The Nonlinear Tuned Vibration Absorber, Part I: Design and Performance Analysis

G. Habib, T. Detroux and G. Kerschen

Space Structures and System Laboratory
Department of Aerospace and Mechanical Engineering
University of Liège, Liège, Belgium.

Introduction

With continual interest in expanding the performance envelope of engineering systems, nonlinear components are increasingly utilized in real-world applications. Mitigating the resonant vibrations of nonlinear structures is therefore becoming a problem of great practical significance; it is the focus of the present study.

Nonlinear vibration absorbers, including the autoparametric vibration absorber [1], the nonlinear energy sink (NES) [2] and other variants [3, 4], can absorb disturbances in wide ranges of frequencies due to their increased bandwidth. However, the performance of existing nonlinear vibration absorbers is known to exhibit marked sensitivity to motion amplitudes. For instance, there exists a well-defined threshold of input energy below which no significant energy dissipation can be induced in an NES [2].

This paper builds upon previous developments [5] to introduce a new nonlinear vibration absorber for mitigating the vibrations around one problem nonlinear resonance. The absorber is termed the nonlinear tuned vibration absorber (NLTVA), because its nonlinear restoring force is determined according to the nonlinear restoring force of the host structure. In other words, we propose to synthesize the absorber's load-deflection characteristic so that the NLTVA can mitigate the considered nonlinear resonance in wide ranges of motion amplitudes.

Furthermore, a nonlinear generalization of Den Hartog's equal-peak method for determining the NLTVA parameters is developed. The basic idea is to select the nonlinear coefficient of the absorber that ensures equal peaks in the nonlinear receptance function for an as large as possible range of forcing amplitudes. We will show that this is only feasible when the mathematical form of the NLTVA's restoring force is carefully chosen, which justifies the proposed synthesis of the absorber's load-deflection curve.

Mathematical Model and Tuning Rule

The dynamics of a Duffing oscillator with an attached NLTVA is considered:

$$\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_{nl1}x_1^3 + c_2(\dot{x}_1 - \dot{x}_2) + g(x_1 - x_2) &= F\cos\omega t \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) - g(x_1 - x_2) &= 0 \end{aligned} \tag{1}$$

where $x_1(t)$ and $x_2(t)$ are the displacements of the primary system and of the NLTVA, respectively. The NLTVA is assumed to have a generic smooth restoring force $g(x_1 - x_2)$ with $g(0) = 0$. Adimensionalizing the equation of motion, the forcing amplitude F disappears from the linear terms, which confirms that the linear part of the absorber is amplitude independent. Furthermore, expanding $g(x_1 - x_2)$ in Taylor series, it can be noted that F appears in the dimensionless coefficients of the nonlinear term with exponent $k - 1$, where k is the order of the corresponding coefficient. This suggests that, if an optimal set of absorber parameters is chosen for a specific value of F, variations of F will detune the nonlinear absorber, unless the nonlinear coefficients of the primary system and of the absorber undergo a similar variation with F. This can be achieved by selecting the same mathematical function for the absorber as that of the primary system. When coupled to a Duffing oscillator, the NLTVA should therefore possess a linear (k_2) and a cubic (k_{nl2}) spring.

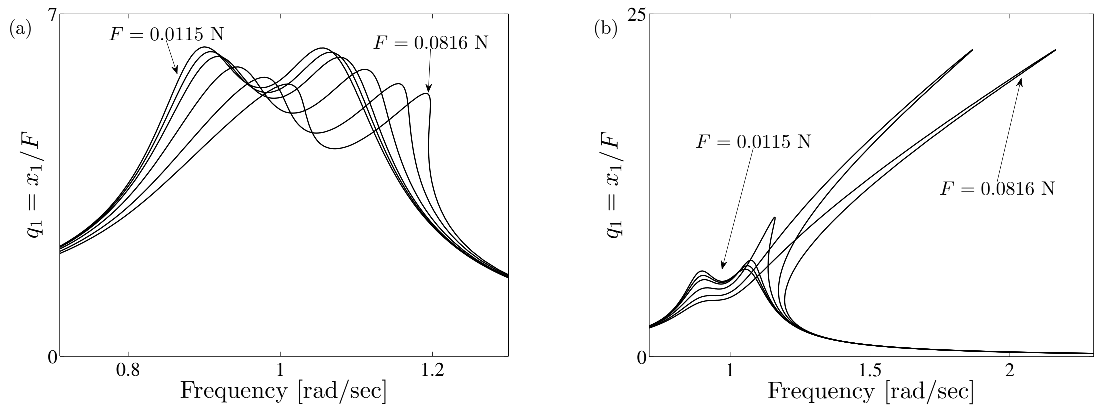

Figure 1: Frequency response of a Duffing oscillator with an attached NLTVA (a) and LTVA (b). For the computation $m_1 = 1$ kg, $c_1 = 0.002$ N.s/m, $k_1 = 1$ N/m, $k_{nl1} = 1$ N/m^3 and $\epsilon = 0.05$. For the different curves $F = 0.0115$ N, $F = 0.0258$ N, $F = 0.0365$ N, $F = 0.0577$ N, and $F = 0.0816$ N.

In order to have an optimal behavior of the system at low forcing amplitudes, the well-known Den Hartog's tuning rule [6] for equal peaks, or alternatively the more precise formulas in [7], should be used. Then, based on a numerical procedure, the optimal value of the coefficient of the nonlinear restoring force of the absorber k_{nl2}, which guarantees equal peaks, can be obtained. Performing the optimization procedure for several values of the Duffing term and of the forcing amplitude, it can be observed that the optimal value of k_{nl2} does not depend on the amplitude and it is linear with respect to variations of k_{nl1}. Through a regression of the dimensionless coefficients of the system, we obtain the formula $k_{nl2} = 2\epsilon^2 k_{nl1}/(1 + 4\epsilon)$, where $\epsilon = m_2/m_1$, which approximates with excellent precision the value obtained through a numerical optimization procedure. This formula allows to easily tune the nonlinear spring restoring force and therefore can be considered as a nonlinear extension of Den Hartog's equal peaks rule.

Interestingly, the Duffing oscillator with an attached NLTVA exhibits linear-like dynamics in the investigated range of forcing amplitude. As shown in Fig. 1 (a), the frequency response increases almost linearly with respect to the forcing amplitude, in spite of the frequency shift of the resonant peaks. On the contrary, a linear tuned vibration absorber (LTVA) is rapidly detuned and its performance is strongly dependent on forcing amplitude (Fig. 1 (b)). For any value of the forcing amplitude in the investigated range, an important results is that the NLTVA has always better performance than the LTVA, which confirms the effectiveness of the proposed device and tuning rule.

Acknowledgments

The authors G. Habib, T. Detroux and G. Kerschen would like to acknowledge the financial support of the European Union (ERC Starting Grant NoVib-307265).

References

[1] S. Oueini, A. Nayfeh, J. Vib. Contr. **6**, (2000) 999-1016.

[2] A.F. Vakakis, O. Gendelman, L.A. Bergman, D.M. McFarland, G. Kerschen, Y.S. Lee, *Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems* (Springer, 2009).

[3] J. Shaw, S.W. Shaw, A.G. Haddow, Int. J. Nonlin. Mech **24**, (1989) 281-293.

[4] N.A. Alexander, F. Schilder, J. Sound Vib. **319**, (2009) 445-462.

[5] R. Viguié, G. Kerschen, J. Sound Vib. **326**, (2009) 780-793.

[6] J.P. Den Hartog, *Mechanical Vibrations* (McGraw-Hill, New York 1934).

[7] T. Asami, O. Nishihara, J. Vib. Acoust. **125**, (2003) 398-405.