

“Or les différents systèmes philosophiques, économiques et politiques qui régissent les hommes sont tous d’ accord sur un point: bernons-les ...”

**Roger Waters
(Pink Floyd, 1974)**



An introduction to optical/IR interferometry

Brief summary of main results obtained during the last lecture:

$$\rho = R / z$$

$$T_{\text{eff}} = (F/\sigma)^{1/4} = (f / \sigma \rho^2)^{1/4}$$

$$E = A(z) \exp[i2\pi\nu t]$$

$$E = A(z, t) \exp[i2\pi\nu t]$$

$$\tau = 1 / \Delta\nu \quad \lambda_{\text{eff}} = \lambda^2 / \Delta\lambda$$

$$I = A A^* = |A|^2 = a^2 .$$



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If $\Delta \geq \lambda / (2B)$, fringe disappearance!

$$I_q = I + I + 2I |\gamma_{12}(0)| \cos(\beta_{12} - 2\pi\nu\tau)$$

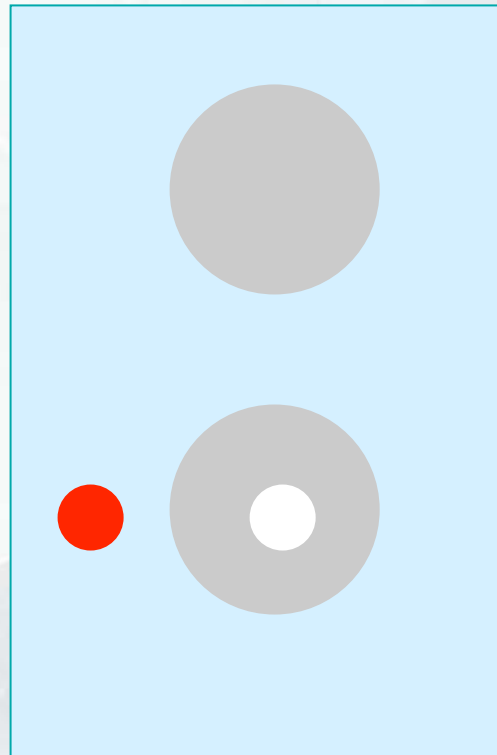
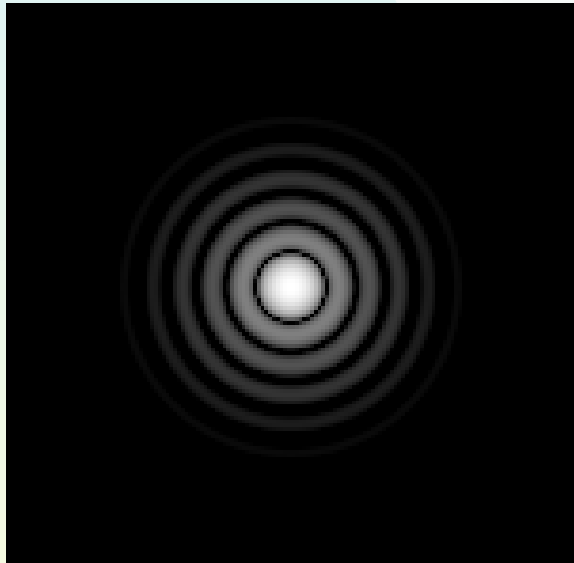
$$\gamma_{12}(\tau) = \langle V_1^*(t) V_2(t - \tau) \rangle / I$$

Fringe visibility:
$$v = \left(\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right) = |\gamma_{12}(0)|$$



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- 2.2. The Huygens-Fresnel principle



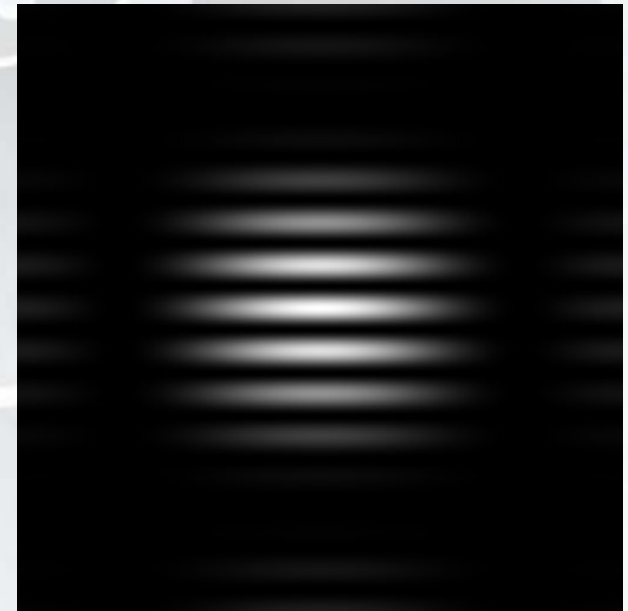
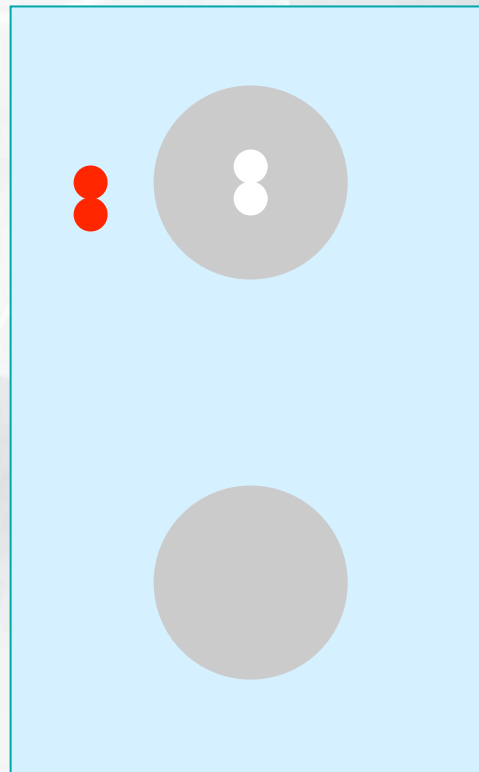
1st experiment!

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- 4 Interferometry with two independent telescopes

b) Fizeau ... the father of stellar interferometry (1868)

2nd experiment!



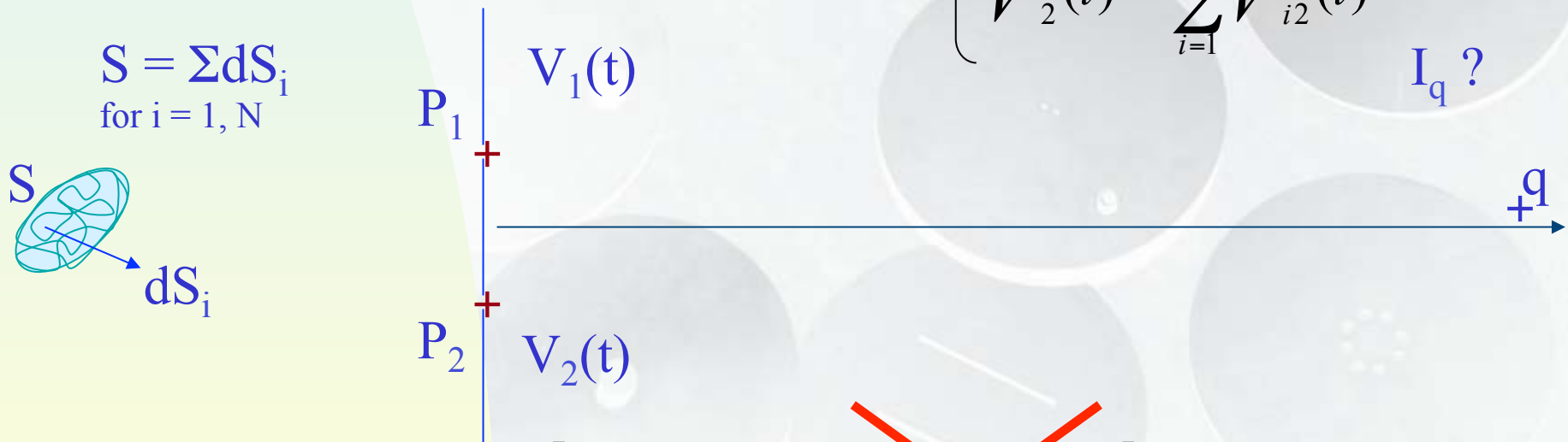
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- 5 Light coherence
- 5.3 Spatial light coherence

?? $\gamma_{12}(\tau = 0) = \langle V_1^*(t) V_2(t) \rangle / I$ (5.3.1)

$$\begin{cases} V_1(t) = \sum_{i=1}^N V_{i1}(t) \\ V_2(t) = \sum_{i=1}^N V_{i2}(t) \end{cases} \quad (5.3.2)$$

$I_q ?$



$$\gamma_{12}(0) = \left[\sum_{i=1}^N \langle V_{i1}^* V_{i2} \rangle + \cancel{\sum_{i \neq j} \langle V_{i1} V_{j2} \rangle} \right] / I \quad (5.3.3)$$

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- 5 Light coherence
- **5.3 Spatial light coherence**

$$\gamma_{12}(0) = \left[\sum_{i=1}^N \langle V_{i1}^* V_{i2} \rangle + \sum_{i \neq j}^N \langle V_{i1} V_{j2} \rangle \right] / I \quad (5.3.3)$$

$$\begin{cases} V_{i1}(t) = \left(a_i(t - r_{i1}/c) / r_{i1} \right) \exp\{i2\pi\nu(t - r_{i1}/c)\} \\ V_{i2}(t) = \left(a_i(t - r_{i2}/c) / r_{i2} \right) \exp\{i2\pi\nu(t - r_{i2}/c)\} \end{cases} \quad (5.3.4)$$

$$V_{i1}^*(t) V_{i2}(t) = \left| a_i(t - r_{i1}/c) \right|^2 / (r_{i1} r_{i2}) \exp\{-i2\pi\nu(r_{i2} - r_{i1})/c\} \quad (5.3.5)$$

as long as: $|r_{i1} - r_{i2}| \leq c / \Delta\nu = \lambda^2 / \Delta\lambda = \ell \quad (5.3.6)$

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- 5 Light coherence
- **5.3 Spatial light coherence**

$$I(s)ds = \left| a_i(t - r/c) \right|^2 \quad (5.3.7)$$

$$\gamma_{12}(0) = \int_S \frac{I(s)}{r_1 r_2} \exp\left\{ -i2\pi(\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{s} / \lambda \right\} ds / I \quad (5.3.8)$$

!!! Theorem of Zernicke-van Cittert !!!

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- 5 Light coherence
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$$|r_2 - r_1| = |P_2P_i - P_1P_i| = |-(X^2 + Y^2) / 2 Z' + (X \zeta + Y \eta)| \quad (5.3.9)$$

where $\zeta = X' / Z'$ and $\eta = Y' / Z'$ (5.3.10)

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- 5 Light coherence
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$$\gamma_{12}(0, X/\lambda, Y/\lambda) = \exp(-i\phi_{X,Y}) \frac{\iint_S I(\xi, \eta) \exp\{-i2\Pi(X\xi + Y\eta)/\lambda\} d\xi d\eta}{\iint_S I(\xi', \eta') d\xi' d\eta'} \quad (5.3.11)$$

$$I'(\xi, \eta) = I(\xi, \eta) / \iint_S I(\xi', \eta') d\xi' d\eta' \quad (5.3.12)$$

Setting $u = X/\lambda, v = Y/\lambda$:

$$\gamma_{12}(0, u, v) = \exp(-i\phi_{u,v}) \iint_S I'(\xi, \eta) \exp\{-i2\Pi(u\xi + v\eta)\} d\xi d\eta \quad (5.3.13)$$

$$I'(\xi, \eta) = \iint \gamma_{12}(0, u, v) \exp(i\phi_{u,v}) \exp\{i2\Pi(\xi u + \eta v)\} d(u) d(v) \quad (5.3.14)$$

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■ 5.4 Fourier transform (cf. Léna 1996)

5.4.1 Definitions:

$$TF _ f(s) = \int_{-\infty}^{\infty} f(x) e^{-2i\pi s x} dx, \quad (5.4.1)$$

$$f(x) = \int_{-\infty}^{\infty} TF _ f(s) e^{2i\pi s x} ds, \quad (5.4.2)$$

$$\int_{-\infty}^{\infty} |f(x)|^2 dx. \quad (5.4.3)$$

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■ 5.4 Fourier transform (cf. Léna 1996)

5.4.1 Definitions: Generalisation:

$$TF_{-} f(\vec{w}) = \int_{-\infty}^{\infty} f(\vec{r}) e^{-2i\pi\vec{r}\vec{w}} d\vec{r} \quad (5.4.4)$$

5.4.2 Some properties:

a) Linearity:

$$TF_{-}(af) = a TF_{-}f, \quad a \in \mathfrak{R}, a \text{ being a constant}, \quad (5.4.5)$$

$$TF_{-}(f+g) = TF_{-}f + TF_{-}g. \quad (5.4.6)$$

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■ 5.4 Fourier transform (cf. Léna 1996)

5.4.2 Some properties: b) Symmetry & parity:

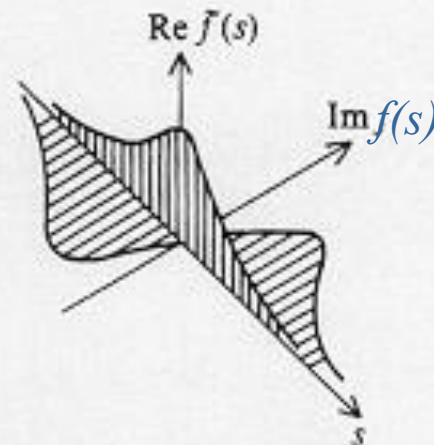
$$f(x) = P(x) + I(x),$$

(5.4.7)

$$TF_f(s) = 2 \int_0^{\infty} P(x) \cos(2\pi xs) dx - 2i \int_0^{\infty} I(x) \sin(2\pi xs) dx .$$

(5.4.8)

Illustration of $TF_f(s)$: $f(x)$ is a real function. The real and imaginary parts of $TF_f(s)$ are shown.



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■ 5.4 Fourier transform (cf. Léna 1996)

c) Similitude:

$$\text{TF}_-(f(x/a))(s) = |a| \text{TF}_-(f(x))(sa), \quad (5.4.9)$$

where $a \in \mathfrak{R}$, is a constant.

d) Translation:

$$\text{TF}_-(f(x - a))(s) = e^{-2i\pi as} \text{TF}_-(f(x))(s) \quad (5.4.10)$$

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■ 5.4 Fourier transform (cf. Léna 1996)

e) Derivation:

$$\text{TF}_{(df/dx)}(s) = 2i\pi s \text{TF}_f(s), \text{TF}_{(d^n f/dx^n)}(s) = (2i\pi s)^n \text{TF}_f(s). \quad (5.4.11)$$

5.4.3 Some important cases (one dimension):

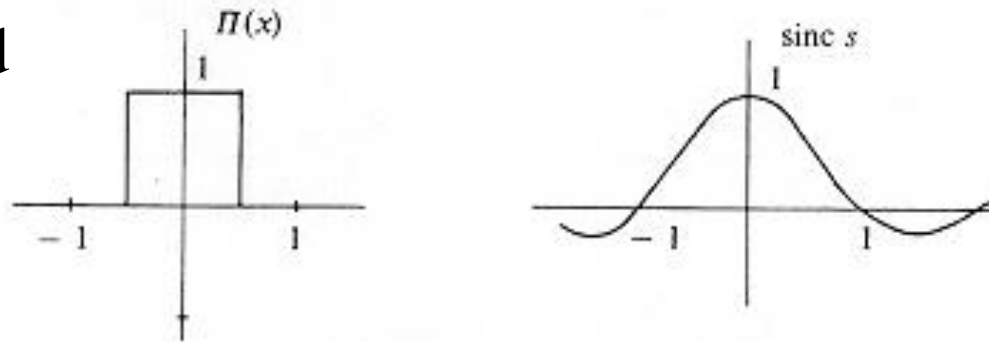
a) Door function:

$$\begin{aligned} \Pi(x) &= 1 \text{ if } x \in]-1/2, 1/2[, \\ &= 0 \text{ if } x \in]-\infty, -1/2] \text{ or } x \in [1/2, \infty[. \end{aligned} \quad (5.4.12)$$

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■ 5.4 Fourier transform (cf. Léna 1996)

The door function and its Fourier transform (cardinal sine)



$$\text{TF}_- (\Pi(x))(s) = \text{sinc}(s) = \sin(\pi s) / \pi s. \quad (5.4.13)$$

$$\text{TF}_- (\Pi(x/a))(s) = |a| \text{sinc}(as) = |a| \sin(\pi as) / \pi as. \quad (5.4.14)$$

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■ 5.4 Fourier transform (cf. Léna 1996)

b) Dirac distribution:

$$\delta(x) = \int_{-\infty}^{\infty} e^{2i\pi sx} ds .$$

(5.4.15)

its Fourier transform is thus unity (= 1) in the interval $]-\infty, \infty[$.

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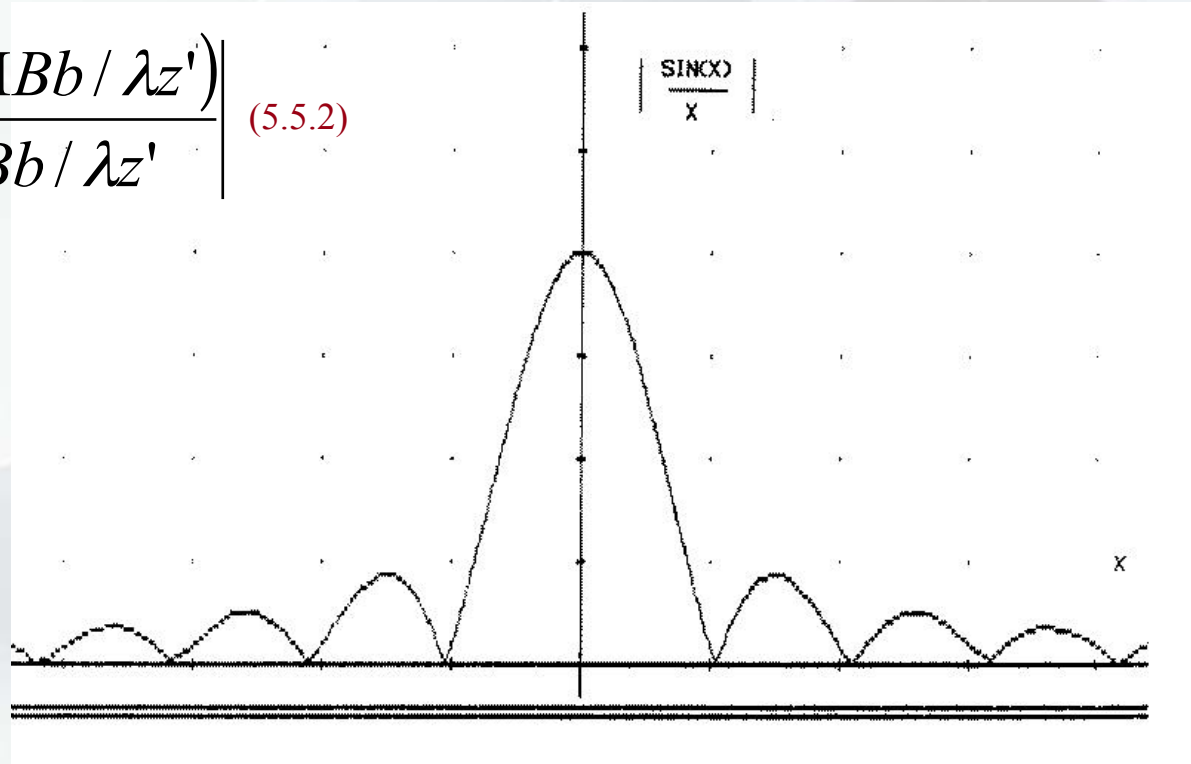
- 5 Light coherence
- 5.5 Aperture synthesis

$$v = \left| \gamma_{12} \left(0, B / \lambda \right) \right| = \left| \frac{\sin \left(\Pi B b / \lambda z' \right)}{\Pi B b / \lambda z'} \right| \quad (5.5.2)$$

$$\Pi B b / \lambda z' = \Pi \quad (5.5.3)$$

$\Delta \sim \lambda / B$, for a rectangular source. (5.5.4)

$\Delta \sim 1.22 \lambda / B$, for a circular source ! (5.5.5)



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Exercises (...): point-like source?, double point-like source with a flux ratio = 1?, gaussian-like source?, uniform disk source?, ...