26th ALERT Doctoral School 2015:

Coupled and multiphysics phenomena

Numerical modelling of a Municipal Waste Disposal as a Bio-Chemo-Thermo-Hydro-Mechanical problem

F. Collin[1], J. Hubert[1], X.F. Liu[2] and B. Cerfontaine[1]
[1]University of Liège, Belgium
[2]University of Newcastle, Australia

Introduction

World population is increasing, leading to new issues:

- Energy demands
- Land reclamation
- Municipal waste management

EU regulation

- EU regulation (Council Directive 1999/31/EC of 26 April 1999 on the landfill of waste)

- The European Union has laid down strict requirements for landfills to prevent and reduce as far as possible the negative effects on the environment, specifically on surface water, groundwater, soil, air and human health.

- The landfill is considered as the ultimate solution

EU regulation

The Directive defines the different categories of waste

- Municipal waste
- Hazardous waste
- Non-hazardous waste
- Inert waste

Landfills are divided into three classes:

- Landfill for hazardous waste
- Landfill for non-hazardous waste: municipal waste; non-hazardous waste of any other origin
- Landfill for inert waste

General requirements for landfills

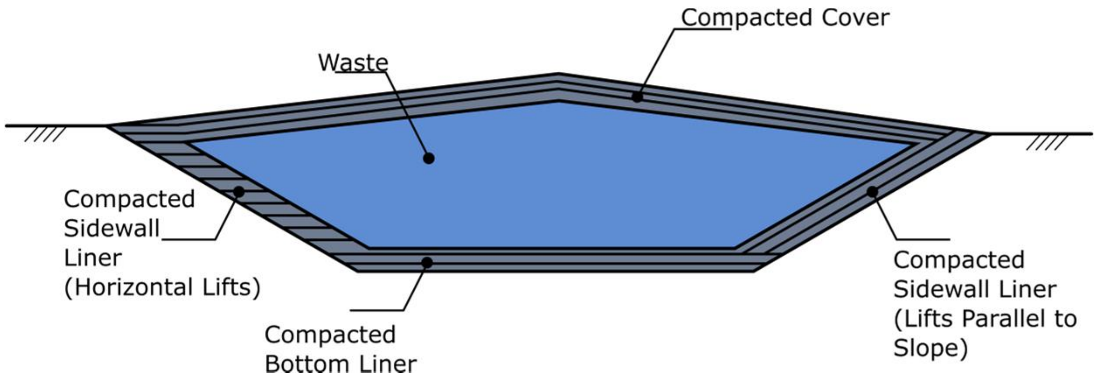
Appropriate measures shall be taken in order to:

- control water from precipitations entering into the landfill body,
- prevent surface water and/or groundwater from entering into the land-filled waste,
- collect contaminated water and leachate.
- treat contaminated water and leachate collected from the landfill to the appropriate standard required for their discharge.

General requirements for landfills

Protection of soil, groundwater and surface water is to be achieved by:

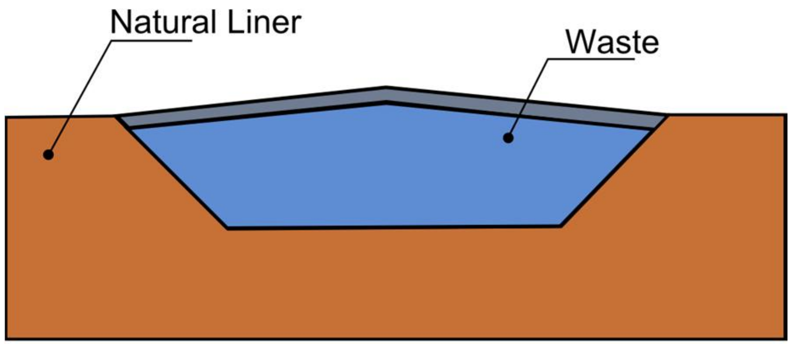
- during the operational/active phase, the combination of a geological barrier and a bottom liner
- during the passive phase/post closure, the combination of a geological barrier and a top liner.

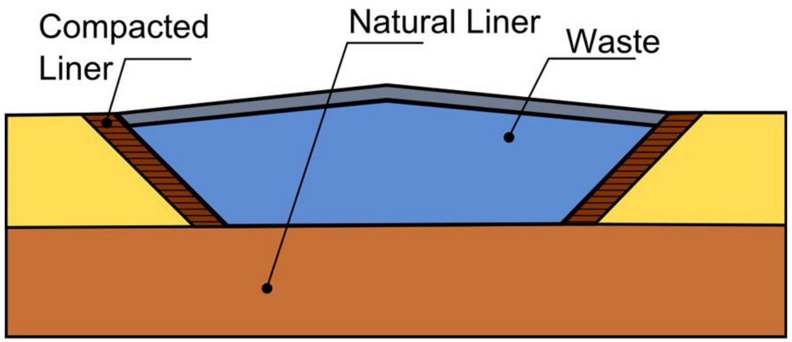


General requirements for landfills

The landfill base and sides shall consist of a mineral layer which satisfies permeability and thickness requirements :

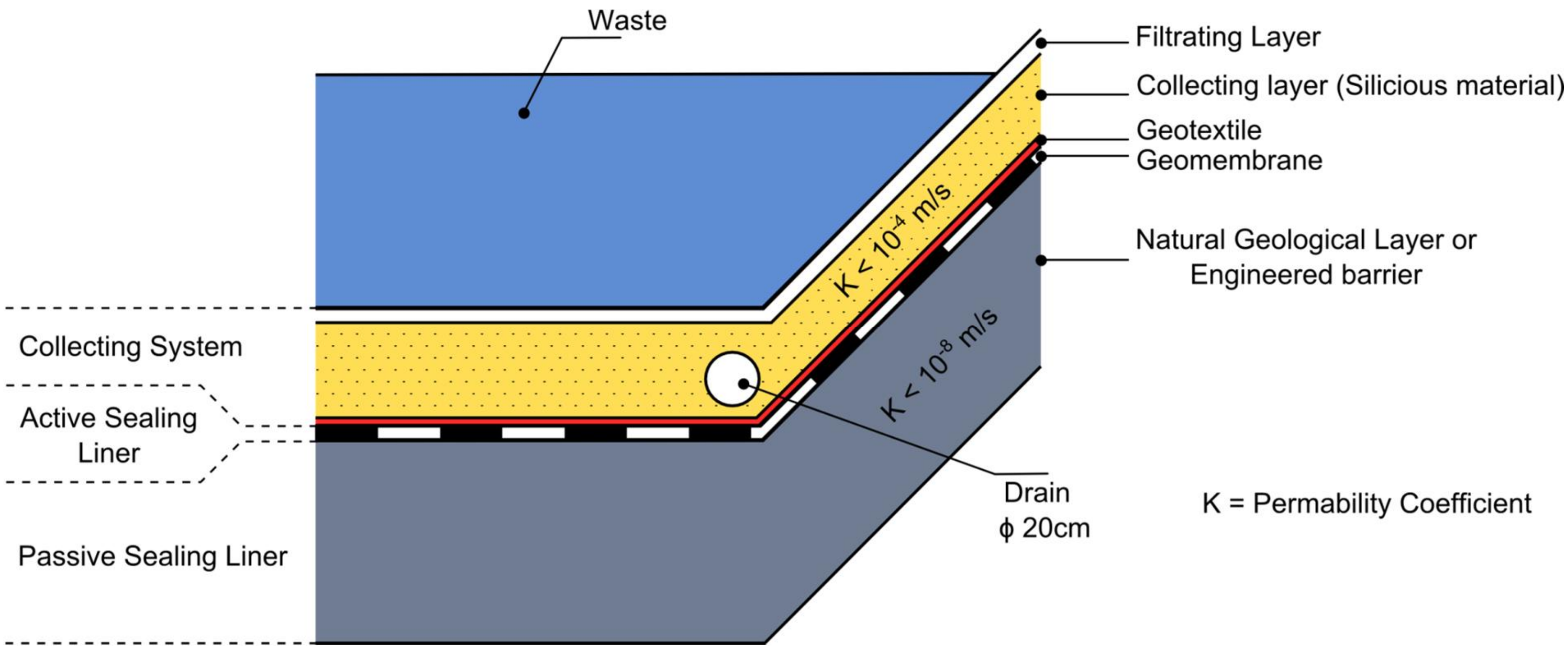
- landfill for hazardous waste: $K \leq 10^{-9}$ m/s; thickness ≥ 5m,
- landfill for non-hazardous waste: $K \leq 10^{-9}$ m/s; thickness ≥ 1m,
- landfill for inert waste: $K \leq 10^{-7}$ m/s; thickness ≥ 1m.





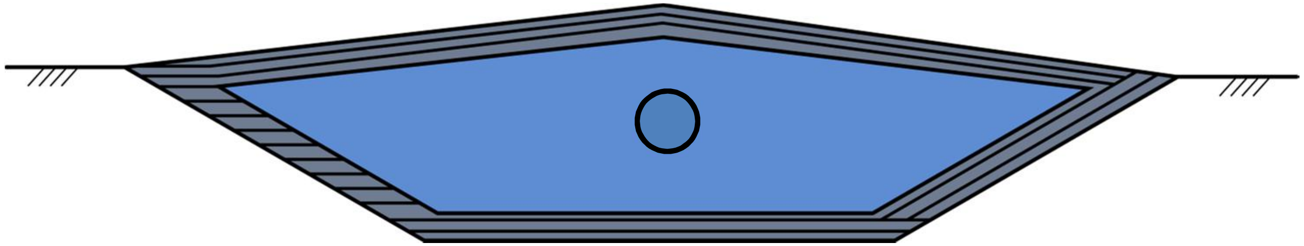
A landfill as a bioreactor

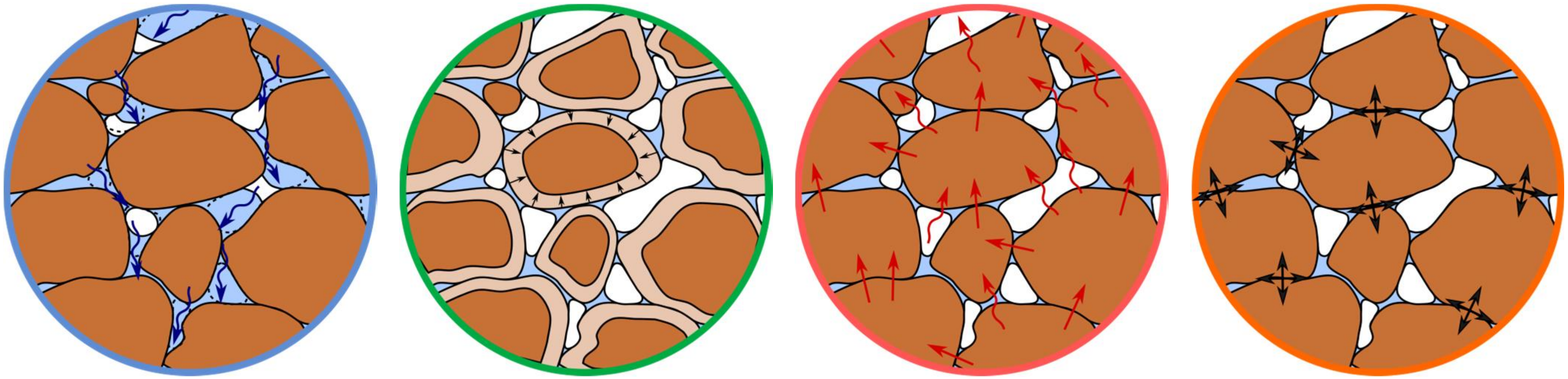
Design of the engineered barrier in a waste disposal



A landfill as a bioreactor

Multiphysical phenomena control the evolution of the landfill





Water and gas flows

Bio-chemo reaction

Heat transfer

Mechanics

Objectives and outcomes

Objectives

- Describe each individual physical phenomenon
- Evidence the coupling effects

Outcomes

- Provide a mathematical formulation for the BCTHM problem
- Develop a numerical model for the long term behaviour of the municipal waste disposal

Table of contents

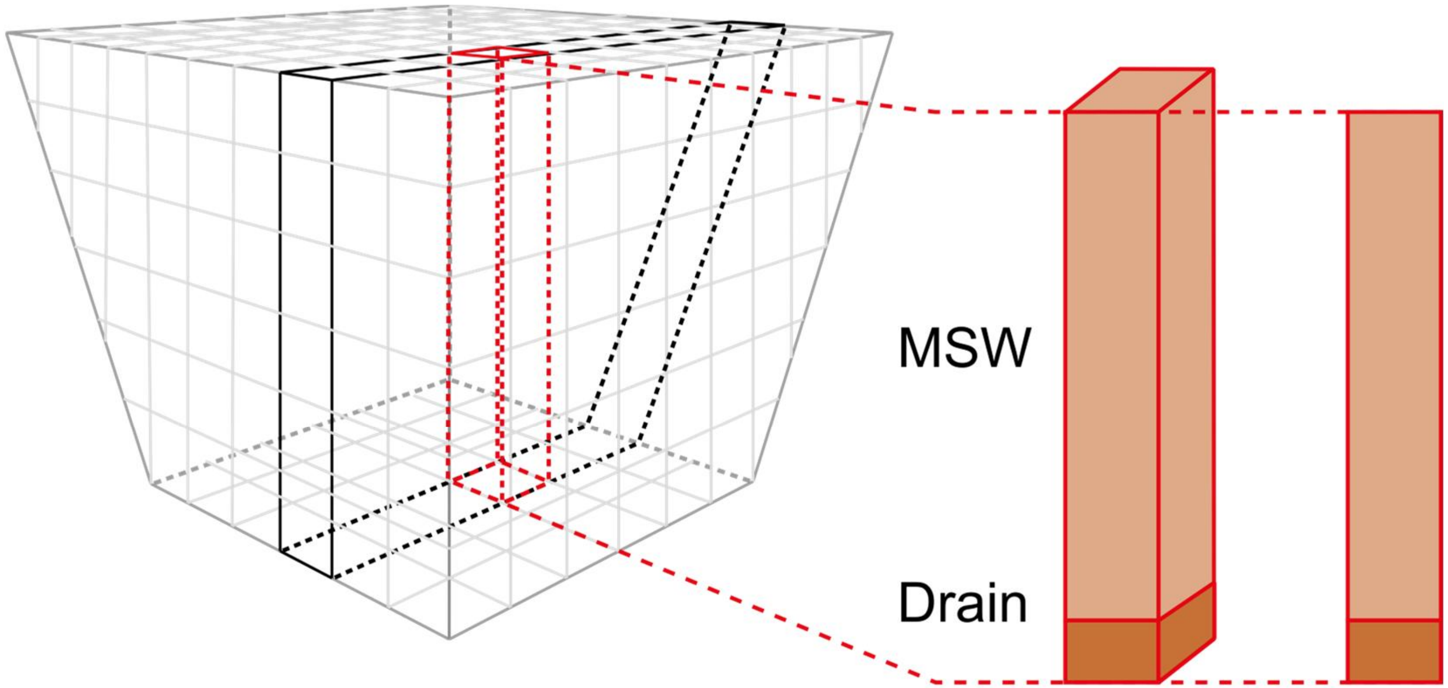
1. Municipal Waste Disposal context
2. Hydraulic model
3. Bio-Chemo-Hydraulic model
4. Bio-Chemo-Thermo-Hydraulic model
5. Bio-Chemo-Thermo-Hydraulic Mechanical model

Statement of the simplified problem

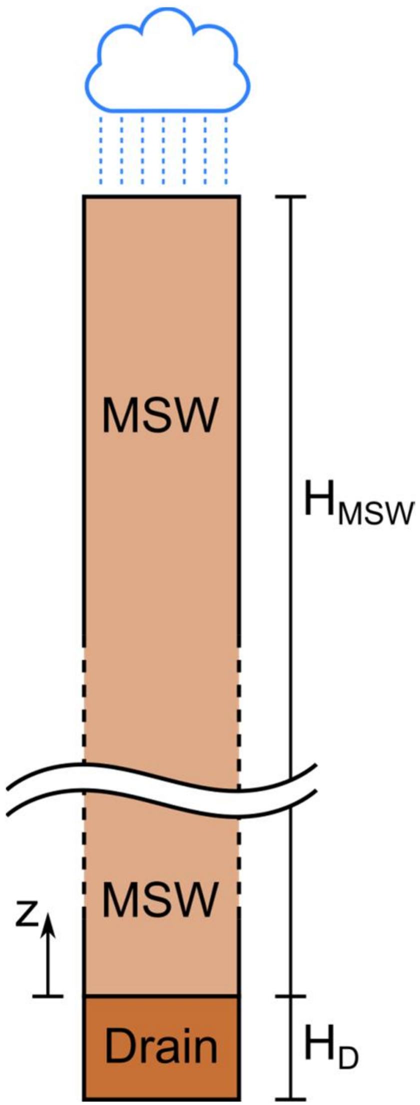
- By essence the wastes are heterogeneous and multiphasic
- Each physical phenomenon will be studied through analytical solution
- Numerical modelling will evidence the coupling effects
- A step by step procedure will be followed

Statement of the simplified problem

- Definition of the geometry (from 3D to 1D problem)



Statement of the simplified problem



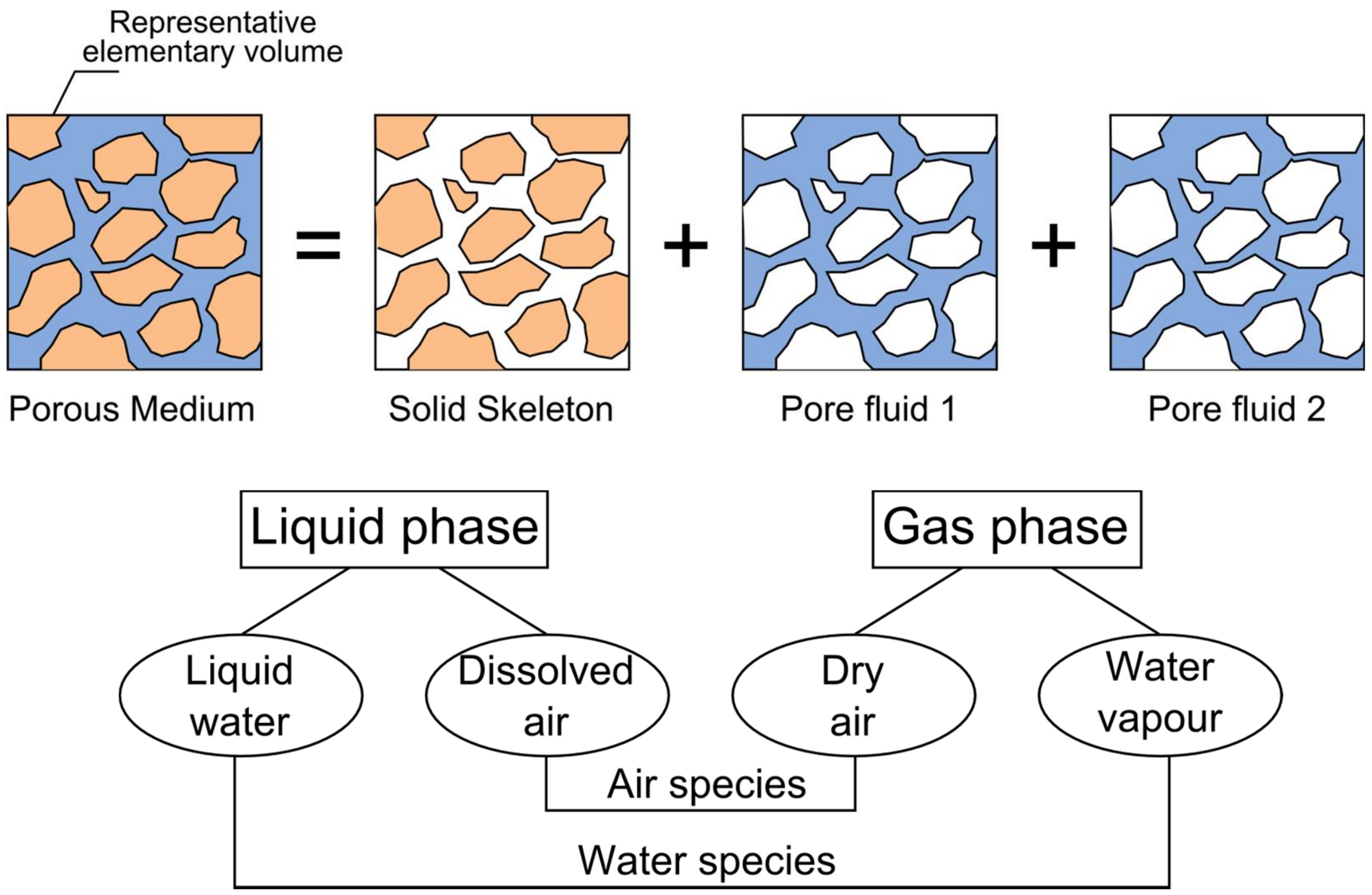
- Incoming water due to precipitations
- Initial stresses within the column
- Geometry limited to the waste and the drain
- Slope stability problem not addressed

Parameters					
H_{MSW}	= 30	[m]	H_D	= 1	[m]
ρ	= 1000	[kg/m³]			

Table of contents

Waste as an unsaturated porous medium

Unsaturated porous medium



Mass balance equation

Hypotheses

- Liquid water and water vapour
- Constant gas pressure

$$\frac{\partial}{\partial t}(\rho_w \, n \, S_{rw}) + \mathrm{div}\left(\underline{f_w}\right) + \frac{\partial}{\partial t}(\rho_v \, n \, S_{rg}) + \mathrm{div}\left(\underline{f_v}\right) - Q_w = 0$$

- Liquid water, S_{rw} water saturation degree
- Water vapour, $S_{rg} = 1 - S_{rw}$ gas saturation degree
- Source term

Liquid water mass flow

$$\underline{f_w} = \rho_w \, \underline{q_l}$$

Advection flow of the liquid phase

$$\underline{q_l} = -\frac{\underline{\underline{K}}_{int}^{sat} \, k_{rw}(S_{rw})}{\mu_w} \left[\underline{\mathrm{grad}}(p_w) + g \, \rho_w \, \underline{\mathrm{grad}}\,(z) \right]$$

where

- $\underline{\underline{K}}_{int}^{sat}$ [m²] is the intrinsic permeability
- $k_{rw}(S_{rw})$ [-] is the water relative permeability
- μ_w [Pa.s] is the water dynamic viscosity
- p_w [Pa] is the pore water pressure
- ρ_w [kg/m³] is the liquid water density

Water vapour mass flow

$$\underline{f_v} = \rho_v \; \underline{q_g} + \underline{i_v}$$

Advection flow of the gas phase

$$\underline{q_g} = -\frac{\underline{\underline{K}}_{int}^{sat} \; k_{rg}(S_{rw})}{\mu_g} \left[\underline{\text{grad}}\left(p_g\right) + g \; \rho_g \; \underline{\text{grad}} \; (z) \right]$$

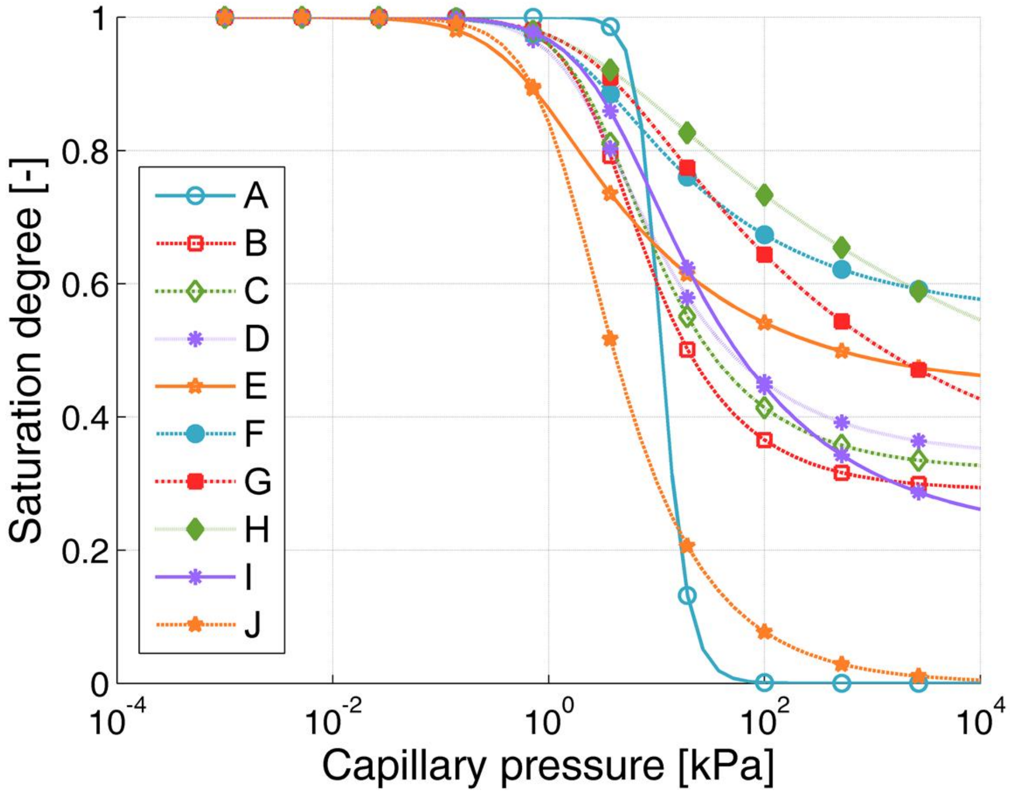
Diffusion within the gaseous phase

$$\underline{i_v} = -n \; S_{rg} \; \tau \; D_{v/a} \rho_g \; \underline{\text{grad}} \; \left(\rho_v / \rho_g\right)$$

- $D_{v/a}$ [m²/s] is the diffusion coefficient of water vapour in dry air
- τ [-] is the tortuosity

Retention properties

Diversity of the retention curves:



Analytical solution

In order to explain the pore pressure evolution, an analytical approach is proposed for a simplified problem.

Hypotheses

- Constant gas pressure p_g
- Liquid properties = liquid water properties
- Water is incompressible
- Stationary problem
- No water vapour

The water mass balance equation becomes:

$$\frac{\partial}{\partial t}(\rho_w\, n\, S_{rw}) + \mathrm{div}\left(\underline{f_w}\right) + \frac{\partial}{\partial t}\left(\rho_v\, n\, S_{rg}\right) + \mathrm{div}\left(\underline{f_v}\right) - Q_w = 0$$

Analytical solution

In order to explain the pore pressure evolution, an analytical approach is proposed for a simplified problem.

Hypotheses

- Constant gas pressure p_g
- Liquid properties = liquid water properties
- Water is incompressible
- Stationary problem
- No water vapour

The water mass balance equation becomes:

$$\mathrm{div}\left(\underline{f_w}\right) = 0$$

Analytical solution

Mass balance equation for 1D problem:

$$\frac{\partial}{\partial z}(q_{lz}\,\rho_w) = 0 \Rightarrow q_{lz}\,\rho_w = q_{in} \qquad (1)$$

Liquid advection flow

$$q_{lz} = -\frac{k_{int}^{sat}\,k_{rw}}{\mu_w}\,(\frac{\partial p_w}{\partial z} + g\,\rho_w) \qquad (2)$$

Analytical solution

Retention curve

$$S_{rw} = \exp\left(-\frac{p_c}{4A}\right) \leq 1 \qquad (3)$$

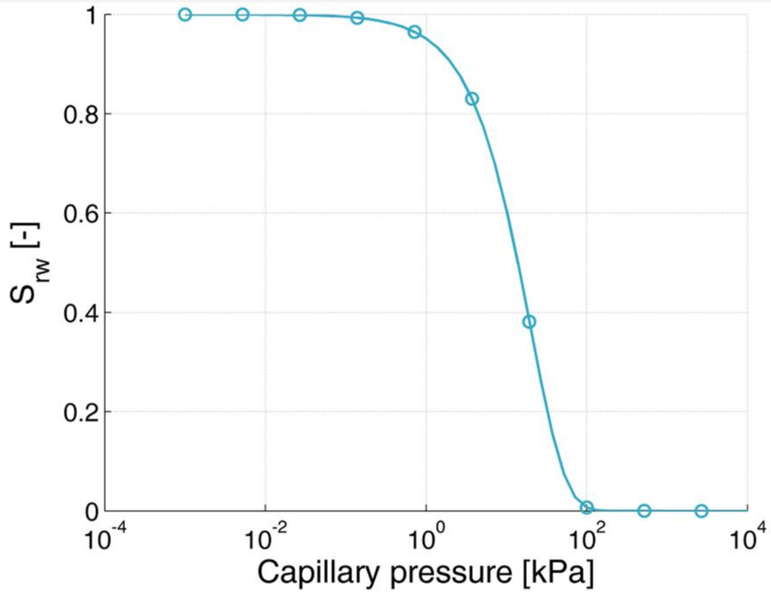
- $p_c = p_g - p_w$ [Pa] capillary pressure
- A [Pa^{-1}] material parameter

Relative permeability curve

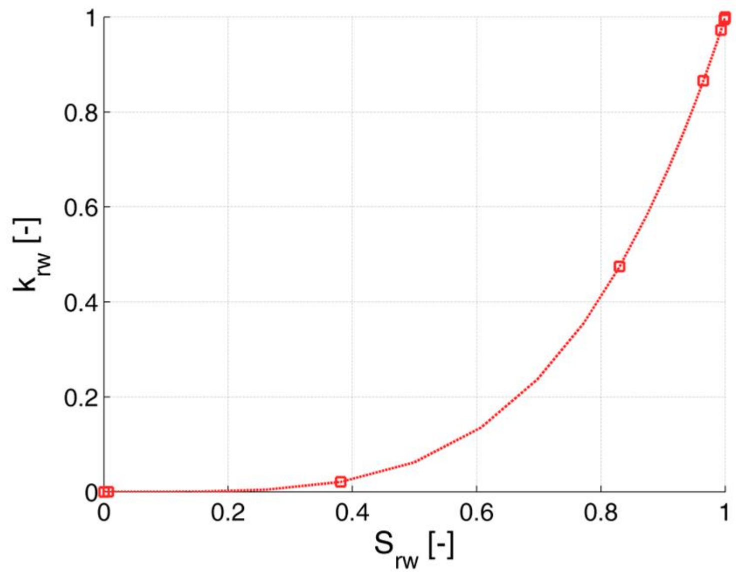
$$k_{rw} = (S_{rw})^4 \qquad (4)$$

Analytical solution

Retention curve



Relative permeability curve



Analytical solution

Introducing Eqs. (2)(3)(4) into (1)

$$\frac{\partial p_w^r}{\partial z} + \frac{q_{in}\,\mu_w}{\rho_w\,k_{int}^{sat}} \exp\left(-\frac{p_w^r}{A}\right) = -\rho_w\,g$$

and $p_w^r = -p_c$ is the relative water pressure (unknown).

Analytical solution

First change of variable $j = \exp\left(-\frac{p_w^r}{A}\right)$

$$\frac{\partial j}{\partial z} = \beta j^2 + \gamma j$$

where

- $\beta = \frac{q_{in}\mu_w}{\rho_w k_{int}^{sat} A}$
- $\gamma = \frac{\rho_w g}{A}$

⇒ Classical Verhulst equation

Analytical solution

Second change of variable $u = 1/j$

$$\frac{\partial u}{\partial z} = -\beta - \gamma u$$

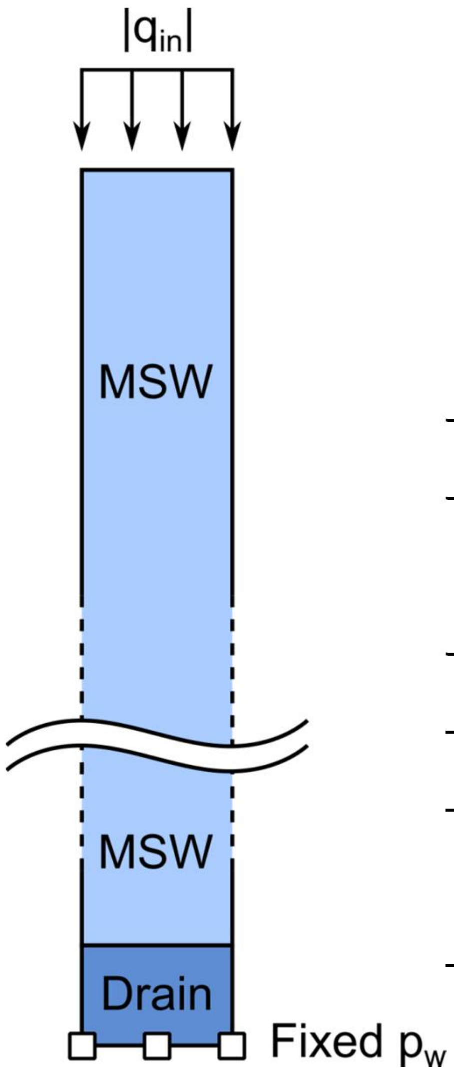
$$\Rightarrow u = \mathrm{C}_1 \exp(-\gamma z) - \frac{\beta}{\gamma}$$

$$\Rightarrow p_w^r(z) = A \ln\left[C_1 \exp(-\gamma z) - \frac{\beta}{\gamma}\right]$$

Relative water pressure p_{w0}^r imposed in $z = 0$

$$C_1 = \frac{\beta}{\gamma} + \exp\left(\frac{p_{w0}^r}{A}\right)$$

Statement of the simplified problem



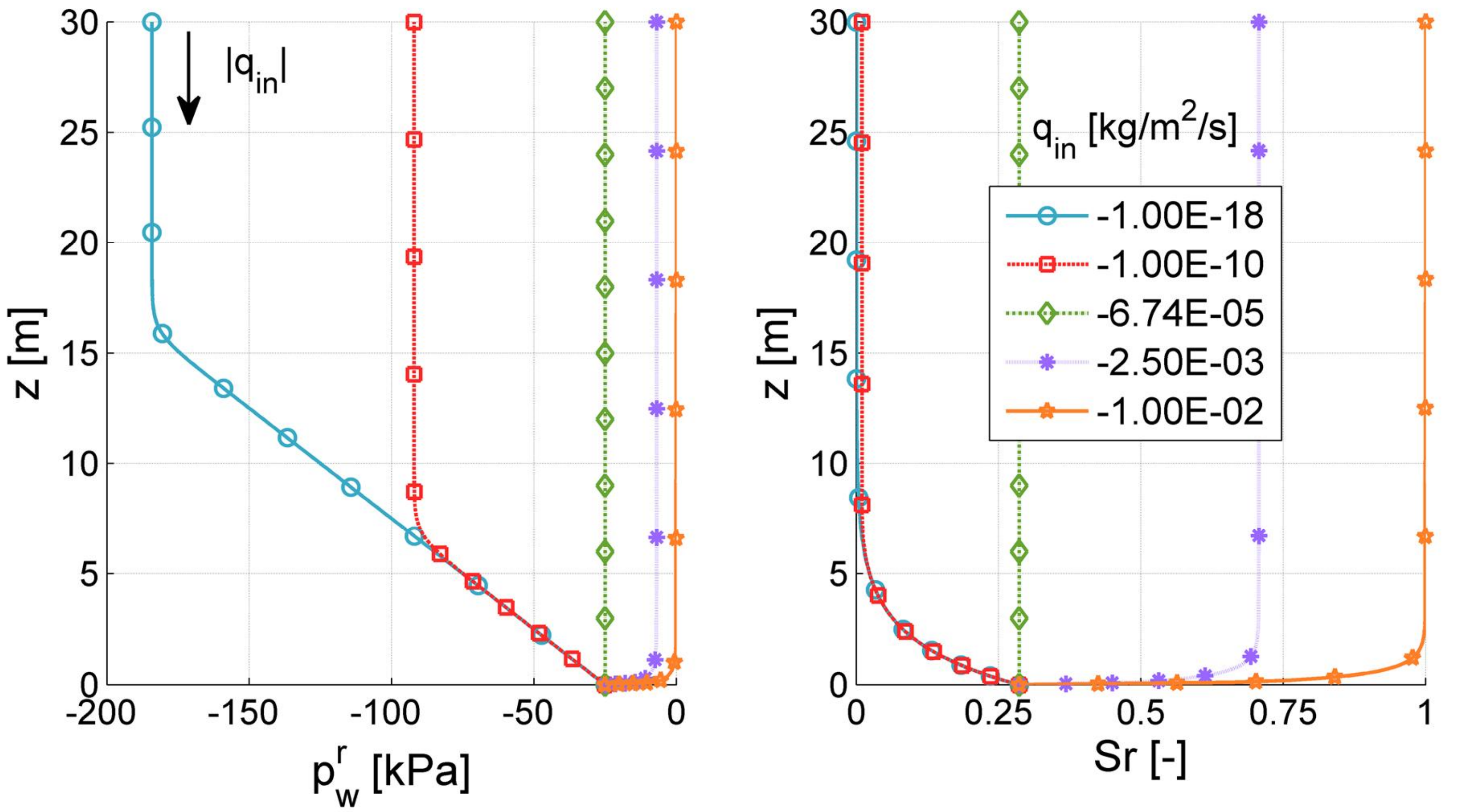
- Imposed water flux at the top
- Imposed pressure under the drain (85 kPa)

Parameters					
n	= 0.5	[-]	k_{sat}	= 10^{-12}	[m²]
A	= 5	[kPa]	τ	= 0.1	[-]

Initial values					
S_{rw}	= 0.6	[-]	p_w	= 90,1	[kPa]

Analytical solution

Variations of incoming flux $|q_{in}|$



Numerical solution

- The mass balance equation is solved numerically

$$\frac{\partial}{\partial t}(\rho_w\, n\, S_{rw}) + \mathrm{div}\left(\underline{f_w}\right) + \frac{\partial}{\partial t}(\rho_v\, n\, S_{rg}) + \mathrm{div}\left(\underline{f_v}\right) - Q_w = 0$$

- The Finite element code Lagamine is used:

✓ 300 elements (8-noded isoparametric elements)
✓ Modeling time: 15 years
✓ Constant injection flow

Numerical solution

Transient solutions $|q_{in}| = 2.5 \cdot 10^{-3} kg/m^2/s$

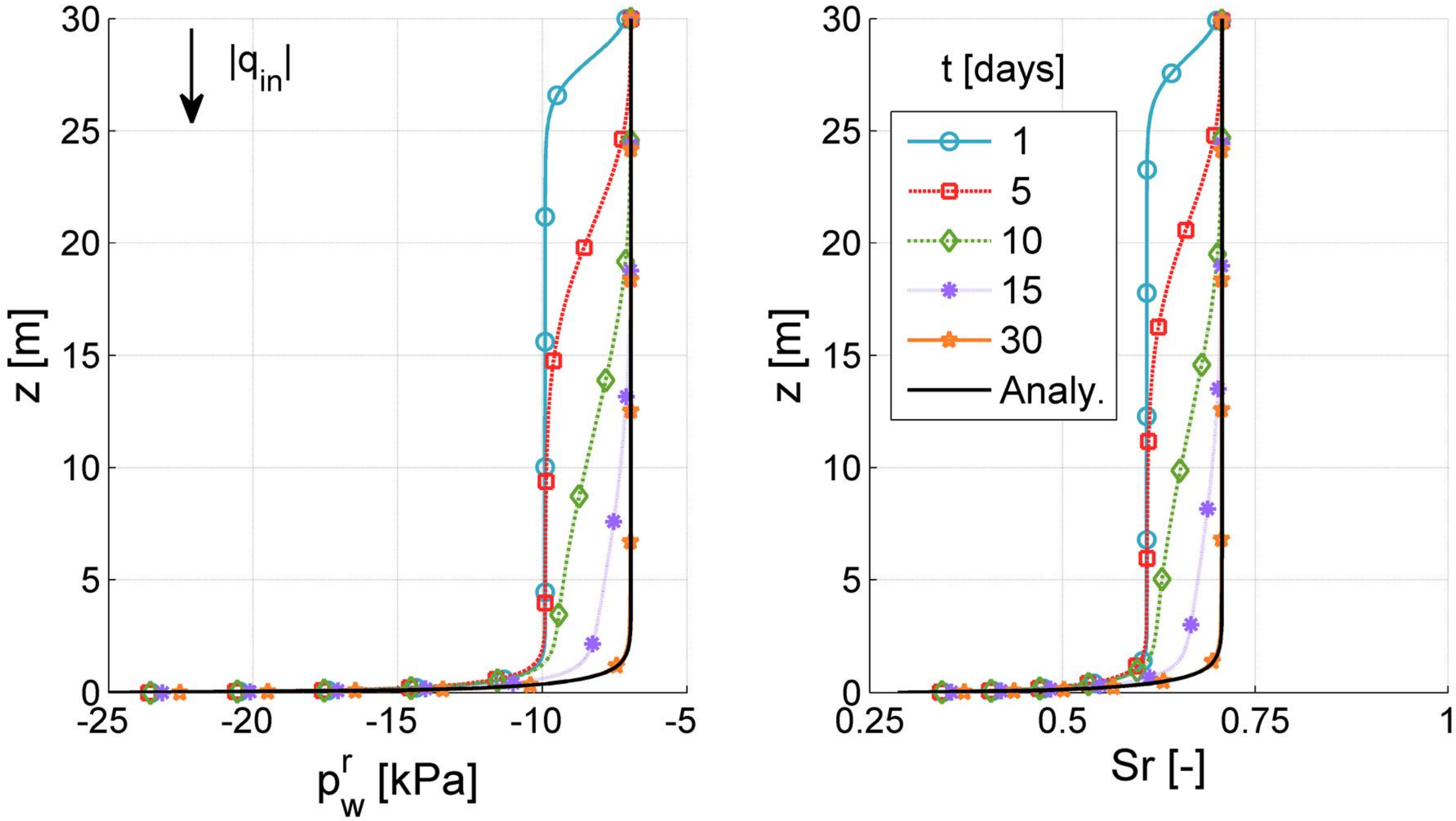
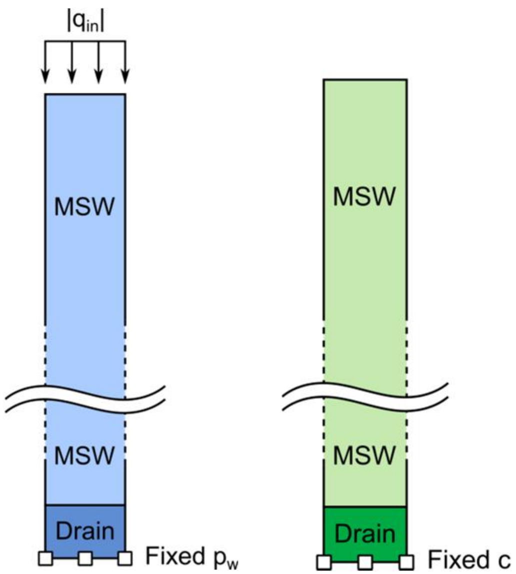


Table of contents



Evolution of the landfill

Landfills are bio-reactors where organic matter is degraded by microorganisms

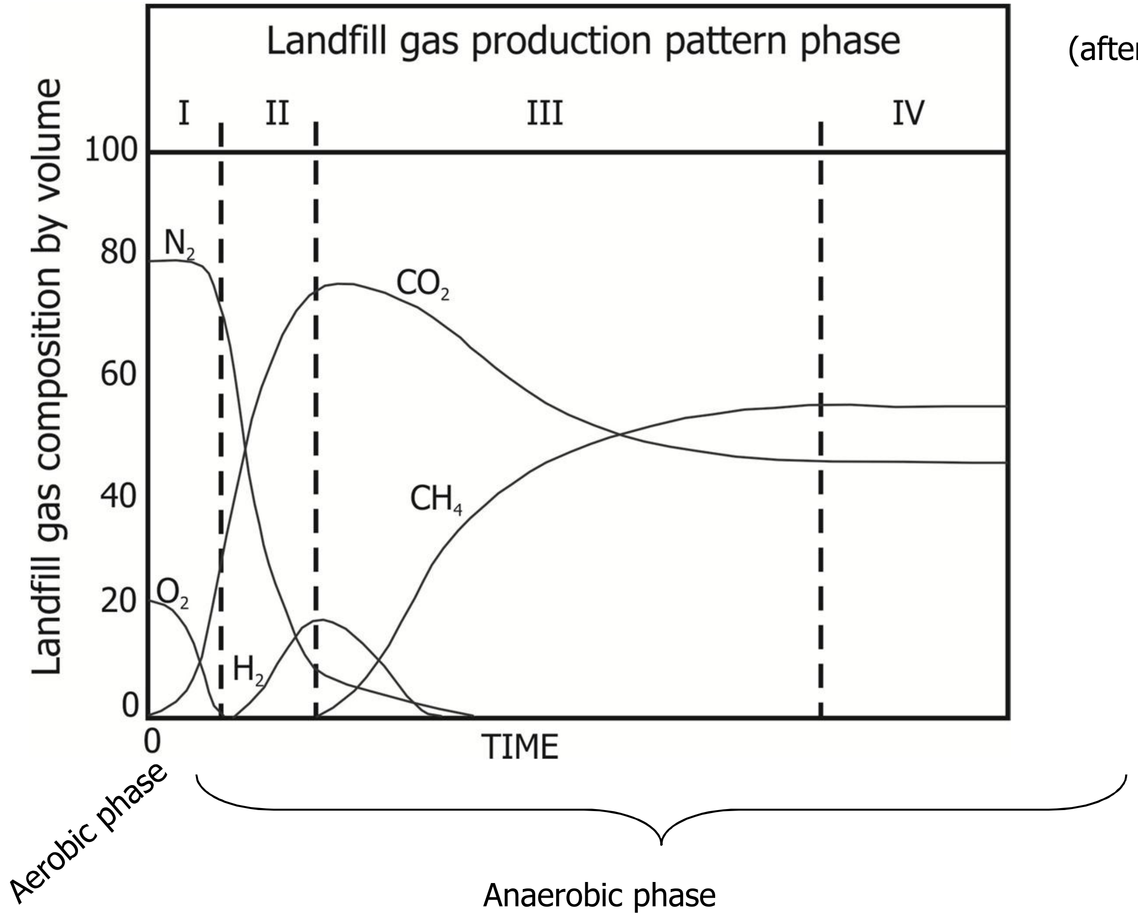
- Temperature: Psychrophile Bacteria (T°<20°C)
  Mesophile Bacteria (20°C<T°<44°C)
  Thermophile Bacteria (T°>44°C)

- pH : almost neutral, when acidity increase -> biological activity decrease

- Water content: Mandatory for bacteria activity
  Minimal water content : 25 - 35%

Evolution of the landfill

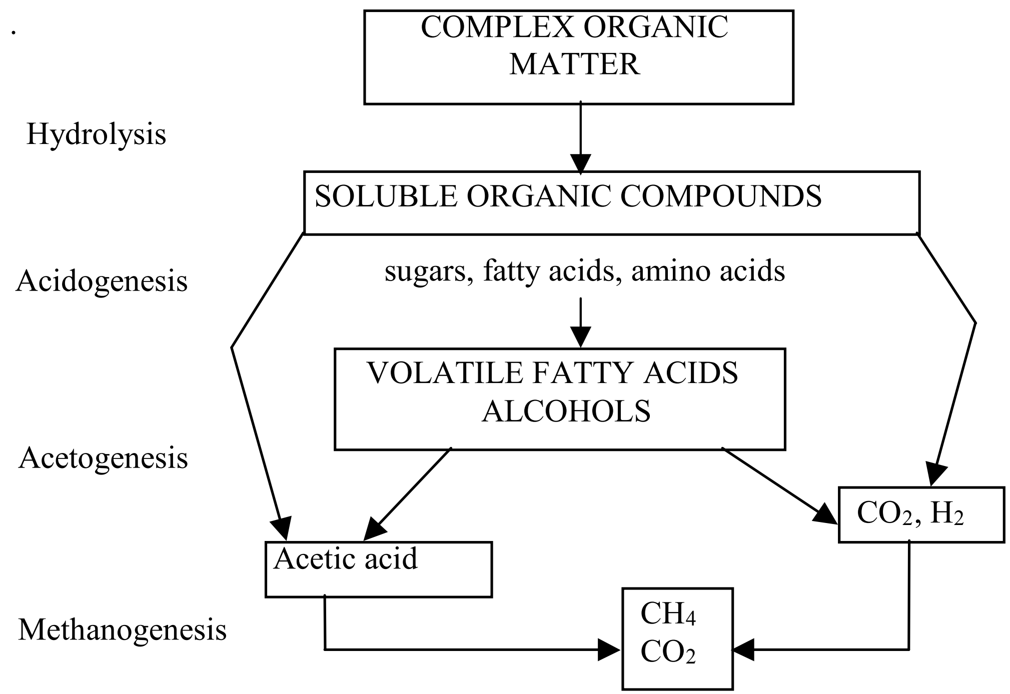
Landfills are bio-reactors where organic matter is degraded by microorganisms



(after Farquhar and Rovers, 1973)

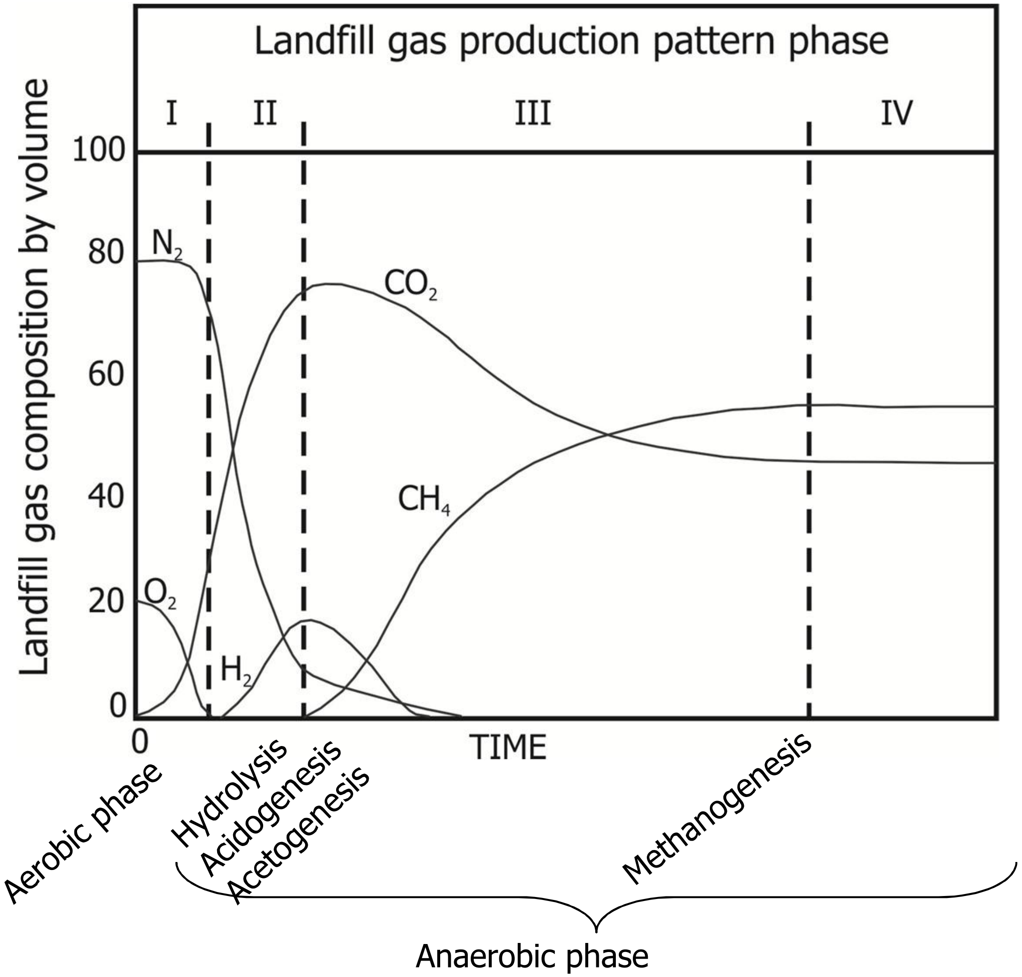
Evolution of the landfill

Landfills are bio-reactors where organic matter is degraded by microorganisms



Evolution of the landfill

Landfills are bio-reactors where organic matter is degraded by microorganisms

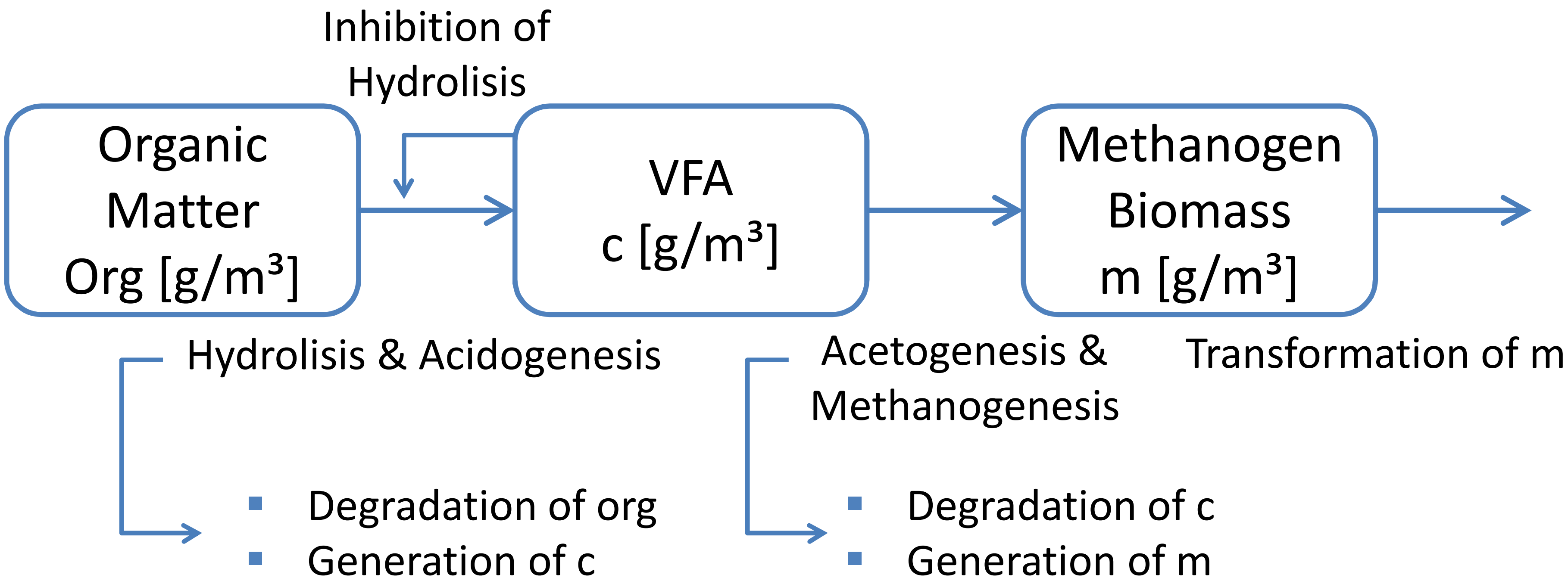


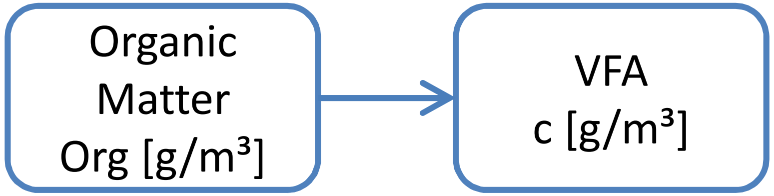
(after Farquhar and Rovers, 1973)

Two-stage model

Two-stage bio-chemical model (McDougal, 2007)

- Aerobic stage neglected
- Three internal variables
- All the reaction rates are defined per water volume



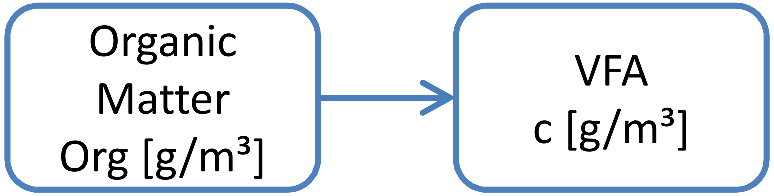


Hydrolisis & Acidogenesis

- Degradation of org
- **Generation rate of c** : r_g [kg/m^3_w/s]

$$r_g = b\,\frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}}\,(1 - \Omega^{\xi})\,\exp(-k_{VFA}\,c)$$

- b [kg/m^3_w/s], maximum VFA growth rate
- θ [-], water content (in volume)
  - θ_{res} [-] residual water content
  - θ_{sat} [-] saturated water content



Hydrolisis & Acidogenesis

- Degradation of org
- **Generation rate of c** : r_g [$kg/m^3_w/s$]

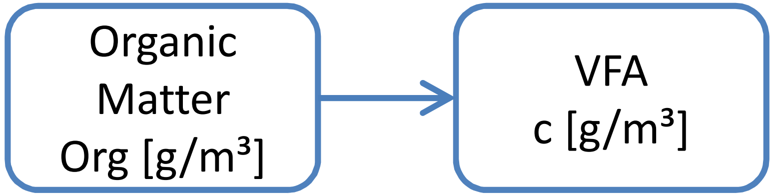
$$r_g = b \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}} \left(1 - \Omega^{\xi}\right) \exp(-k_{VFA}\, c)$$

- Ω [-], organic matter ratio

$$\Omega = 1 - \frac{org}{org_0} = \begin{cases} 0 \ (t = 0) \\ 1 \ (t = \infty) \end{cases}$$

- org [g/m³], organic content
- ξ [-], parameter

Bio-Chemical model

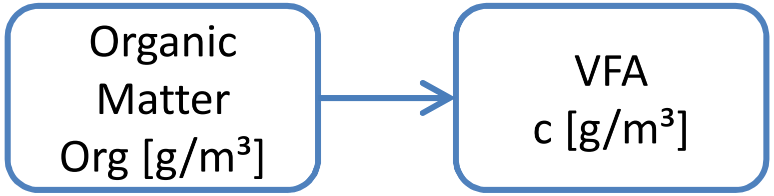


Hydrolisis & Acidogenesis

- Degradation of org
- **Generation rate of c** : r_g [kg/$m^3{}_w$/s]

$$r_g = b\,\frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}}\left(1 - \Omega^{\xi}\right)\exp(-k_{VFA}\, c)$$

- k_{VFA} [-], parameter
- c [g/m³], the VFA concentration in water

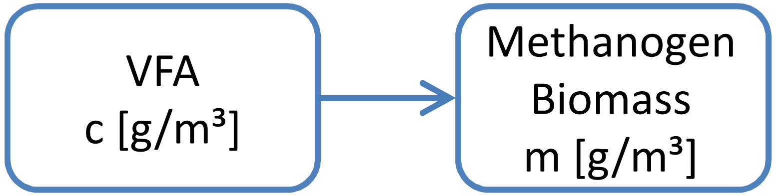


Hydrolisis & Acidogenesis

- **Degradation of org**
- Generation rate of c : r_g [kg/m^3_w/s]

$$Z\, r_g$$

- Z [-], substrate yield coefficient

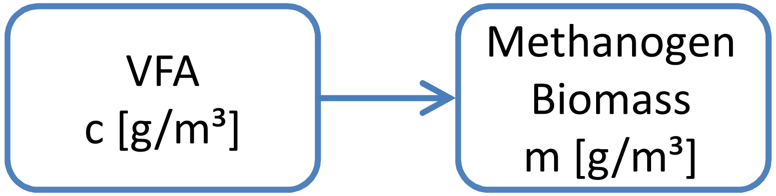


Acetogenesis & Methanogenesis

- **Generation rate of m** r_j [kg/m³/s]
- Degradation of c

$$r_j = \frac{k_0}{k_{MC}\,\theta^2/c + \theta}\, m$$

- θ [-], water content
- m [g/m³], methanogen biomass in water
- k_0 [s^{-1}], the specific growth rate
- k_{MC} [g/m³], the half saturation constant

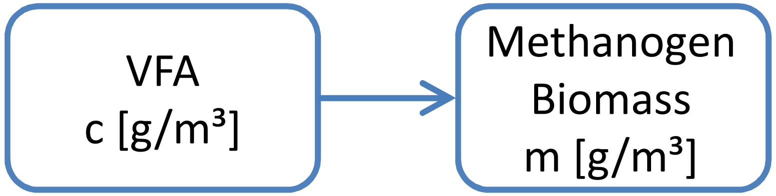


Acetogenesis & Methanogenesis

- **Generation rate of m** r_j [kg/m³/s]
- Degradation of c

$$r_j = \frac{k_0}{k_{MC}\,\theta^2/c + \theta}\, m$$

- c [g/m³], the VFA concentration in water



Acetogenesis & Methanogenesis

- Generation of m
- **Degradation rate of c** r_h [kg/m³/s]

$$r_h = \frac{r_j}{Y}$$

- Y [-], substrate yield coefficient

Methanogen
Biomass
m [g/m³]

Transformation of m

- **Degradation rate of m** r_k [kg/m³/s]

$$r_k = k_2 \frac{m}{\theta}$$

- k_2 [s^{-1}], methanogen death coefficient

Balance equations

VFA concentration

$$\mathrm{div}(\underline{u}\, c) - div\left(\underline{\underline{D_h}} \cdot \underline{\nabla} c\right) + (r_g - r_h)\,\theta = \frac{\partial c}{\partial t}$$

- Advective flux, where

$$\underline{u} = \underline{q_l}/(S_{rw}\, n_e)$$

  - $\underline{q_l}$ [m/s], water Darcy velocity
  - S_{rw} [-] water saturation degree
  - n_e [-], effective porosity
- Diffusion flux (mechanical dispersion and molecular diffusion)
  - D_h [m²/s], apparent diffusion coefficient
- Reaction term

Balance equations

Methanogen concentration

$$\mathrm{div}(\underline{u}\, m) - div\left(\underline{\underline{D_h}} \cdot \underline{\nabla} m\right) + \left(r_j - r_k\right)\theta = \frac{\partial m}{\partial t}$$

- Advective flux, where

$$\underline{u} = \underline{q_l}/(S_{rw}\, n_e)$$

  - $\underline{q_l}$ [m/s], water Darcy velocity
  - S_{rw} [-] water saturation degree
  - n_e [-], effective porosity
- Diffusion flux (mechanical dispersion and molecular diffusion)
  - D_h [m²/s], apparent diffusion coefficient
- Reaction term

Balance equations

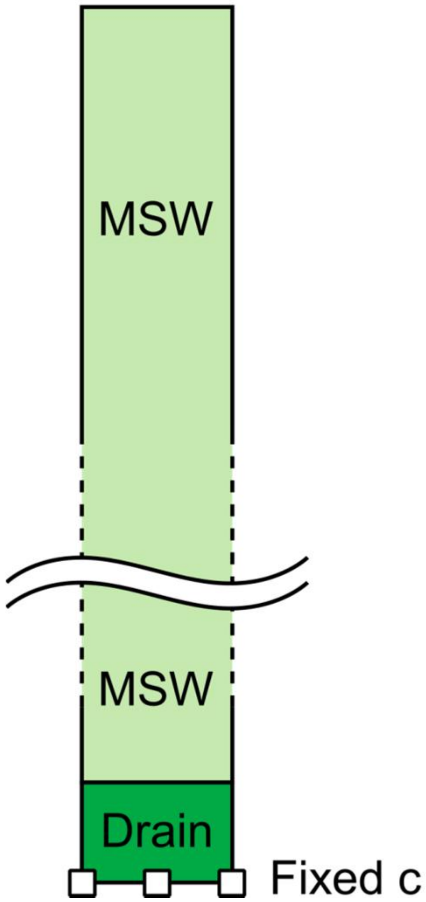
Organic content

$$-\mathrm{Z}\,\mathrm{r_g}\,\theta = \frac{\partial org}{\partial t}$$

- Z [-] substrate yield coefficient
- r_g [kg/m³/s] generation rate of c
- θ [-] water content

Statement of the simplified problem

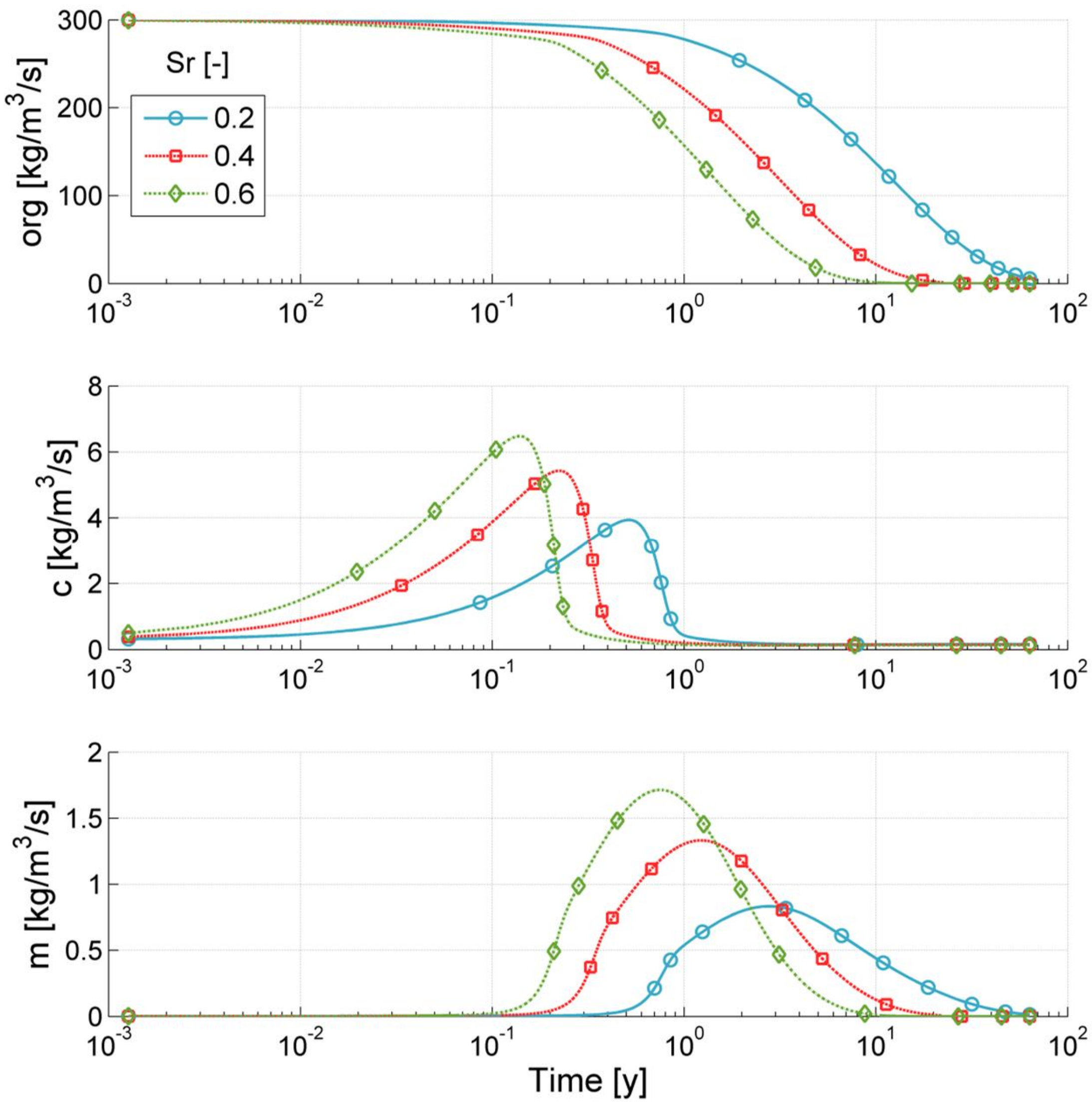
- Imposed VFA concentration at the bottom of the drain (c = 0)



Parameters					
b	= 2.9 10^{-5}	[kg/m³/s]	ξ	= 0.36	[-]
k_0	= 5.7 10^{-6}	[s^{-1}]	k_{VFA}	= 0.2	[m³/kg]
k_{MC}	= 4.2	[kg/m³]	Y	= 0.08	[-]
k_2	= 2.3 10^{-7}	[s^{-1}]	Z	= 2.7	[-]

Initial values					
org	= 300	[kg/m³]	c	= 0.3	[kg/m³]
m	= 0.0025	[kg/m³]			

Internal variable couplings



- Single point bioreactor
- Constant saturation
- No advection
- No diffusion

Analytical solution

Degradation of organic matter

$$\frac{\partial org}{\partial t} + \theta\, \theta_e\, Z\, b \exp(-k_{VFA}\, c) \left[1 - \left(1 - \frac{\mathrm{org}}{\mathrm{org}_0}\right)^{\xi}\right] = 0$$

Change of variable $\Omega = 1 - org/org_0$

$$\frac{\partial \Omega}{\partial t} = C_2 \left(1 - \Omega^{\xi}\right)$$

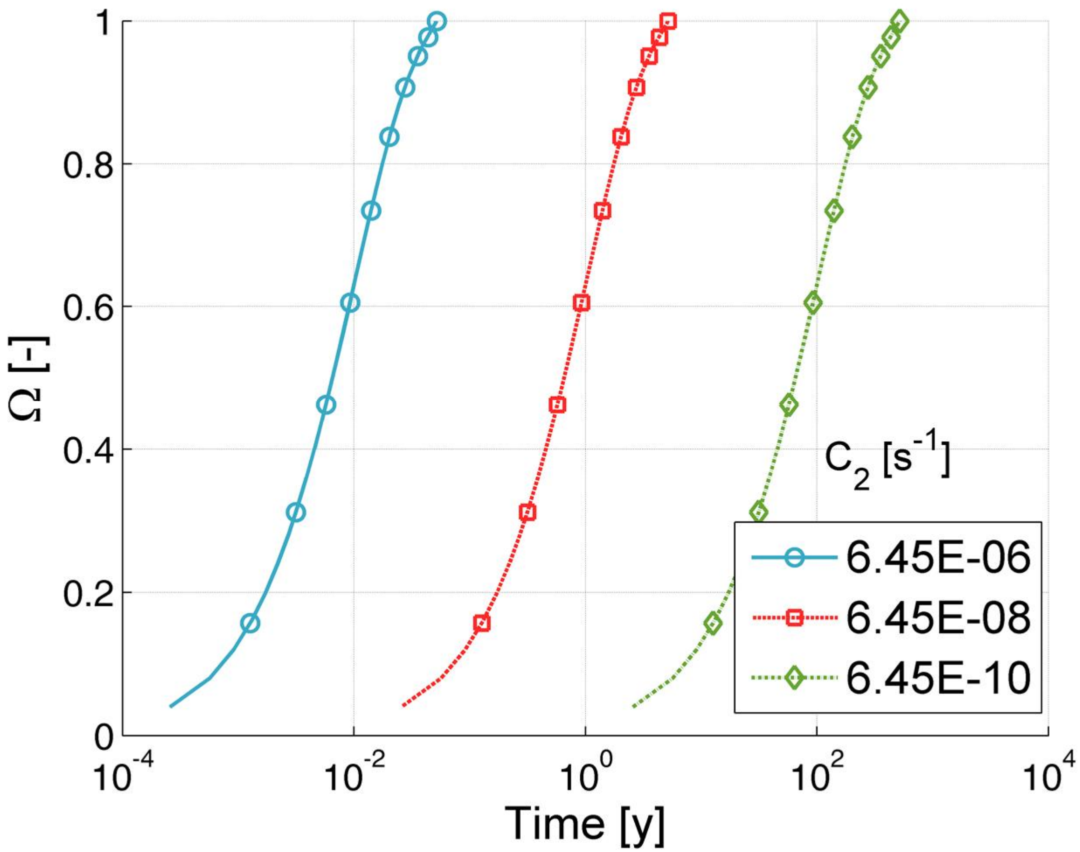
Where $C_2 = \theta\, \theta_e\, Z\, b \exp(-k_{VFA}\, c)/org_0$, solved by Mathematica

$$t = \frac{\Omega}{C_2} \sum_{n=1}^{\infty} \frac{(1)_n \left(\frac{1}{\xi}\right)_n}{\left(1 + \frac{1}{\xi}\right)_n} \frac{\Omega^{n\xi}}{n!}$$

where $(x)_n = x\,(x+1)(x+2)\ldots(x+n-1)$

Analytical solution

Influence of the time constant $C_2 = \theta\ \theta_e\ Z\ b\ \exp(-k_{VFA}\ c)/org_0$



Numerical solution

Non linear system of equations to be solved:

$$\frac{\partial}{\partial t}(\rho_w\, n\, S_{rw}) + \mathrm{div}\left(\underline{f_w}\right) + \frac{\partial}{\partial t}(\rho_v\, n\, S_{rg}) + \mathrm{div}\left(\underline{f_v}\right) - Q_w = 0$$

$$\mathrm{div}(\underline{u}\, c) - div\left(\underline{\underline{D_h}} \cdot \underline{\nabla} c\right) + (r_g - r_h)\,\theta = \frac{\partial c}{\partial t}$$

$$\mathrm{div}(\underline{u}\, m) - div\left(\underline{\underline{D_h}} \cdot \underline{\nabla} m\right) + (r_j - r_k)\,\theta = \frac{\partial m}{\partial t}$$

$$-\mathrm{Z}\,\mathrm{r_g}\,\theta = \frac{\partial org}{\partial t}$$

2 nodal unknowns (p_w and c) and two internal variables (m and org)

Numerical solution

- Sub-stepping procedure for the time integration of the reaction rate.

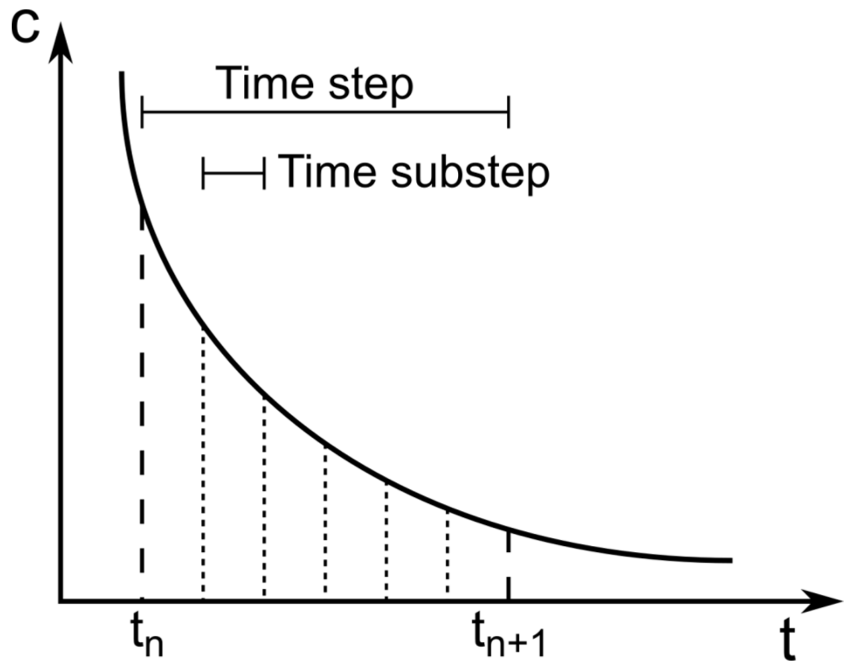
$$\mathrm{div}(\underline{u}\, c) - div\left(\underline{\underline{D_h}} \cdot \underline{\nabla} c\right) + \left(r_g - r_h\right)\theta = \frac{\partial c}{\partial t}$$

As proposed by Wessling et al., 2008, the calculation of VFA accumulation can be split into a transport-only part (V) and a production-only part (Q).

Numerical solution

- Sub-stepping procedure for the time integration of the reaction rate.

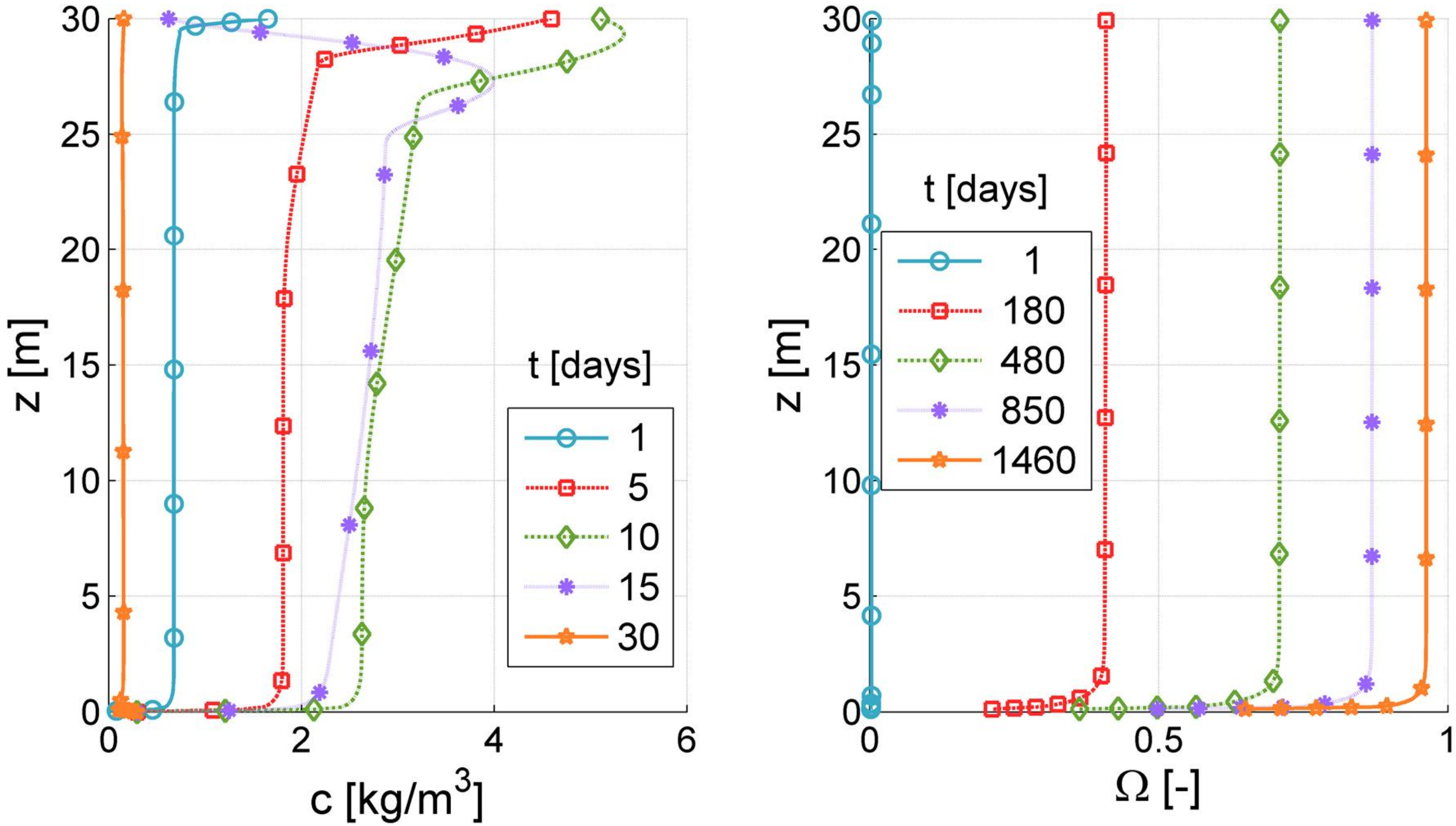
The reaction rate is highly non linear:



$$\Delta t_{ch} = 0.1\,\frac{1}{VFA\ production\ rate}$$

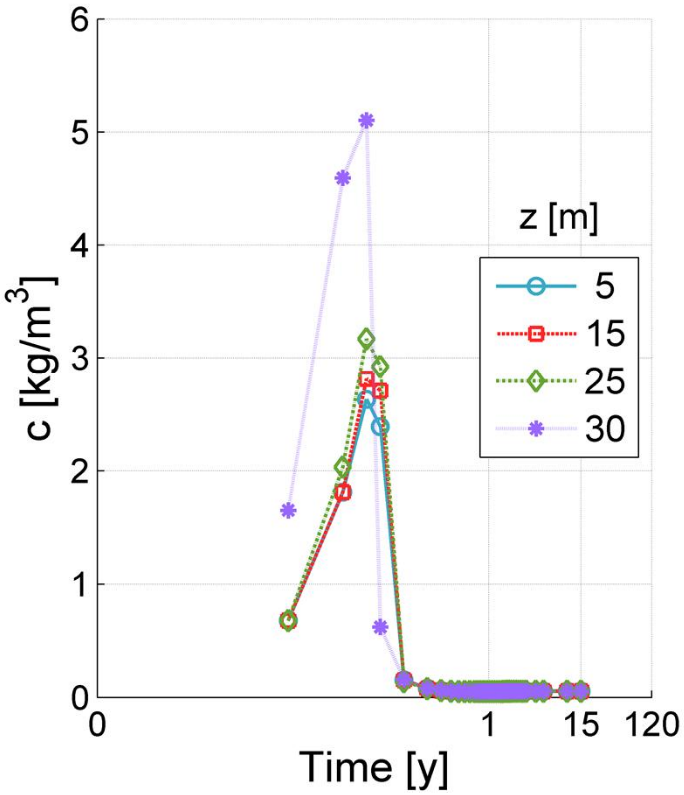
Numerical solution

Evolution of Ω almost uniform over the waste column



Numerical solution

Results for the analytical solution					
Sr	= 0.7071	[-]	c	= 0.055	[kg/m³]



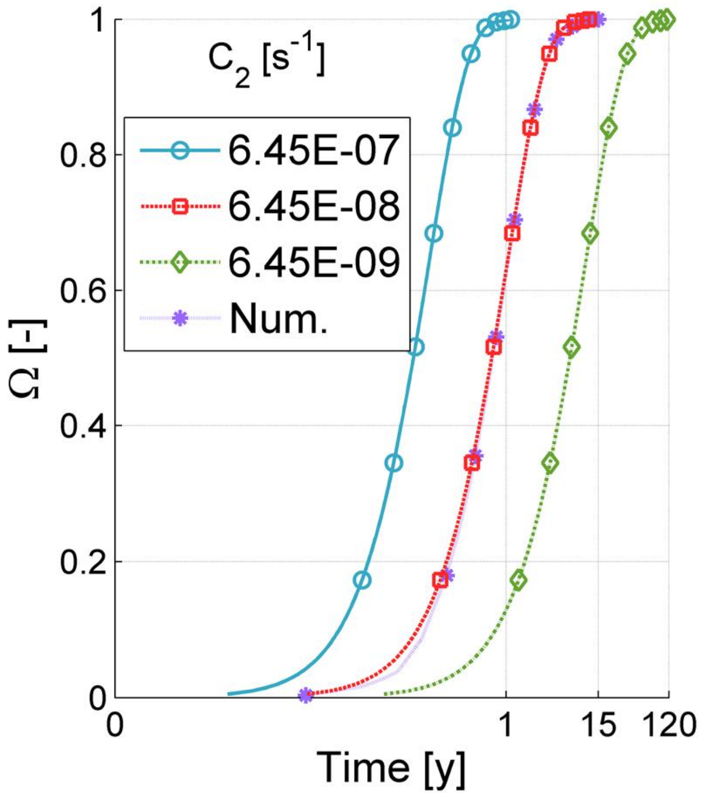
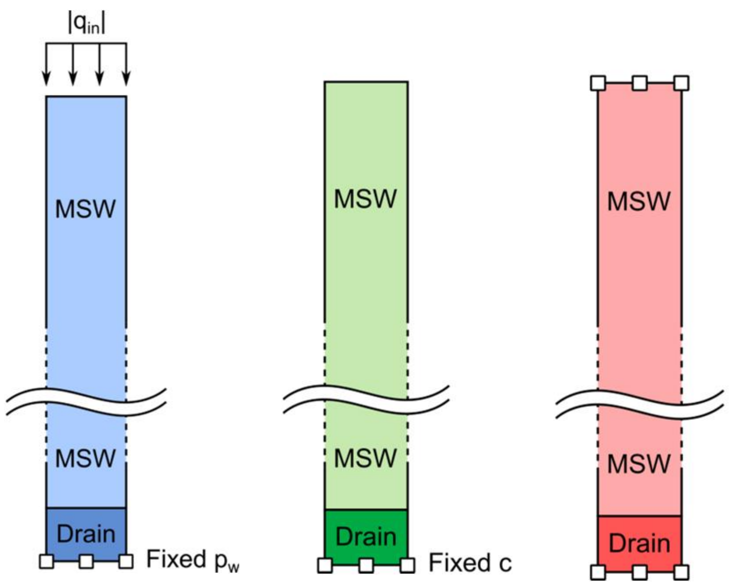


Table of contents

1. Municipal Waste Disposal context
2. Hydraulic model
3. Bio-Chemo-Hydraulic model
4. Bio-Chemo-Thermo-Hydraulic model
5. Bio-Chemo-Thermo-Hydraulic Mechanical model



Energy balance equation

$$\frac{\partial S_T}{\partial t} + \mathrm{div}\left(\underline{V_T}\right) - Q_T = 0$$

- Heat storage
- Heat flux
- Heat production

Heat storage

$$\boxed{\frac{\partial S_T}{\partial t}} + \mathrm{div}\left(\underline{V_T}\right) - Q_T = 0$$

$$\begin{aligned} S_T = {} & n\, S_{rw}\, \rho_w\, c_{pw}\, (\mathrm{T} - \mathrm{T}_0) + n\, S_{rg}\, \rho_a\, c_{pa}\, (\mathrm{T} - \mathrm{T}_0) \\ & + (1-n)\, \rho_s\, c_{ps}\, (\mathrm{T} - \mathrm{T}_0) + n\, S_{rg}\, \rho_v\, c_{pv}\, (\mathrm{T} - \mathrm{T}_0) \\ & + n\, S_{rg}\, \rho_v\, L \end{aligned}$$

- c_{pi} [J/kg/K] specific heat of the component
  - Liquid water: w
  - Water vapour: v
  - Dry air: a
  - Solid waste: s
- ρ_i [kg/m³] density
- L [J/kg] latent heat of water vaporisation
- T & T_0 [°K] Temperature and initial temperature

Heat flux

$$\frac{\partial S_T}{\partial t} + \boxed{\mathrm{div}(\underline{V_T})} - Q_T = 0$$

$$V_T = -\Gamma\,\nabla \mathrm{T} + c_{pw}\,\rho_w\,\underline{q_l}\,(\mathrm{T}-\mathrm{T}_0) + c_{pv}\left(\rho_v\,\underline{q_g} + \underline{i_v}\right)(\mathrm{T}-\mathrm{T}_0)$$
$$+ \left(\rho_v\,\underline{q_g} + \underline{i_v}\right)L + c_{pa}\left(\rho_a\,\underline{q_g} + \underline{i_a}\right)(\mathrm{T}-\mathrm{T}_0)$$

- Conduction
  - Γ [W/m/K] thermal conductivity of waste (including contributions from different phases)
- Advection

Heat source

$$\frac{\partial S_T}{\partial t} + \mathrm{div}(\underline{V_T}) - \boxed{Q_T} = 0$$

$$Q_T = \frac{\partial \mathrm{org}(t)}{\partial t} H_m$$

- Energy release from exothermal biochemical reactions
- H_m[J/kg] heat produced by the degradation of 1kg of waste

Analytical solution

1D heat balance equation (no advection/water vapour)

$$'\rho c_p' \frac{\partial T(z,t)}{\partial t} - \Gamma \frac{\partial^2 T(z,t)}{\partial z^2} - Q_T(t) = 0$$

Dividing by $'\rho c_p'$

$$\frac{\partial T(z,t)}{\partial t} - \alpha \frac{\partial^2 T(z,t)}{\partial z^2} - Q^*(t) = 0$$

Where $\alpha = \frac{\Gamma}{\rho c_p}$ and $Q^*(t) = \frac{Q_T(t)}{\rho c_p}$

Analytical solution

Assuming the following evolution of the organic content:

$$\Omega(t) = 1 - \exp(-\zeta\, t)$$

where ζ = 2.8 10^{-8} [s^{-1}] is fitted from previous results

Heat source expression is given by:

$$Q^*(t) = \frac{H_m}{\rho\, c_p} \frac{\partial \mathrm{org}(t)}{\partial t}$$

$$Q^*(t) = -\frac{H_m\, \zeta}{\rho\, c_p} org_0 \exp(-\zeta\, t)$$

Analytical solution

Classical solution of non homogeneous heat equation

$$T(z,t) = T(z,0) + \sum_{n=1}^{\infty} \sin(\frac{n\pi}{H} z) \int_0^t B_n(s) \exp\big(-\alpha\lambda_n(t-s)\big) \mathrm{d}s$$

Initial and boundary conditions

$$T(z,t=0) = 20° \quad \& \; T(z=0,t) = T(z=30m,t) = 20°$$

where

$$\lambda_n = \left(\frac{n\pi}{H}\right)^2$$

$$B_n(s) = \frac{2}{H}\int_0^H Q^*(s)\sin\left(\frac{n\pi}{H} z\right)\mathrm{d}z$$

$$= \begin{cases} \frac{H_m\zeta}{\rho c_p} or\, g_0 \frac{4}{n\pi} \exp(-\zeta s) & n\ odd \\ 0 & n\ even \end{cases}$$

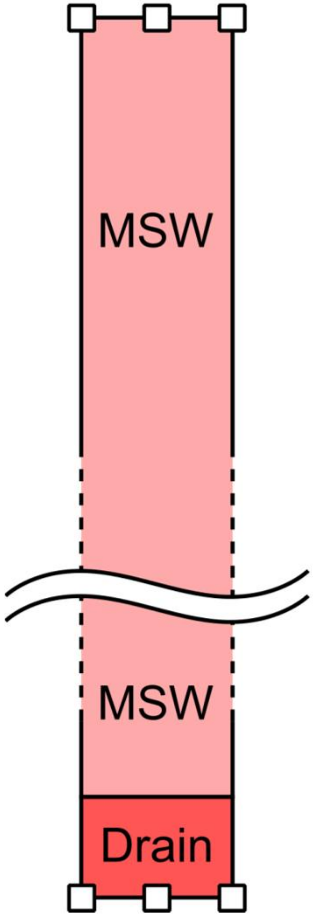
Analytical solution

Classical resolution of non homogeneous heat equation

$$T(z,t) = T(z,0) +$$

$$\sum_{n=1,2}^{\infty} \frac{4}{\mathrm{n}\pi} \frac{H_m \zeta}{\rho c_p} org_0 \frac{1}{-\zeta + \alpha\lambda_n} \sin\left(\frac{n\pi}{H}\, z\right) [\exp(-\zeta t) - \exp(-\alpha\lambda_n t)]$$

Statement of the simplified problem



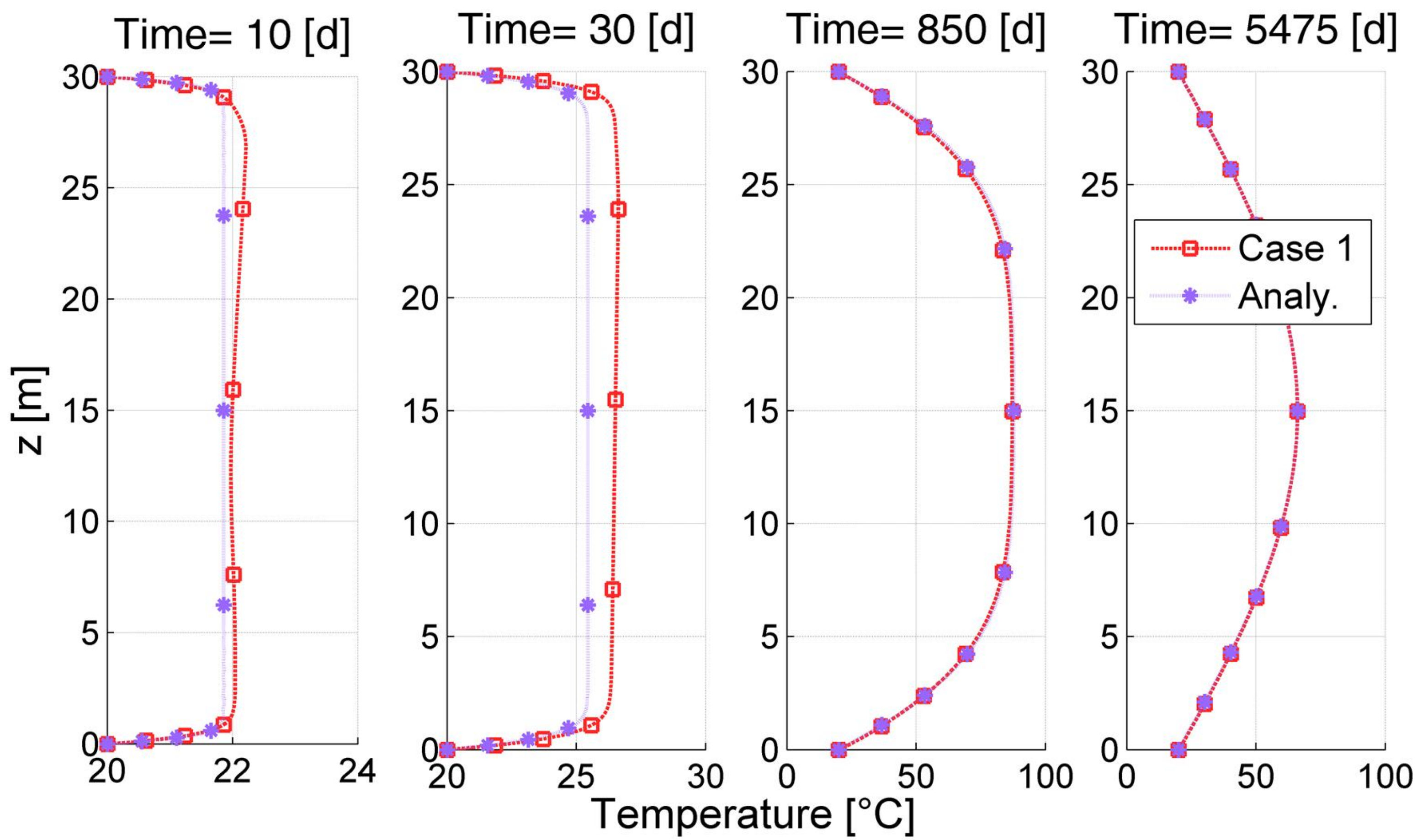
- Imposed temperature at the top
- Imposed temperature at the bottom

Parameters					
c_s	= 1939	[J/kg/K]	ρ_s	= 1000	[kg/m³]
c_w	= 4185	[J/kg/K]	ρ_w	= 1000	[kg/m³]
c_a	= 1004	[J/kg/K]	ρ_a	= 1.2	[kg/m³]
Γ_s	= 0.35	[W/m/K]	Γ_w	= 0.6	[W/m/K]
Γ_a	= 0.025	[W/m/K]	Q_M	= 632	[kJ/kg]

Initial values		
T_0	= 20	[°]

Numerical solution

Case 1: no convection



Numerical solution

Non linear system of equations to be solved:

$$\frac{\partial}{\partial t}(\rho_w\, n\, S_{rw}) + \text{div}\left(\underline{f_w}\right) + \frac{\partial}{\partial t}(\rho_v\, n\, S_{rg}) + \text{div}\left(\underline{f_v}\right) - Q_w = 0$$

$$\text{div}(\underline{u}\, c) - div\left(\underline{\underline{D_h}} \cdot \underline{\nabla} c\right) + (r_g - r_h)\,\theta = \frac{\partial c}{\partial t}$$

$$\frac{\partial S_T}{\partial t} + \text{div}(\underline{V_T}) - Q_T = 0$$

$$(r_j - r_k)\,\theta = \frac{\partial m}{\partial t} \text{ and } -\mathrm{Z}\, \mathrm{r_g}\,\theta = \frac{\partial org}{\partial t}$$

3 nodal unknowns (p_w , T and c) and two internal variables (m and org)

Numerical solution

Case 1: no convection

Case 2: convection ($|q_{in}|$)

Case 3: convection ($|q_{in}|/100$)

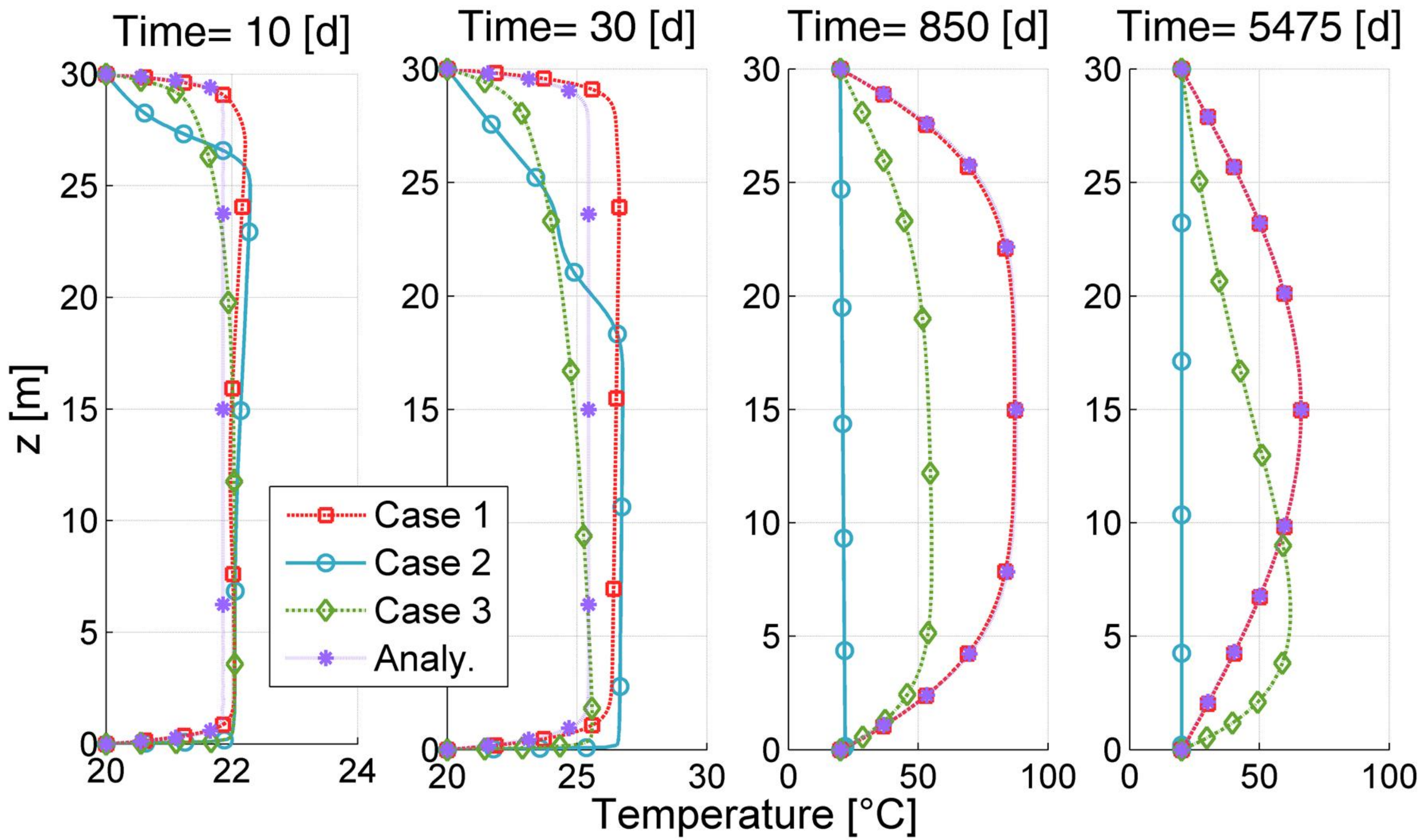
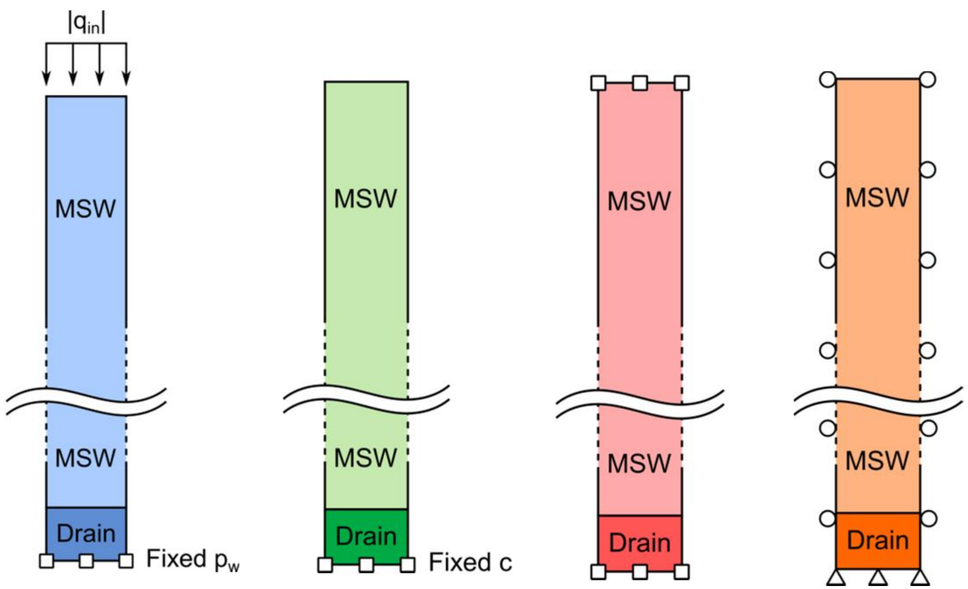


Table of contents



Chemo-mechanical behaviour: experiments

The wastes are highly heterogeneous media

- Large experimental set-up (around 1 m^3) (Olivier 2003, Gandolla et al 1992, Jessberger 1993, …)
- The settlement evolves for several tenth of years
- The biodegradation of the waste concerns mainly the organic component of the wastes (paper, wood …)

Bio-chemo-mechanical behaviour:

- Reactive process (Hueckel, 2009)
- Time-dependent phenomena
- Complex processes

Chemo-mechanical behaviour: experiments

Experimental set-up

- Choice of the material:

  Material with a high organic content: **wood**

  Good biodegradability: **leafy tree (sawdust)**

- Choice of a tree species: beech

  (ρ_s = 742 kg/m³, n= 0.77)

Chemo-mechanical behaviour: experiments

Experimental set-up

- Parameters
  - Bacteria activity
  - Water content
  - Temperature
- Oedometer cell at controlled temperature

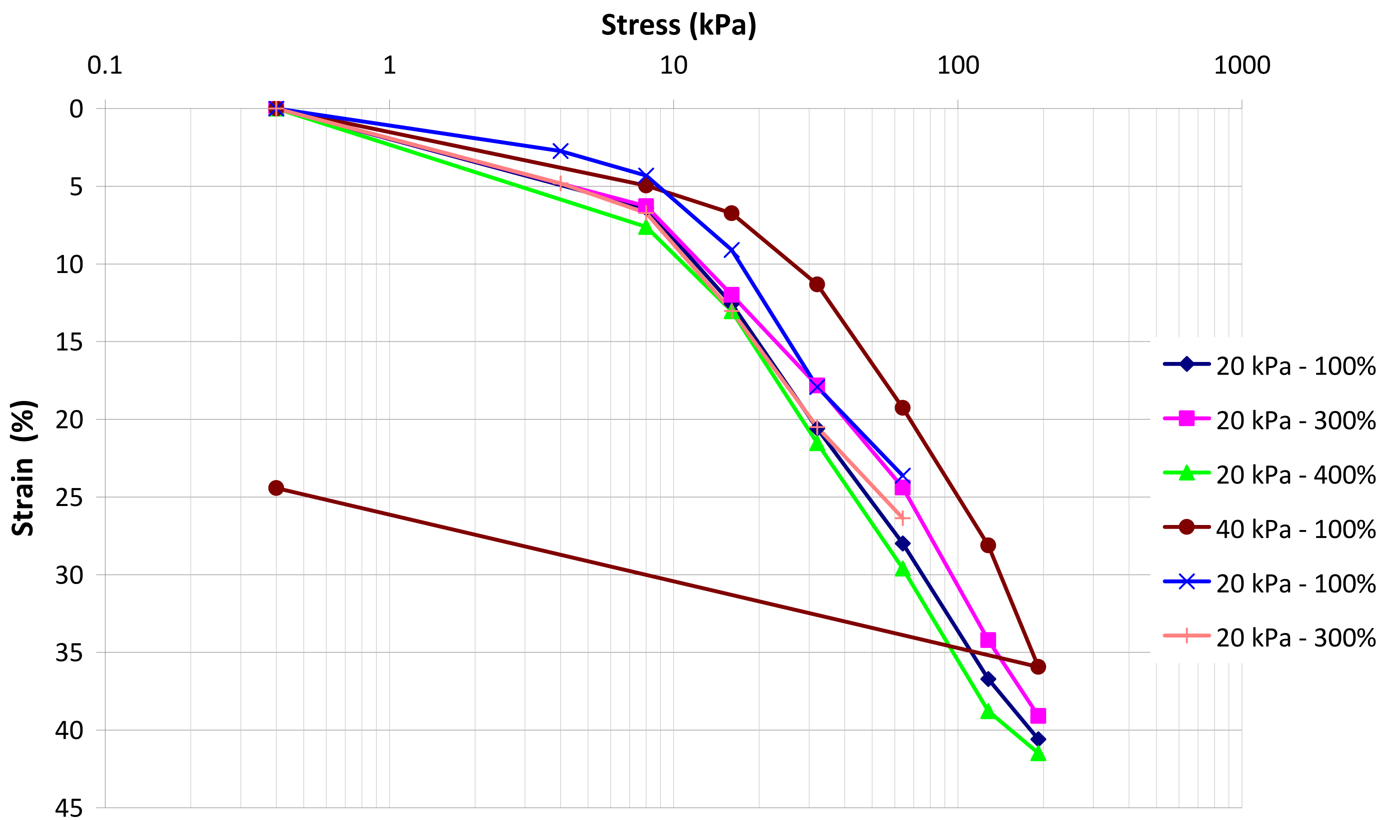
Chemo-mechanical behaviour: experiments

- First series (without leachates)

✓ Sterilization in the autoclave
✓ Water content between 100% and 400%
✓ Precompaction at 20 kPa (or 40 kPa)
✓ Temperature at 36°C

Chemo-mechanical behaviour: experiments

- First series (without leachates)



Chemo-mechanical behaviour: experiments

- First series (without leachates)

Observations

- ✓ No influence of the water content
- ✓ Clear influence of the preconsolidation pressure
- ✓ Compaction higher than the one observed in the landfill.

C_c = 1.0 – 1.1

C_s = 0.2 (0.13 for p_0=40 kPa)

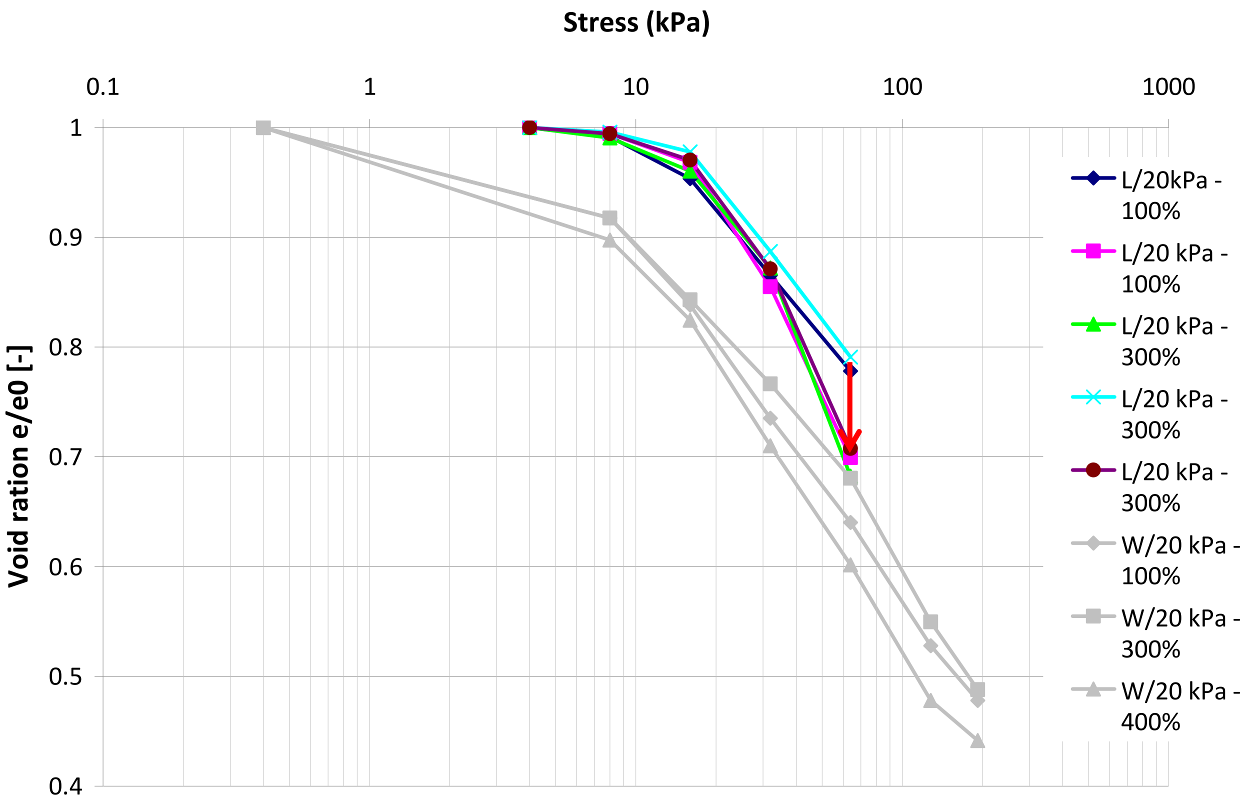
Chemo-mechanical behaviour: experiments

- Second series (with leachates)

- ✓ Sterilisation in the autoclave
- ✓ Leachate content at 100% and 300% (under Natrium flux)
- ✓ Precompaction at 20 kPa (under Natrium flux)
- ✓ Temperature at 36°C

Chemo-mechanical behaviour: experiments

- Second series (with leachates)



Chemo-mechanical behaviour: experiments

- Second series (with leachates)

Observations

- ✓ Influence of the biodegradation: additional compaction
- ✓ No influence of the water content
- ✓ Initial biodegradation seems to have an influence
- ✓ Improve the reproducibility

Chemo-mechanical behaviour: constitutive model

- Features of the material

When loaded, a additional compaction is observed for degraded material

This behaviour is similar to unsaturated soil under wetting path (pore collapse)

The framework proposed by Hueckel is adopted in the following.

Deformation rate decomposition

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{e} + \dot{\epsilon}_{ij}^{p} = \dot{\epsilon}_{ij}^{em} + \dot{\epsilon}_{ij}^{e\Omega} + \dot{\epsilon}_{ij}^{pm}$$

Elastic components

- $\dot{\epsilon}_{ij}^{em}$ mechanical, classical Hooke's law
- $\dot{\epsilon}_{ij}^{e\Omega}$ chemical

$$\epsilon_{ij}^{e\Omega} = -\frac{1}{3}\beta\dot{\Omega}\delta_{ij}$$

$$\beta = F_0\beta_0 \exp(\beta_0[1 - \Omega + \ln\Omega])\left(\frac{1}{\Omega} - 1\right)$$

where F_0 [-] and β_0 [-] are material parameters

Plastic component

- $\dot{\epsilon}_{ij}^{pm}$ mechanical

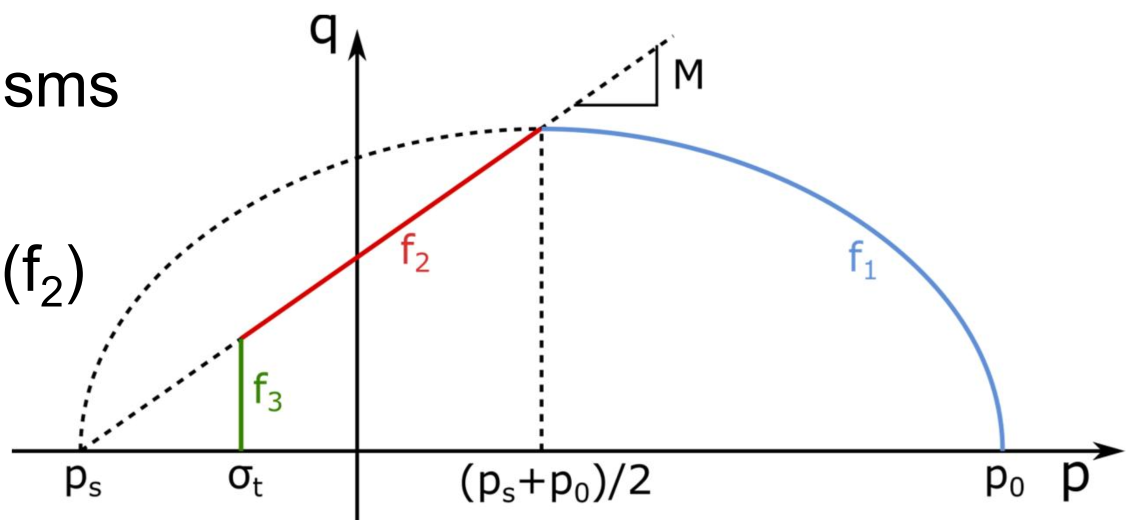
Yield surface

Bishop's stress definition

$$\sigma'_{ij} = \sigma_{ij} - p_g \delta_{ij} + S_{rw}(p_g - p_w)\delta_{ij}$$

Three plastic mechanisms

- Pore collapse (f_1)
- Frictional-cohesive (f_2)
- Tensile (f_3)



$$f_1 \equiv q^2 + M^2(p + p_s)(p - p_0) = 0 \qquad p \geq \frac{p_0 - p_s}{2}$$

$$f_2 \equiv q - M(p - p_s) = 0 \qquad \sigma_t < p \leq \frac{p_0 - p_s}{2}$$

$$f_3 \equiv p + \sigma_t = 0$$

Chemical hardening/softening

Evolution of plastic internal variables with mechanical loading and biodegradation $\Omega \in [0,1]$ for oedometric stress path

- Classical hardening of p_0^* for the CamClay model

$$\mathrm{d}p_0^* = \frac{1+e_0}{\lambda-\kappa}\, p_0^* \,\mathrm{d}\epsilon_v^p$$

- Effect of biodegradation on pore collapse mechanism

$$p_0(\Omega) = p_0^* \exp(-\alpha\Omega)$$

  - p_0^* [Pa] preconsolidation pressure for initial organic content
  - α [-] material parameter

Analytical solution

Hypotheses

- Constant stress state $\dot{\sigma}'_{ij} = 0$
- Biodegradation of plastic deformation only
- Pore collapse mechanism (f_1) only

Consistency condition

$$\frac{\partial f}{\partial \Omega}\,\mathrm{d}\Omega + \frac{\partial f}{\partial p_0^*}\,\mathrm{d}p_0^* = 0$$

$$\frac{\partial f}{\partial \Omega} = \frac{\partial f}{\partial p_0}\frac{\partial p_0}{\partial \Omega} \qquad \frac{\partial p_0}{\partial \Omega} = -p_0^*\,\alpha\exp(-\alpha\Omega) \qquad \frac{\partial f}{\partial p_0^*} = \frac{\partial f}{\partial p_0}\frac{\partial p_0}{\partial p_0^*} \qquad \frac{\partial p_0}{\partial p_0^*} = \exp(-\alpha\Omega)$$

$$\Rightarrow \mathrm{d}p_0^* = \mathrm{d}\Omega\,\frac{\frac{\partial p_0}{\partial \Omega}}{\frac{\partial p_0}{\partial p_0^*}} = -p_0^*\,\alpha\mathrm{d}\Omega$$

Analytical solution

$$\mathrm{d}p_0^* = -p_0^*\,\alpha \mathrm{d}\Omega$$

Classical hardening of p_0^* for the CamClay model

$$\mathrm{d}p_0^* = \frac{1+e_0}{\lambda-\kappa}\,p_0^*\,\mathrm{d}\epsilon_v^p$$

- e_0 [-] initial void ratio
- λ [-] material parameter
- κ [-] material parameter

Relation between biodegradation and plastic deformation

$$\mathrm{d}\epsilon_v^p = -\frac{\lambda-\kappa}{1+e_0}\alpha\,\mathrm{d}\Omega$$

Numerical solution

Non linear system of equations to be solved:

$$\frac{\partial}{\partial t}(\rho_w\, n\, S_{rw}) + \mathrm{div}\left(\underline{f_w}\right) + \frac{\partial}{\partial t}(\rho_v\, n\, S_{rg}) + \mathrm{div}\left(\underline{f_v}\right) - Q_w = 0$$

$$\mathrm{div}(\underline{u}\, c) - div\left(\underline{\underline{D_h}} \cdot \underline{\nabla} c\right) + \left(r_g - r_h\right)\theta = \frac{\partial c}{\partial t}$$

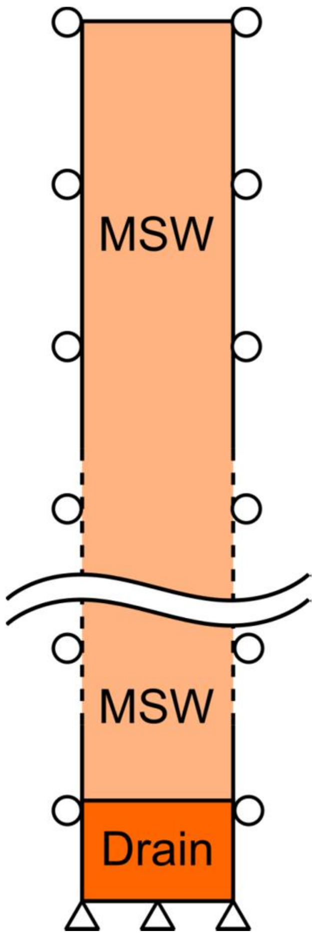
$$\frac{\partial S_T}{\partial t} + \mathrm{div}(\underline{V_T}) - Q_T = 0$$

$$\mathrm{div}(\underline{\sigma}) - \rho g = 0$$

$$\left(r_j - r_k\right)\theta = \frac{\partial m}{\partial t} \text{ and } -\mathrm{Z}\,\mathrm{r_g}\,\theta = \frac{\partial org}{\partial t}$$

4 nodal unknowns (z, p_w , T and c) and two internal variables (m and org)

Statement of the simplified problem

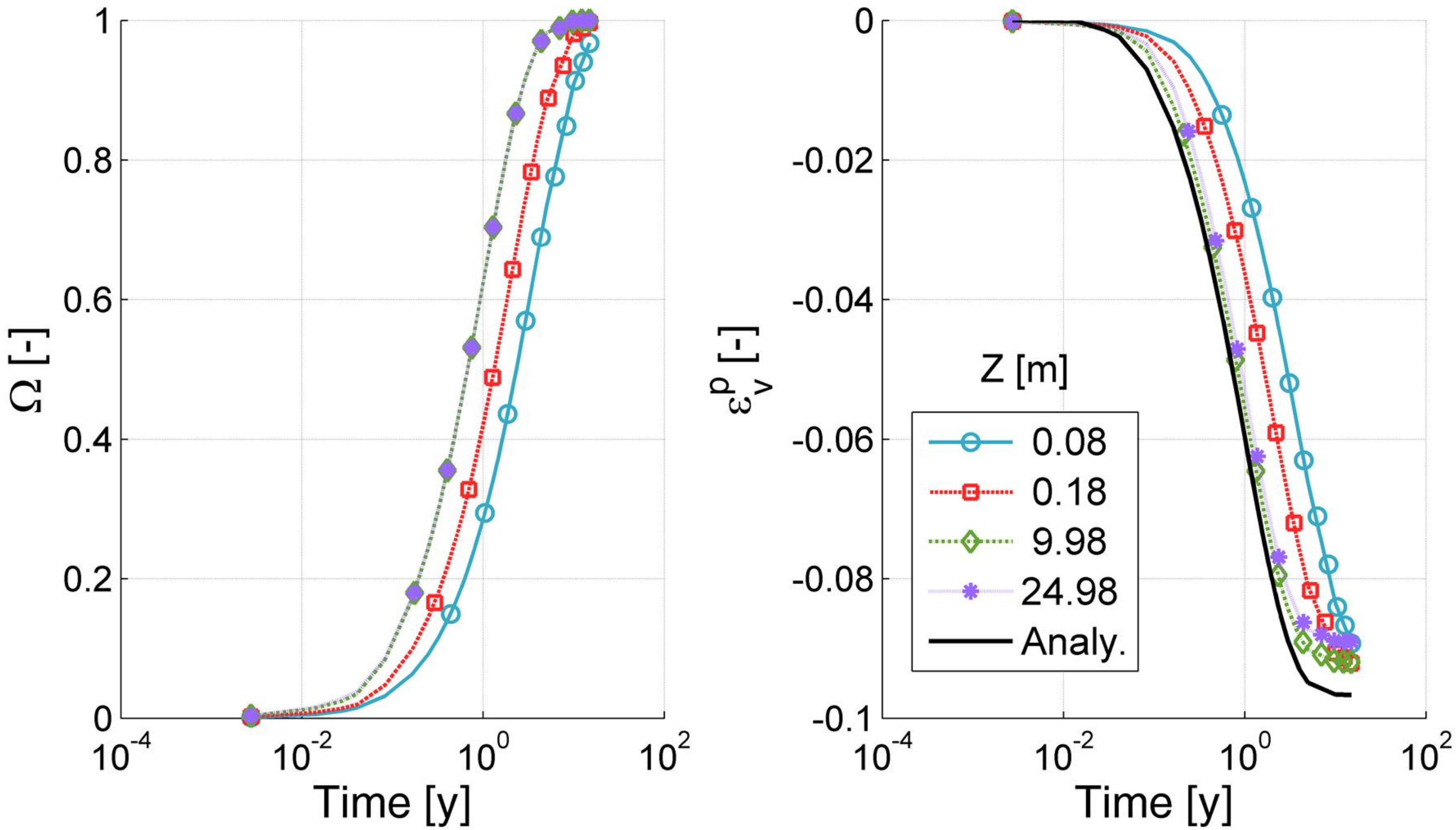


- Fixed displacements at the base
- No lateral displacements on the sides

Parameters					
λ	= 0.0648	[-]	κ	= 0.00792	[-]
α	= 3.45	[-]			
c	= 20	[kPa]	ϕ	= 35	[°]
k_2	= 2.3 10^{-7}	[s^{-1}]			

Initial values					
OCR	= 1.01	[-]	e_0	= 1.0	[-]

Numerical solution



Numerical solution

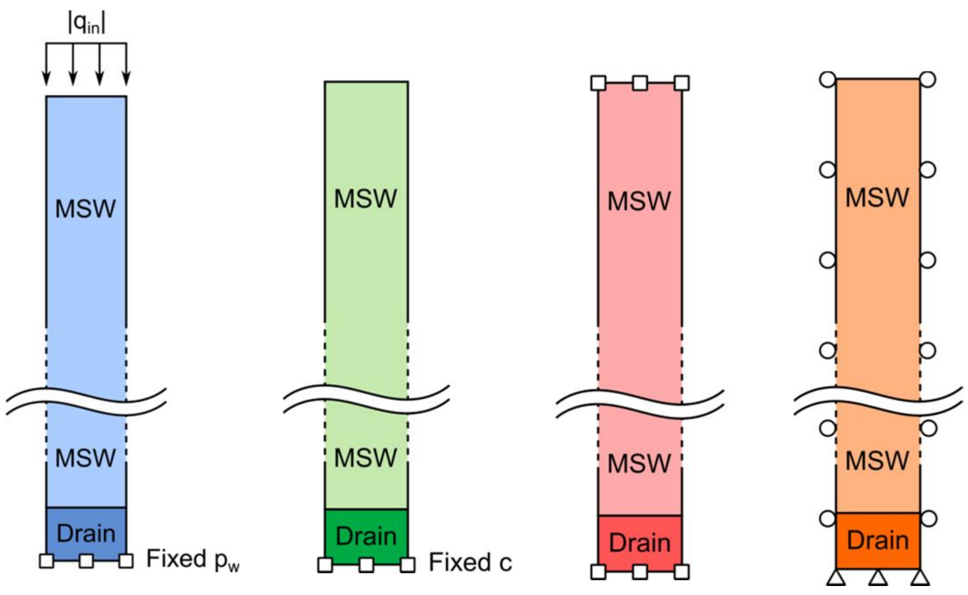
- Problems related to the waste settlement

✓ Differential settlement of the capping system
✓ Increase the shear stress on the confining system along the slope
✓ Additional friction stress on the well tubing (leachate and biogas collection)

- Advantages

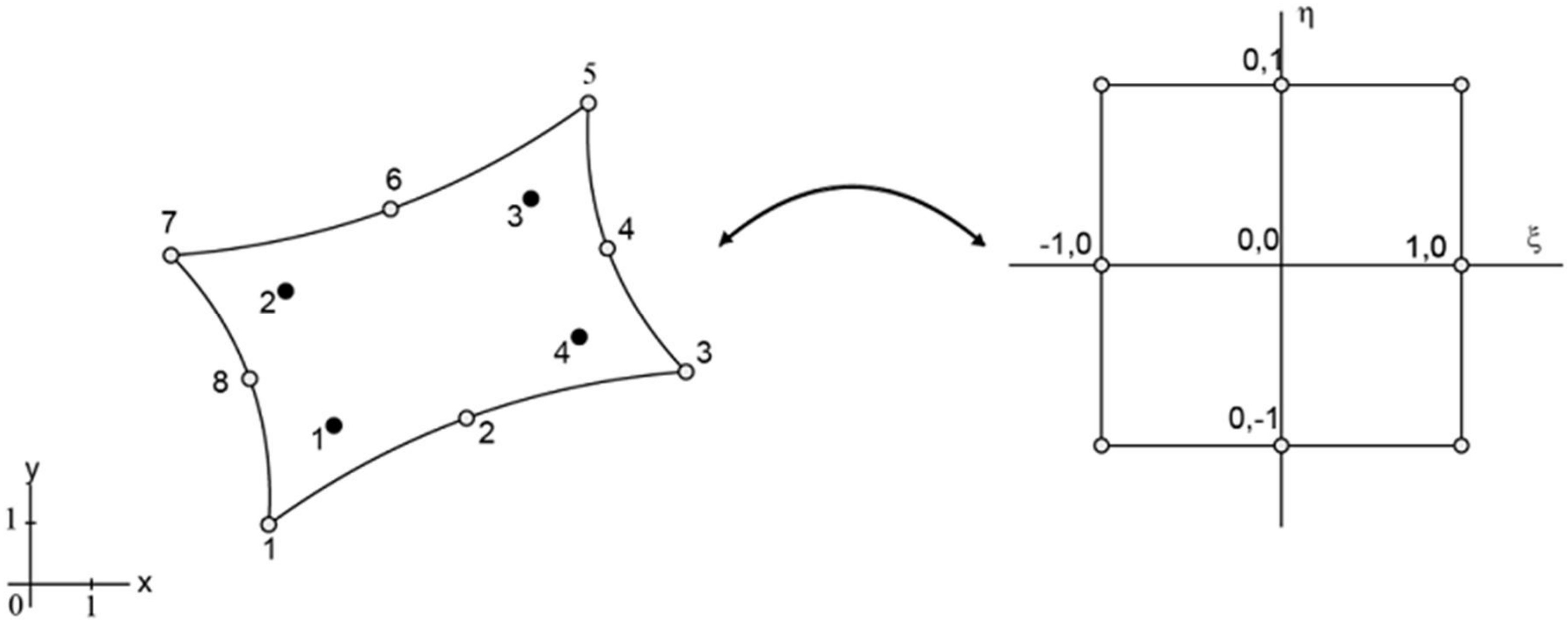
✓ Optimization of the final waste height (maximization of the landfill capacity)

Table of contents



Finite element formulation

- 8-noded isoparametric finite element



- 6 d.o.f : x, y, p_w, p_g, T, c (4 nodes)

Finite element formulation

- 8-noded isoparametric finite element

$$F_{L,1}^{Int} = \sum_{IP} \left(\sigma_{11} \frac{\partial N_L}{\partial x_j} + \sigma_{12} \frac{\partial N_L}{\partial x_j} \right) h \, |J| \, W_{IP}$$

$$F_{L,2}^{Int} = \sum_{IP} \left(\sigma_{12} \frac{\partial N_L}{\partial x_j} + \sigma_{22} \frac{\partial N_L}{\partial x_j} \right) h \, |J| \, W_{IP}$$

$$F_{L,pw}^{Int} = \sum_{IP} \dot{S_w} \; N_L - \left(V_{w1} \frac{\partial N_L}{\partial x_1} + V_{w2} \frac{\partial N_L}{\partial x_2} \right) h \, |J| \, W_{IP}$$

$$F_{L,T}^{Int} = \sum_{IP} \dot{S_T} \; N_L - \left(V_{T1} \frac{\partial N_L}{\partial x_1} + V_{T2} \frac{\partial N_L}{\partial x_2} \right) h \, |J| \, W_{IP}$$

$$F_{L,c}^{Int} = \sum_{IP} \dot{S_c} \; N_L - \left(V_{c1} \frac{\partial N_L}{\partial x_1} + V_{c2} \frac{\partial N_L}{\partial x_2} \right) h \, |J| \, W_{IP}$$

Finite element formulation

- 8-noded isoparametric finite element

$$\begin{bmatrix} K_{MM} & K_{WM} & K_{TM} & K_{CM} \\ K_{MW} & K_{WW} & K_{TW} & K_{CW} \\ K_{MT} & K_{WT} & K_{TT} & K_{CT} \\ K_{MC} & K_{WC} & K_{TC} & K_{CC} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial p_w} & \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial c} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial p_w} & \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial c} \\ \frac{\partial F_{p_w}}{\partial x_1} & \frac{\partial F_{p_w}}{\partial x_2} & \frac{\partial F_{p_w}}{\partial p_w} & \frac{\partial F_{p_w}}{\partial T} & \frac{\partial F_{p_w}}{\partial c} \\ \frac{\partial F_T}{\partial x_1} & \frac{\partial F_T}{\partial x_2} & \frac{\partial F_T}{\partial p_w} & \frac{\partial F_T}{\partial T} & \frac{\partial F_T}{\partial c} \\ \frac{\partial F_c}{\partial x_1} & \frac{\partial F_c}{\partial x_2} & \frac{\partial F_c}{\partial p_w} & \frac{\partial F_c}{\partial T} & \frac{\partial F_c}{\partial c} \end{bmatrix}$$

Conclusions

- Each individual phenomenon has been studied
- Concerning the coupling:

✓ Biodegradation controlled by the water saturation
✓ Temperature increase induced by the biodegradation
✓ Mechanical compaction related to biodegradation

- But some additional coupling are not taken into account

✓ The permeability is reduced by the compaction
✓ The biodegradation depends on the temperature