Why the Defined-Benefit Pension System Is Unfriendly to Growth in an Ageing Society

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Abstract

This paper develops an OLG growth model with human capital, public education and two types of PAYG pension systems (defined benefit and defined contribution). The crucial characteristic of the model is the integration of an explicit exogenous age structure in order to study its growth effects. The paper's main results are threefold. First, a decomposition of the income growth rate per capita is proposed to separate demographic from technological effects in a growth setup without a PAYG pension system. Second, although it reduces potential income growth, a defined-contribution PAYG pension system is shown to be "demographically neutral" in the sense that its negative effect on income growth does not vary with population ageing. Third, it is shown that a defined-benefit PAYG pension system is the most unfriendly to economic growth as the population ages.

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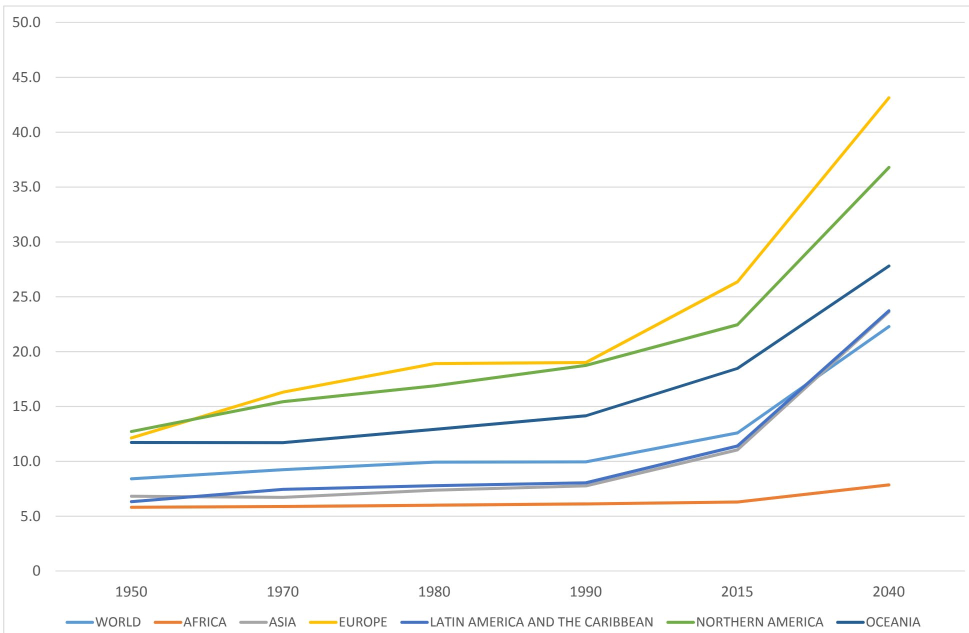
1 Introduction

The world population is ageing (UN 2015). Almost everywhere, both mortality and fertility rates are declining. The advanced economies already have aged populations while the less developed countries are now experiencing population ageing. According to the UN Population Statistics, 12% of the world population is aged 60 or over in 2015 and the share of this segment of the population is the fastest growing. The old-age dependency ratio, which is the ratio of population aged 65 or over to population aged 15-64, is expected to increase rapidly in the next generation (Figure 1). The main source of this ageing process is the decline in the fertility rate observed in the past for advanced countries and more recently for less developed countries (Weil 2008). The steady rise in life expectancy reinforces this process.

Population ageing has two main economic effects. First, the labor supply becomes scarce as newborn generations entering the labor market are in smaller numbers than old generations exiting it. The macroeconomic effects are uncertain. If the scarcity of the labor supply implies an increase in the price of labor, it also provides the firms with an incentive to be more productive (Cutler et al. (1990)). Empirical research has shown that a decline in fertility has a positive effect on economic growth in line with the predictions of theoretical growth models (Li and Zhang (2007) and Ashraf, Weil, and Wilde (2013)). Most studies assessing the impact of ageing on economic growth are not particularly dim (see, for instance, Cutler et al. (1990), Tyers and Shi (2007) and Bloom, Canning, and Fink (2010)). The second effect is the burden of the intergenerational transfers to elderly people and, in particular, pensions. There is a lot of concern about the consequences of population ageing on the financial sustainability of pension systems.[1] Many advanced countries introduced PAYG pension systems at a time when the labor force was rising and life expectancy was not as long as it is nowadays. As the demographic trend reverses, the

[1] See, for instance, Barr and Diamond (2006), EC (2014) and OECD (2013).



Source: UN Population Division (2015)
UN Population estimates for 1950 to 2015 and projections for 2040 (Medium Variant)

Figure 1: Old-Age Dependency Ratio (%) in the Five Continents (1950-2040)

cost and the sustainability of the pension system now is at the top of the political agenda. Many pension reforms have been introduced in advanced countries amidst popular disapproval. They all consist in rebalancing the imputation of the cost of ageing from the working population to the pensioners. Many diverse and complex pension systems now are in place in those countries mixing pay-as-you-go (PAYG) defined-benefit (DB) and defined-contribution (DC) schemes as well as funded schemes. In OECD countries, 18 have a DB PAYG system while 10 have a DC PAYG system (OECD 2013). The question is: which pension system is the most favorable to income growth?

The objective of this paper is to provide a theoretical appraisal of the most sustainable pension system in ageing society. Standard growth models are not well equipped to tackle this question. They assume that population grows at a constant rate in line with conventional demographic theory, which presumed that fertility would stabilize at the end of the demographic transition (Bongaarts (2002) and Lee (2003)). Despite a post-war baby boom followed by a baby bust at the end of the 1960s in Western countries, it was widely believed that the stabilization of fertility would reach the replacement level of 2.1 births per woman. In fact, the fertility rate in advanced societies declined well below 2.0 in the 1990s implying shrinking and rapidly ageing future populations. However, since the beginning of the new century, the fertility rate in most of these societies has reversed (Myrskylä, Kohler, and Billari 2009). Therefore, the fertility rate appears to be quite fluctuating over time while life expectancy rises steadily. These two indicators entail that a non-constant demographic structure is more the rule rather the exception. Given the dependence of public policies such as education, health care and pensions on intergenerational transfers, the demographic structure is a very important determinant of public finance management and economic development. In this paper, we build an OLG model whose growth engine is human capital accumulation. It extends the analysis of Artige, Cavenaile, and Pestieau (2014) to a growth framework with an explicit demographic structure. Parents pay a labor income tax, which finances public education, the main input of human capital formation. We first decompose the income growth rate per capita into demographic and technological growth effects in an economy without pension (or with a fully-funded pension system). This decomposition allows to measure the effect of population ageing on income growth in this benchmark economy. We then compare it with the decompositions of two economies, one with a defined-contribution (DC) PAYG pension system and one with a defined-benefit (DB) PAYG pension system. Our results show that the DC pension system is "demographically neutral" in the sense that the demographic effects on income growth are not different from those of the benchmark economy. In contrast, income growth in an economy with DB pension system is specifically sensitive to population ageing. The pension cost increases with the old-age dependency ratio. On a policy point of view, DB pension systems in ageing societies require reforms such a rise in the retirement age or higher education spending to guarantee their financial sustainability and offset their specific impact on income growth.

The rest of the paper is organized as follows. Section 2 presents the OLG growth model with public education. Section 3 describes the demographic structure and dynamics. Section 4 describes the decomposition between demographic and technological effects on income growth in the benchmark economy. Section 5 describes the balanced growth path. Section 6 introduces the PAYG pension system with two types: defined contribution (DC) and defined benefit (DB) and assess the impact of ageing on income growth and welfare. Section 7 discusses the results and concludes.

2 An OLG Model with Public Education

We consider a discrete-time deterministic model of an economy producing a single aggregate good under perfect competition from date $t = 0$ to infinity as in Diamond (1965).

The economy is populated by overlapping generations living for three periods. The third period is of length $l \in [0,1]$ so that life expectancy at birth is equal to $2+l$. The population size at time t is composed of three generations: $Z_t = lN_{t-2} + N_{t-1} + N_t$ where N_t is the size of generation t, N_{t-1} is the working-age population and $lN_{t-2} + N_t$ is the dependent population. We assume that the growth rate of population is positive or null. When young, the individuals benefit from education spending and build their human capital. Their consumption is included in their parent's consumption. When middle-aged, individuals supply inelastically one unit of labor to the firms in a perfectly competitive labor market, receive a wage and allocate their net of tax income between consumption and saving. In their old age, they retire and consume their resources.[2] The representative firm of this economy produces a single good using a Cobb-Douglas technology

$$Y_t = K_t^{\alpha}(L_t h_t)^{1-\alpha}, \tag{1}$$

where Y_t is the output at time t, K_t is physical capital and h_t is the average stock of human capital and L_t is the labor input at time t which is equal to the working-age generation N_{t-1}. Physical capital is assumed to be fully depreciated after one period. The parameter $\alpha < 1$ is the output elasticity of physical capital. At time t, the representative firm chooses the stock of capital K_t and the labor input L_t and maximizes its profits

$$\pi_t = \max_{K_t, L_t} K_t^{\alpha}(L_t h_t)^{1-\alpha} - w_t h_t L_t - (1+r_t)K_t, \tag{2}$$

where w_t is the wage per unit of effective labor, $(1+r_t)K_t$ is the return distributed to the owners of physical capital (households) and r_t the real interest rate. The maximization of (2) with respect to K_t and L_t by the representative firm yields the wage rate and the

[2]For the sake of simplicity, we do not consider optimal retirement age. Assuming it would not change the results qualitatively.

interest rate. The expressions for the wage rate and the interest rate are thus the optimal return of labor and capital respectively:

$$w_t = (1-\alpha)K_t^{\alpha}(L_t h_t)^{-\alpha} \tag{3}$$

$$r_t = \alpha K_t^{\alpha-1}(L_t h_t)^{1-\alpha} - 1 \tag{4}$$

The consumers can invest in physical capital but also in human capital. The production function for the human capital accumulation is defined by

$$h_{t+1} = \left(\frac{L_t}{L_{t+1}}\right)\Psi e_t^{\theta} h_t^{1-\theta}, \quad 0 < \theta < 1, \quad \Psi > 0, \tag{5}$$

where θ is the elasticity of average human capital production with respect to average education spending and Ψ is a scale technological parameter. The average stock of human capital at time $t+1$ is assumed to depend on contemporaneous average education spending, e_t, financed by the middle-aged individuals, and on the average human capital stock of the previous period, h_t. As in Lucas (1988), it is assumed that the production of human capital does not require physical capital because education is known to be relatively intensive in human capital. The human capital dilution effect due to the increase in the labor force is captured by $\frac{L_t}{L_{t+1}}$. In other words, at given education spending, the higher the growth rate of the working-age population, the slower the growth of the average human capital. Education is assumed to be publicly financed by a labor-income tax levied on the middle-aged (working-age) generation

$$N_t \varepsilon w_t h_t = \tau_h w_t h_t N_{t-1} \tag{6}$$

where $\varepsilon w_t h_t = e_t$ and ε is the child's education spending rate per labor income. In other words, ε measures the percentage of labor income spent on education for each child across generations and is equal to

$$\varepsilon = \frac{N_{t-1}}{N_t} \tau_h \tag{7}$$

where τ_h is the contribution rate by each middle-aged individual to education spending. The representative consumer's preferences are represented by a logarithmic life-cycle utility function

$$u = \ln c_t + \beta l \ln d_{t+1} \tag{8}$$

The consumer maximizes (8) with respect to the budget constraints

$$\begin{aligned} c_t + s_t &= (1 - \tau_h)\, w_t h_t & (9) \\ d_{t+1} &= \frac{(1 + r_{t+1}) s_t}{l} & (10) \end{aligned}$$

where w_t, c_t, s_t are respectively the wage, the consumption when middle-aged and the individual saving at time t. When old, the individuals consume d_{t+1} per unit of time during their period l. The parameter $\beta \in (0, 1)$ is the psychological discount factor. Optimal saving is therefore

$$s_t = \frac{\beta l}{1 + \beta l} (1 - \tau_h)\, w_t h_t. \tag{11}$$

where $\frac{\beta l}{1+\beta l}$ is the propensity to save. The higher the longevity, the higher the propensity to save.

3 The Demographic Structure and its Dynamics

The population is composed of the young, the middle-aged and the old generations at each period of time. In many advanced societies with PAYG pension systems, the young and the old generations benefit from intergenerational transfers: education for the young and pensions for the old. The working population bears the cost of both transfers. The cost per worker therefore depends on the size of the dependent and self-supporting populations. Two ratios have been used to measure this cost: the support ratio and the total dependency ratio. The support ratio is calculated as the effective labor force divided by the effective number of consumers (Cutler et al. 1990). The denominator - the effective number of consumers - is identical to the entire population if consumers' needs are assumed to be the same across ages. The numerator - the effective labor force - depends on the workers' relative needs of consumption across ages, their income, the hours worked, the employment rate and the age retirement. The total dependency ratio is less-data demanding. It divides the dependent populations (0-14 years old and 65 years or over) by the working-age population (15-64 years old). The unit is no longer economic (effective number of consumers) but demographic (number of people) and the dependency criterion is no longer economic (labor occupation) but age. First, let us consider the total dependency ratio, which is decomposed as

$$TDR_t = CDR_t + ODR_t \tag{12}$$

where TDR_t is the total dependency ratio, $CDR_t = \frac{N_t}{N_{t-1}}$ the child-dependency ratio and $ODR_t = \frac{lN_{t-2}}{N_{t-1}}$ the old-age dependency ratio at time t. Population ageing can be defined as a decrease in the child-dependency ratio (due to a decline in the fertility rate) or/and an increase in the old-age dependency ratio (due to a decline in fertility or/and a decline in the old-age mortality rate). The total dependency ratio is increasing over time whenever the population growth rate is higher than the growth rate of the working-age population as

$$g_{1+TDR,t+1} = g_{Z,t+1} - g_{L,t+1} \tag{13}$$

where $g_{1+TDR,t+1}$ is the growth rate of the total dependency ratio, $g_{Z,t+1} = \ln Z_{t+1} - \ln Z_t$ is the growth rate of population, $g_{L,t+1} = \ln L_{t+1} - \ln L_t$ is the growth rate of the working-age population (=labor supply) between time t and $t+1$. We can also write that

$$g_{1+TDR,t+1} = -g_{\frac{L}{Z},t+1} \tag{14}$$

where $g_{\frac{L}{Z},t+1} = \ln(L_{t+1}/Z_{t+1}) - \ln(L_t/Z_t)$ is the growth rate of the share of the working-age population in total population (or, equivalently in this model, the support ratio). Equation (14) means that if the total dependency ratio increases by 1% then the share of working-age population in total population must decrease by the same rate. The dynamics of the share of the working-age population in total population between time t and $t+1$ can be written in terms of dependency ratios:

$$g_{\frac{L}{Z},t+1} = -g_{ODR/CDR,t+1} - g_{CDR,t+1} \tag{15}$$

where $g_{ODR/CDR,t+1} = \ln\left(1 + \frac{1+ODR_{t+1}}{CDR_{t+1}}\right) - \ln\left(1 + \frac{1+ODR_t}{CDR_t}\right)$ is the growth rate of the ratio ODR/CDR in the population[3] and $g_{CDR,t+1} = \ln CDR_{t+1} - \ln CDR_t$ is the growth rate of the child-dependency ratio between time t and $t+1$. Population ageing occurs whenever the fertility rate decreases or/and longevity increases. The former affects both $g_{ODR/CDR,t+1}$ and $g_{CDR,t+1}$ while the latter only influences $g_{ODR/CDR,t+1}$. When the fertility rate decreases between time t and $t+1$, $g_{ODR/CDR,t+1}$ is positive while $g_{CDR,t+1}$ is

3

$$\begin{aligned}
g_{\frac{L}{Z},t+1} &= \ln(L_{t+1}/Z_{t+1}) - \ln(L_t/Z_t) \\
&= \ln\left(\frac{1}{1+CDR_{t+1}+ODR_{t+1}}\right) - \ln\left(\frac{1}{1+CDR_t+ODR_t}\right) \\
&= -\ln\left[CDR_{t+1}\left(1+\frac{1+ODR_{t+1}}{CDR_{t+1}}\right)\right] + \ln\left[CDR_t\left(1+\frac{1+ODR_t}{CDR_t}\right)\right] \\
&= -g_{ODR/CDR,t+1} - g_{CDR,t+1}
\end{aligned}$$

negative. The net effect on $g_{\frac{L}{Z},t+1}$ is hence ambiguous. When the mortality rate goes down between t and $t+1$, ODR_{t+1} increases and $g_{ODR/CDR,t+1}$ is positive, implying a negative $g_{\frac{L}{Z},t+1}$. The "demographic dividend"[4] is the temporary situation when population ages but $g_{\frac{L}{Z},t+1}$ remains positive because the positive effect of the decrease in the child dependency ratio is temporarily higher than the negative effect of the increase in the ratio ODR/CDR. Once the baby-bust generation has reached the working age, the ratio ODR/CDR increases further, especially if the mortality rate also decreases. In developed countries, the share of the working population is now decreasing.

Instead, if we consider the support ratio of Cutler et al. (1990), its rate of variation is

$$g_{\frac{LF}{CON},t+1} = g_{LF,t+1} - g_{CON,t+1} \tag{16}$$

where $g_{\frac{LF}{CON},t+1}$, $g_{LF,t+1}$ and $g_{CON,t+1}$ are the growth rates of, respectively, the support ratio, the effective labor income and the effective number of consumers between t and $t+1$.

4 Economic Growth and Population Ageing

The dynamics of the OLG economy will be analyzed in terms of three stationary variables: the physical-human capital ratio k_t, the growth rate of average human capital $g_{h,t+1} = \ln h_{t+1} - \ln h_t$, and the growth rate of income per capita $g_{y,t+1} = \ln y_{t+1} - \ln y_t$ where $y_t = \frac{Y_t}{Z_t}$. Equilibrium requires a stationary wage and a capital return, i.e., a physical-human capital ratio that should be defined as:

$$k_t \equiv \frac{K_t}{L_t h_t}. \tag{17}$$

[4]For a description of the "demographic dividend" see Lee and Mason (2006).

It is calculated from equation (4) for instance. From Equation (1), we can calculate the growth rate of income per capita between t and $t+1$ is

$$g_{y,t+1} = \alpha g_{k,t+1} + g_{h,t+1} + g_{\frac{L}{Z},t+1} \tag{18}$$

where $g_{k,t+1} = \ln k_{t+1} - \ln k_t$ is the growth rate of the stock of physical capital per effective labor, $g_{\frac{L}{Z},t+1}$ is the growth rate of the share of the working-age population in total population (defined in the previous section) and $g_{h,t+1} = \ln h_{t+1} - \ln h_t$ is the growth rate of the average stock of human capital. The accumulation of both types of capital are endogenous. Aggregate physical capital builds on the aggregation of saving of the middle-aged individuals, $K_{t+1} = L_t s_t$, which yields the following accumulation rule of the physical capital stock per effective labor

$$\frac{k_{t+1}}{k_t} = \left(\frac{L_{t+1}}{L_t}\right)^{\theta} \left(\frac{\beta l(1-\alpha)^{1-\theta}}{\psi(1+\beta l)}\right) \frac{1-\tau_h}{\tau_h^{\theta}} k_t^{\alpha(1-\theta)-1} \tag{19}$$

where $\frac{k_{t+1}}{k_t}$ is the growth factor of physical capital per effective labor between t and $t+1$. In order to separate exogenous demographic and endogenous economic variables, we rewrite (19) as

$$\frac{k_{t+1}}{k_t} = \left(\frac{L_{t+1}}{L_t}\right)^{\theta} \frac{\tilde{k}_{t+1}}{\tilde{k}_t} \tag{20}$$

where $\frac{\tilde{k}_{t+1}}{\tilde{k}_t} = \left(\frac{\beta l(1-\alpha)^{1-\theta}}{\psi(1+\beta l)}\right) \frac{1-\tau_h}{\tau_h^{\theta}} k_t^{\alpha(1-\theta)-1}$ is the growth factor of physical capital net of the growth effect of the labor force. As for the average stock of human capital, it accumulates according to

$$\frac{h_{t+1}}{h_t} = \left(\frac{L_{t+1}}{L_t}\right)^{-1} \psi \left[\left(\frac{N_{t-1}}{N_t}\right) \tau_h (1-\alpha) k_t\right]^{\theta} \tag{21}$$

where $\frac{h_{t+1}}{h_t}$ is the growth factor of average human capital between t and $t+1$. Again, in order order to separate demographic and technological variables and given the fact that

$L_t = N_{t-1}$, we rewrite Equation (21) as

$$\frac{h_{t+1}}{h_t} = \left(\frac{L_{t+1}}{L_t}\right)^{-1-\theta} \frac{\tilde{h}_{t+1}}{\tilde{h}_t} \tag{22}$$

where $\frac{\tilde{h}_{t+1}}{\tilde{h}_t} \equiv \psi[\tau_h(1-\alpha)k_t]^\theta$ is the growth factor of average human capital net of the growth effect of the labor force. By using Equations (20) and (22), we can rewrite (18), the growth rate of income per capita between t and $t+1$, as

$$g_{y,t+1} = \alpha g_{\tilde{k},t+1} + g_{\tilde{h},t+1} + g_{D,t+1} \tag{23}$$

where $g_{D,t+1}$ is the total demographic effect on the growth rate of income per capita between time t and $t+1$:

$$g_{D,t+1} = -g_{ODR/CDR,t+1} - g_{CDR,t+1} - [1+\theta(1+\alpha)]g_{L,t+1} \tag{24}$$

The last term on the right-hand side of Equation (24) is the standard dilution effect of a variation in the size of the newborn generation on the accumulation of physical and human capital. The first two terms, which equal the rate of variation in the share of the working-age population in total population, are related to the effect of the age structure of the population on income growth per capita. In an economy with ageing population, $g_{D,t+1}$ is unambiguously negative and the more the population ages, the higher the negative demographic effect will be on income growth.

Equation (23) gives the decomposition of the growth rate of income per capita into growth effects of technology and growth effects of demographics. In contrast, Equation (18) does not take the effect of demographic variables out of the productivity growth process. The growth rate $g_{k,t+1}$ and $g_{h,t+1}$ include demographic effects due to the intertemporal allocation for physical capital accumulation and the intergenerational transfer for human capital formation. Therefore, if one wants to separate demographic and technological effects on per-capita income growth, one should use Equation (23) rather than Equation (18).

If we use the support ratio, we can rewrite (16) as

$$g_{\frac{LF}{CON},t+1} = g_{L,t+1} + g_{w,t+1} + g_{h,t+1} - g_{CON,t+1} \tag{25}$$

where $g_{L,t+1} + g_{w,t+1} + g_{h,t+1} = \ln(L_{t+1}w_{t+1}h_{t+1}) - \ln(L_t w_t h_t)$ is the growth rate of the effective labor income. By using the wage equation and Equation (22), we can rewrite (25) and separate demographic and technological variables in the same way as for the dependency ratio:

$$g_{\frac{LF}{CON},t+1} = \alpha g_{\tilde{k},t+1} + g_{\tilde{h},t+1} + g_{D',t+1} \tag{26}$$

where $g_{D',t+1} = [\alpha(1+\theta)]g_{L,t+1} - g_{CON,t+1}$.

5 Balanced Growth Path

At the steady state (along the balanced growth path), the productivity growth rate is constant and equal to the growth rate of human capital accumulation since the physical-human capital ratio becomes constant (i.e., $g_{\tilde{k},t+1} = 0$):

$$\bar{g}_{\tilde{h}} = \left[\Psi[\tau_h(1-\alpha)]^{\theta}\right]^{\frac{1-\alpha}{1-\alpha(1-\theta)}} \left(\frac{\beta l(1-\alpha)(1-\tau_h)}{1+\beta l}\right)^{\frac{\alpha\theta}{1-\alpha(1-\theta)}} \tag{27}$$

For a given sequence of values for the demographic variables, the growth rate of income per capita and the growth rate of the support ratio respectively become at the steady state

$$\bar{g}_y = \bar{g}_{\tilde{h}} + \bar{g}_D \quad (28)$$

$$\bar{g}_{\frac{LF}{CON}} = \bar{g}_{\tilde{h}} + \bar{g}_{D'} \quad (29)$$

where $\bar{g}_D = -g_{CDR/ODR} - g_{1+CDR} - (1+\theta)g_L$ and $\bar{g}_{D'} = -\theta g_L - g_{CON}$. Again, when the population is ageing, $\bar{g}_D$ and $\bar{g}_{D'}$ are unambiguously negative. The higher the negative demographic growth rate, the lower the income growth rate per capita and the growth rate of the support ratio. To counteract this negative effect of population ageing, a straightforward policy consists in increasing the number of self-supporting people and their productivity by inciting them to accumulate more human capital and work longer.

6 PAYG Pension

We assume that the government introduces a pay-as-you-go (PAYG) pension system. The first old generation thus benefits from a free lunch. The other generations pay a wage tax when middle-aged and receive a pension when old. The pension financing constraint at time t is

$$pw_t h_t l N_{t-2} = \tau_p w_t h_t N_{t-1}, \quad (30)$$

i.e., the cost of the pension system per unit of effective labor is

$$p = \left(\frac{N_{t-1}}{lN_{t-2}}\right)\tau_p, \quad 0 < p, \tau_p < 1, \quad (31)$$

where p is the pension rate and τ_p is the tax rate applied by the government to each middle-aged worker's wage. The pension is thus an intergenerational transfer from the middle-aged to the old individuals. We will consider two PAYG pension systems: a defined-benefit

(DB) pension system in which the pension rate is constant across all generations and the tax rate adjusts to the value of the old-age dependency ratio; a defined-contribution (DC) pension system in which the tax rate is constant across all generations and the pension rate adjusts to the value of the the old-age dependency ratio.

The objective of this section is to carry out a comparative analysis of the cost of both pension systems (DC and DB). We will pay particular attention to their effects through demographic variables. The introduction of a PAYG pension system in the model requires a modification in the budget constraints:

$$
\begin{aligned}
c_t + s_t &= (1 - \tau_h - \tau_p)\, w_t h_t && (32) \\
d_{t+1} &= \frac{\left(\frac{N_t}{lN_{t-1}}\right)\tau_p w_{t+1} h_{t+1} + (1 + r_{t+1}) s_t}{l} && (33)
\end{aligned}
$$

Equations (32) and (33) refer to the DC pension system. In the DB pension system, τ_p must be replaced by $\left(\frac{plN_{t-2}}{N_{t-1}}\right)$ in Equation (32) and $\left(\frac{\tau_p N_t}{lN_{t-1}}\right)$ by p in Equation (33). The optimal saving levels in both systems are

$$
\begin{aligned}
s_t^{DC} &= \frac{\beta l}{1+\beta l}(1 - \tau_h - \tau_p)(1-\alpha)k_t^{\alpha} h_t - \frac{(1-\alpha)k_{t+1}h_{t+1}}{\alpha(1+\beta l)}\left(\frac{\tau_p}{ODR_{t+1}}\right) && (34) \\
s_t^{DB} &= \frac{\beta l}{1+\beta l}\left(1 - \tau_h - pODR_t\right)(1-\alpha)k_t^{\alpha} h_t - \frac{p(1-\alpha)k_{t+1}h_{t+1}}{\alpha(1+\beta l)} && (35)
\end{aligned}
$$

where $ODR_t = \left(\frac{lN_{t-2}}{N_{t-1}}\right)$. The effect of an increase in the old-age dependency ratio is, *ceteris paribus*, positive on s_t^{DC} and negative on s_t^{DB}.

6.1 Economic Growth and Defined-Contribution Pension

The introduction of a PAYG pension system implies that the working-age population pays a tax to finance the pension of the old generation. This tax is assumed to apply to labor income. The life-cycle resources of the individuals are thus modified since their net-of-tax wage is reduced but they benefit from a pension when old. When the pension is of the defined contribution type, the pension rate p adjusts to the demographic structure of the population and the contribution rate τ_p remains fixed. When the working-age population decreases, the pension rate diminishes. The accumulation rules of physical and human capital in this setup are the following:

$$\frac{k_{t+1}}{k_t} = \left(\frac{L_{t+1}}{L_t}\right)^{\theta} \frac{\beta l(1-\tau_h-\tau_p)k_t^{\alpha(1-\theta)-1}}{\Psi\left[\tau_h(1-\alpha)\right]^{\theta}\left(\frac{1+\beta l}{1-\alpha}+\frac{\tau_p}{\alpha}\right)} \tag{36}$$

$$\frac{h_{t+1}}{h_t} = \left(\frac{L_{t+1}}{L_t}\right)^{-1-\theta} \Psi\left(\tau_h(1-\alpha)k_t^{\alpha}\right)^{\theta} \tag{37}$$

The demographic structure is identical to the one in the model without PAYG pension. Therefore, the growth rate of income per capita in an OLG economy with a DC pension system between t and $t+1$ is

$$g_{y,t+1}^{DC} = \alpha g_{\tilde{k},t+1}^{DC} + g_{\tilde{h},t+1}^{DC} - g_{D,t+1} \tag{38}$$

where the productivity growth rate equal to $\alpha g_{\tilde{k},t+1}^{DC} + g_{\tilde{h},t+1}^{DC}$ now is lower than in the model with PAYG pension due to the new labor income tax levied to finance the pension system. This tax reduces investment in physical and human capital. Along the balanced growth path, the growth rate of income per capita becomes

$$\bar{g}_y^{DC} = \bar{g}_{\tilde{h}}^{DC} - \bar{g}_D \tag{39}$$

where

$$\bar{g}_{\tilde{h}}^{DC} = \left[\Psi[\tau_h(1-\alpha)]^{\theta}\right]^{\frac{1-\alpha}{1-\alpha(1-\theta)}} \left(\frac{\beta l(1-\tau_h-\tau_p)}{\frac{1+\beta l}{1-\alpha}+\frac{\tau_p}{\alpha}}\right)^{\frac{\alpha\theta}{1-\alpha(1-\theta)}} \tag{40}$$

which is clearly lower than Equation (27). The difference between the two can be easily computed:

$$\bar{g}_{\tilde{h}} = (1+\mu)\,\bar{g}_{\tilde{h}}^{DC} \tag{41}$$

where $\mu = \frac{\tau_p\left(\frac{1-\tau_h}{\alpha}+\frac{1+\beta l}{1-\alpha}\right)}{\frac{1+\beta l}{1-\alpha}(1-\tau_h-\tau_p)} > 0$.

The same demographic structure is also obviously obtained for the growth rate of the support ratio:

$$g^{DC}_{\frac{LF}{CON},t+1} = \alpha g^{DC}_{\tilde{k},t+1} + g^{DC}_{\tilde{h},t+1} - g_{D',t+1} \tag{42}$$

Along the balanced growth path, the growth rate of the support ratio becomes

$$\bar{g}^{DC}_{\frac{LF}{CON}} = \bar{g}^{DC}_{\tilde{h}} - \bar{g}_{D'}. \tag{43}$$

The DC pension system crowds out investment in physical and human capital relative to an economy without pension system (or with a fully-funded pension system) but does not modify the growth effects of the demographic structure. We could say that the DC pension system is "demographically neutral" in the sense that it does not generate its own demographic effects on income growth. In other words, population ageing will have the same demographic effects on income growth in an economy without pension system (or with a fully-funded pension system) and an economy with a DC PAYG pensions system.

Nevertheless, the more the population ages, the stronger the negative demographic effect on income growth and the more controversial the cost of the PAYG pension system. If the policy objective is to maintain the pension system unchanged (i.e. unchanged contribution rate), there are two options to enhance income growth per capita and offset the negative growth effect of the PAYG pension system: a demographic mechanism or a technological mechanism. The demographic mechanism consists in inciting people to work longer. This would change the demographic structure by increasing the working-age population. The success of this policy depends on the effective labor demand for senior workers. The technological mechanism proceeds by increasing the tax rate financing education provided that the initial rate is lower than its optimal level equal to $1-\alpha$. The success of this mechanism rests on the efficiency (measured, in this model, by θ, the productivity elasticity of education spending) of the human capital formation.

6.2 Economic Growth and Defined-Benefit Pension

In this subsection, the PAYG pension system is of the defined benefit type. The pension rate p is fixed while the contribution rate τ_p adjusts to the demographic structure of the population. The accumulation rules of physical and human capital become:

$$\frac{k_{t+1}}{k_t} = \left(\frac{L_{t+1}}{L_t}\right)^{\theta} \frac{\beta l\left(1-\tau_h-pODR_t\right)k_t^{\alpha(1-\theta)-1}}{\Psi\left[\tau_h(1-\alpha)\right]^{\theta}\left(\frac{1+\beta l}{1-\alpha}+\frac{p}{\alpha}ODR_t\right)} \tag{44}$$

$$\frac{h_{t+1}}{h_t} = \left(\frac{L_{t+1}}{L_t}\right)^{-1-\theta} \Psi\left(\tau_h(1-\alpha)k_t^{\alpha}\right)^{\theta} \tag{45}$$

where $ODR_t = lN_{t-2}/N_{t-1}$. From Equation (44), we can observe that the demographic structure in this economy with a DB pension system is different from those of the economies with a DC pension or without pension. The cost of the pension system now depends on the age structure of the population. Now it is not possible to neatly separate the demographic from the technological variables in Equation (44) while Equation (45) remains the same as previously. Let us rewrite both equations as

$$\frac{k_{t+1}}{k_t} = \left(\frac{L_{t+1}}{L_t}\right)^{-(1-\theta)} \Omega_t \left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right) \tag{46}$$

$$\frac{h_{t+1}}{h_t} = \left(\frac{L_{t+1}}{L_t}\right)^{-\theta} \frac{\tilde{h}_{t+1}}{\tilde{h}_t} = \left(\frac{L_{t+1}}{L_t}\right)^{-\theta} \psi[\tau_h(1-\alpha)k_t]^{\theta} \tag{47}$$

where $\frac{\hat{k}_{t+1}}{\hat{k}_t} = \frac{k_t^{\alpha(1-\theta)-1}}{\Psi[\tau_h(1-\alpha)]^\theta}$ and $\Omega_t \equiv \left[\frac{\beta l(1-\tau_h - pODR_t)}{\frac{1+\beta l}{1-\alpha}+\frac{p}{\alpha}ODR_t}\right]$.

Equation (46) now includes three distinctive elements. The first on the right-hand side of the equality is the dilution effect. The second element is the propensity to invest in physical capital (Ω_t), which is a decreasing function of the old-age dependency ratio, and the third element is the part of capital accumulation that does not depend on demographics. The growth rate of income per capita in an OLG economy with a DB pension system between t and $t+1$ is

$$g^{DB}_{y,t+1} = \alpha[g^{DB}_{\hat{k},t+1} + \ln \Omega^{DB}_t] + g^{DB}_{\tilde{h},t+1} - g_{D,t+1} \tag{48}$$

where Ω^{DB}_t is the propensity to invest in physical capital and $g_{D,t+1}$ is the same as previously. The cost of the pension system is directly dependent on the old-age dependency ratio. This is the specific demographic effect of the DB pension system on income growth per capita. Every time this ratio increases, the propensity to invest in physical capital Ω_t decreases and private investment is crowded out. Therefore, The DB PAYG pension system is not "demographically neutral". As the population ages, the DB pension system gains on income growth per capita and, hence, raises doubts on its sustainability in ageing societies. Along the balanced growth path, the income growth rate per capita becomes

$$\bar{g}^{DB}_y = \left(\frac{\alpha\theta}{1-\alpha(1-\theta)}\right) \ln \Omega^{DB}_t + \bar{g}^{DB}_{\hat{h}} - \bar{g}_D \tag{49}$$

where $\bar{g}^{DB}_{\hat{h}} = [\Psi\left(\tau_h(1-\alpha)\right)^\theta]^{\frac{1-\alpha}{1-\alpha(1-\theta)}}$.

Again, we obtain a comparable expression for the growth rate of the support ratio:

$$g^{DB}_{\frac{LF}{CON},t+1} = \alpha[g^{DB}_{\hat{k},t+1} + \ln \Omega^{DB}_t] + g^{DB}_{\tilde{h},t+1} - g_{D',t+1} \tag{50}$$

which, along the balanced growth path, becomes

$$\bar{g}^{DB}_{\frac{LF}{CON}} = \left(\frac{\alpha\theta}{1-\alpha(1-\theta)}\right)\ln\Omega^{DB}_{t} + \bar{g}^{DB}_{\hat{h}} - \bar{g}_{D'} \tag{51}$$

In the long term, population ageing eats away the growth rate of income per capita through the demographic dynamics and the declining propensity to invest. If the policy objective is to maintain the DB pension system unchanged (i.e., a constant pension rate p), the two mechanisms described in the previous section (incentive for working longer and financing more education) remain valid with the DB pension system. However, the reforms must be more serious due to the negative effect of population ageing on the propensity to invest.

7 Discussion and Concluding Remarks

The objective of this paper is to provide a theoretical answer to the finding of the most sustainable PAYG pension system in an ageing society. We proposed an endogenous growth model with overlapping generations to focus on the growth effects of exogenous variations in the demographic structure. By separating demographic and technological variables, we are able to compare the growth performance of three institutional pension arrangements: the no-pension economy (or, equivalently, the fully-funded pension system), the defined-contribution and the defined-benefit pension systems. We show that the defined-benefit scheme is the only institutional pension arrangement having a specific growth effect when the demographic structure changes. PAYG pension systems are widespread in advanced societies. They were introduced when the share of the working population was increasing and the life expectancy of retirees was low. The defined-benefit scheme is the most desirable one as pensioners enjoy retirement income streams with certainty. As our model shows also, the defined-benefit scheme was the most growth-friendly institutional arrangement of the two PAYG pension systems. Due to its sensitivity to the demographic structure, it raises doubt about its sustainability as the population ages. In our model, the specific negative growth effect of the scheme increases with the old-age dependency ratio. Therefore, in advanced ageing societies, there is now a tradeoff between the certainty of income streams of future retirees, the sustainability of the PAYG pension systems and the macroeconomic implications of these intergenerational transfers. The conclusions of our model derive from a closed-economy framework. A natural extension would be to study the same question in an open economy, in which young emerging countries could provide saving to the old advanced societies.

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