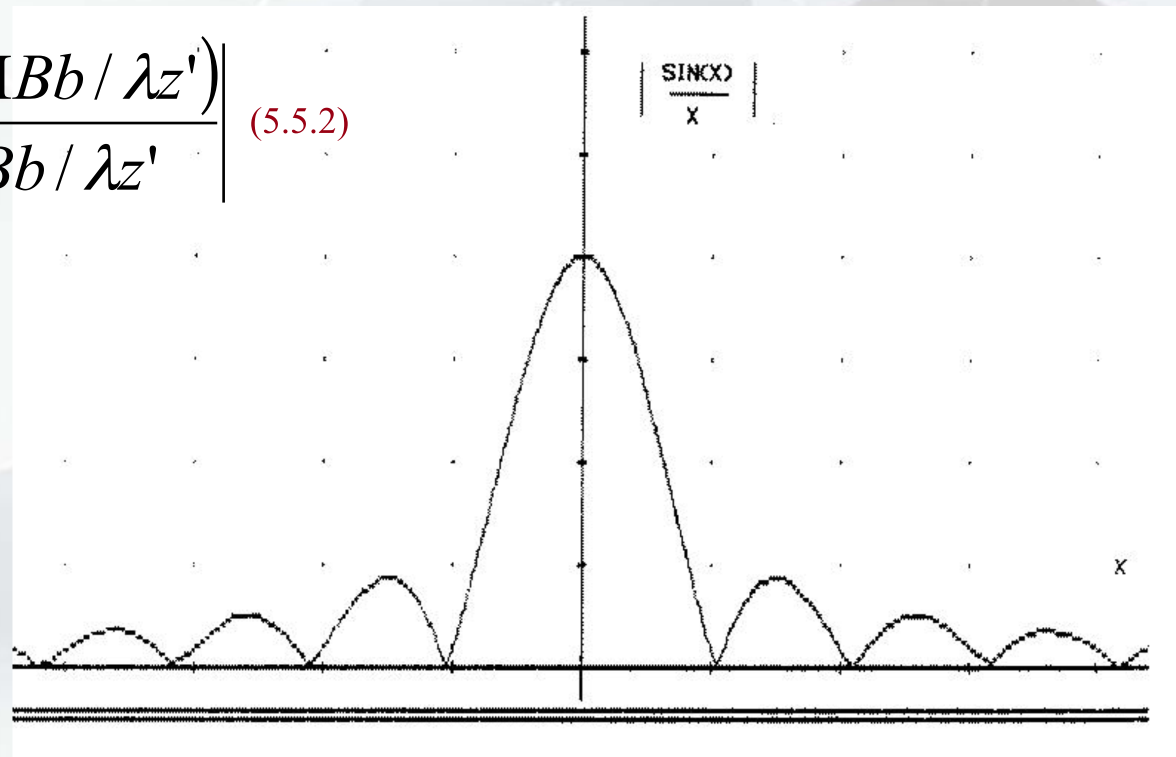
- 5 Light coherence
- **5.5 Aperture synthesis**

$$\upsilon = \left|\gamma_{12}(0, B/\lambda)\right| = \left|\frac{\sin\left(\Pi Bb/\lambda z'\right)}{\Pi Bb/\lambda z'}\right| \quad (5.5.2)$$

$$\Pi Bb/\lambda z' = \Pi \quad (5.5.3)$$

$\Delta \sim \lambda / B$, for a rectangular source. (5.5.4)

$\Delta \sim 1.22\ \lambda / B$, for a circular source ! (5.5.5)



An introduction to optical/IR interferometry

- 5 Light coherence
- **5.5 Aperture synthesis**

Exercises (…): point-like source?, double point-like source with a flux ratio = 1?, gaussian-like source?, uniform disk source?, …