Exploiting frame-invariant operators for the efficient numerical simulation of flexible multibody systems

Valentin Sonneville*, **Alberto Cardona**# **and Olivier Brüls***

*Department of Aerospace and Mechanical Engineering (LTAS)
University of Liège
Chemin des Chevreuils 1 (B52/3), 4000 Liège, Belgium
v.sonneville,o.bruls@ulg.ac.be

#CIMEC
Universidad Nacional Litoral - Conicet
Colectora Ruta Nac 168 / Paraje El Pozo, 3000 Santa Fe, Argentina
acardona@intec.unl.edu.ar

Abstract

Recently, the authors have presented a library of generic elements formulated in the special Euclidean group $SE(3)$ formalism for the numerical simulation of flexible multibody systems. This library includes an implicit time integration method [1], a rigid body and kinematic joints [2], a geometrically exact flexible beam [3, 4], a geometrically exact flexible shell [5] and a geometrically exact super-element [6]. The geometric description of the elements is based on the representation of frame transformations as 4×4 homogeneous transformation matrices

$$\mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} \tag{1}$$

where $\mathbf{x}$ is a 3×1 vector and $\mathbf{R}$ is a 3×3 rotation matrix whose meaning is related to the element kinematics. For instance, for a beam, $\mathbf{x}$ accounts for the position of the neutral axis and $\mathbf{R}$ for the orientation of the cross-sections whereas, for a kinematic joint, $\mathbf{x}$ accounts for the relative displacements and $\mathbf{R}$ for the relative rotations in the joint.

The proposed framework exhibits reduced non-linearities compared to classical formulations for two reasons

- The equations of motion are expressed, both at position and rotation level, in local frames. Accordingly, the numerical expressions are invariant with respect to superimposed rigid body transformations. As a consequence, the tangent matrices and the constraint gradient are insensitive to large amplitude motions and depend on local transformations only, i.e. deformations and relative motions.

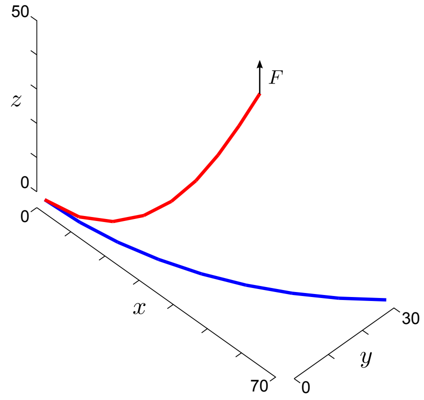
- The equations of motion, which take the form of second order differential-algebraic equations on a Lie group, are solved without introducing a global parametrization of the motion, and in particular of the rotation.

Several important properties also appear as a consequence of the formalism. For instance, the representation of large rotations is naturally singularity-free, the flexible elements are naturally locking-free and the initially curvature of flexible elements is trivially accounted for at any additional costs.

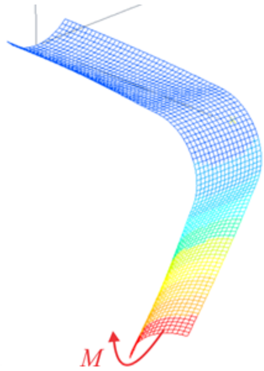
In this work, computational strategies taking advantages of these convenient numerical properties are presented and several applications in flexible multibody systems are considered.

As an example of the importance of the reduction of non-linearites, the 45°-bend (Fig. 1a) presented in [7] and modelled with flexible beam elements can be solved in one load step using the material part of tangent stiffness matrix only.

In the case of small deformations, most parts of the iteration matrix used in the implicit integration method do not have to be updated thanks to the invariance with respect to superimposed rigid body transformation, which leads to a significant reduction in computational time. Indeed, both the evaluation and the inversion of these parts can be done once for an entire simulation. This feature is applied to the analysis of the dynamic behaviour of a tape-spring (see representation in Fig. 1b) modelled with flexible shell elements and to the shape-optimization of a flexible robot.



(a) 45°-bend modelled with geometrically exact flexible beam elements.



(b) Tape-spring modelled with geometrically exact flexible shell elements.

Figure 1: Examples of applications considered in this work.

Acknowledgement
Valentin Sonneville would like to acknowledge the Belgian National Fund for Scientific Research (FRIA) for its financial support.

References

[1] O. Brüls, A. Cardona, and M. Arnold. Lie group generalized-α time integration of constrained flexible multibody systems. *Mechanism and Machine Theory*, (48):121–137, 2012.

[2] V. Sonneville and O. Brüls. A formulation on the special Euclidean group for dynamic analysis of multibody systems. *Journal of Computational and Nonlinear Dynamics*, 9(4):041002, 2014.

[3] V. Sonneville, A. Cardona, and O. Brüls. Geometrically exact beam finite element formulated on the special Euclidean group SE(3). *Computer Methods in Applied Mechanics and Engineering*, 268:451–474, January 2014.

[4] V. Sonneville, A. Cardona, and O. Brüls. Geometric interpretation of a non-linear beam finite element on the Lie group SE(3). *Archive of Mechanical Engineering*, 64(2):305–329, August 2014.

[5] V. Sonneville and O. Brüls. Formulation of a non-linear shell finite element on the Lie group SE(3). In *11th World Congress on Computational Mechanics (WCCM XI) - Book of Abstract*, 2014.

[6] V. Sonneville and O. Brüls. Model reduction of geometrically exact structures formulated on the Lie group SE(3). In *The 3rd Joint International Conference on Multibody System Dynamics - Book of Abstract*, 2014.

[7] K.-J. Bathe and S. Bolourchi. Large displacement analysis of three-dimensional beam structures. *International Journal for Numerical Methods in Engineering*, 14:961–986, 1979.