26th ALERT Doctoral School 2015: Coupled and multiphysics phenomena

Numerical modelling of a Municipal Waste Disposal as a Bio-Chemo-Thermo-Hydro-Mechanical problem

F. Collin¹, J. Hubert¹, X.F. Liu² and B. Cerfontaine¹

¹University of Liège, Belgium
²University of Newcastle, Australia
Introduction

World population is increasing, leading to new issues:

- Energy demands
- Land reclamation
- Municipal waste management

- The European Union has laid down strict requirements for landfills to prevent and reduce as far as possible the negative effects on the environment, specifically on surface water, groundwater, soil, air and human health.

- The landfill is considered as the ultimate solution
The Directive defines the different categories of waste:

- Municipal waste
- Hazardous waste
- Non-hazardous waste
- Inert waste

Landfills are divided into three classes:

- Landfill for hazardous waste
- Landfill for non-hazardous waste: municipal waste; non-hazardous waste of any other origin
- Landfill for inert waste
General requirements for landfills

Appropriate measures shall be taken in order to:

- control water from precipitations entering into the landfill body,
- prevent surface water and/or groundwater from entering into the land-filled waste,
- collect contaminated water and leachate.
- treat contaminated water and leachate collected from the landfill to the appropriate standard required for their discharge.
Protection of soil, groundwater and surface water is to be achieved by:

- during the operational/active phase, the combination of a geological barrier and a bottom liner
- during the passive phase/post closure, the combination of a geological barrier and a top liner.
General requirements for landfills

The landfill base and sides shall consist of a mineral layer which satisfies permeability and thickness requirements:

- landfill for hazardous waste: $K \leq 10^{-9}$ m/s; thickness $\geq 5$ m,
- landfill for non-hazardous waste: $K \leq 10^{-9}$ m/s; thickness $\geq 1$ m,
- landfill for inert waste: $K \leq 10^{-7}$ m/s; thickness $\geq 1$ m.

![Diagram of landfill structures]
A landfill as a bioreactor

Design of the engineered barrier in a waste disposal

- Waste
- Filtrating Layer
- Collecting layer (Silicious material)
- Geotextile
- Geomembrane
- Natural Geological Layer or Engineered barrier
- Collecting System
- Active Sealing Liner
- Passive Sealing Liner
- Drain φ 20cm

K = Permability Coefficient
A landfill as a bioreactor

Multiphysical phenomena control the evolution of the landfill

Water and gas flows
Bio-chemo reaction
Heat transfer
Mechanics
Objectives and outcomes

Objectives

- Describe each individual physical phenomenon
- Evidence the coupling effects

Outcomes

- Provide a mathematical formulation for the BCTHM problem
- Develop a numerical model for the long term behaviour of the municipal waste disposal
# Table of contents

1. Municipal Waste Disposal context  
2. Hydraulic model  
5. Bio-Chemo-Thermo-Hydraulic Mechanical model
Definition of the problem

Statement of the simplified problem

- By essence the wastes are heterogeneous and multiphasic
- Each physical phenomenon will be studied through analytical solution
- Numerical modelling will evidence the coupling effects
- A step by step procedure will be followed
Definition of the geometry (from 3D to 1D problem)
Statement of the simplified problem

- Incoming water due to precipitations
- Initial stresses within the column
- Geometry limited to the waste and the drain
- Slope stability problem not addressed

### Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{\text{MSW}}$</td>
<td>30</td>
<td>[m]</td>
</tr>
<tr>
<td>$H_{D}$</td>
<td>1</td>
<td>[m]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1000</td>
<td>[kg/m$^3$]</td>
</tr>
</tbody>
</table>
Table of contents

1. Municipal Waste Disposal context
2. Hydraulic model
5. Bio-Chemo-Thermo-Hydraulic Mechanical model
Waste as an unsaturated porous medium

Unsaturated porous medium

Liquid phase
- Liquid water
- Dissolved air

Gas phase
- Dry air
- Water vapour

Air species

Water species
Hydraulic model

Mass balance equation

Hypotheses

- Liquid water and water vapour
- Constant gas pressure

\[
\frac{\partial}{\partial t} (\rho_w n S_{rw}) + \text{div} \left( f_w \right) + \frac{\partial}{\partial t} (\rho_v n S_{rg}) + \text{div} \left( f_v \right) - Q_w = 0
\]

- **Liquid** water, \( S_{rw} \) water saturation degree
- **Water vapour**, \( S_{rg} = 1 - S_{rw} \) gas saturation degree
- **Source** term
Hydraulic model

Liquid water mass flow

\[ f_w = \rho_w q_l \]

Advection flow of the liquid phase

\[ q_l = -\frac{K_{int}^{sat} k_{rw}(S_{rw})}{\mu_w} \left[ \text{grad}(p_w) + g \rho_w \text{grad} (z) \right] \]

where

- \( K_{int}^{sat} [m^2] \) is the intrinsic permeability
- \( k_{rw}(S_{rw}) [-] \) is the water relative permeability
- \( \mu_w [Pa.s] \) is the water dynamic viscosity
- \( p_w [Pa] \) is the pore water pressure
- \( \rho_w [kg/m^3] \) is the liquid water density
Hydraulic model

Water vapour mass flow

\[ f_v = \rho_v \ q_g + i_v \]

Advection flow of the gas phase

\[ q_g = - \frac{K_{\text{int}} \ k_{rg}(S_{rw})}{\mu_g} \left[ \text{grad}(\rho_g) + g \ \rho_g \ \text{grad} \ (z) \right] \]

Diffusion within the gaseous phase

\[ i_v = -n \ S_{rg} \ \tau \ D_{v/a} \rho_g \ \text{grad} \ (\rho_v/\rho_g) \]

- \( D_{v/a} \ [\text{m}^2/\text{s}] \) is the diffusion coefficient of water vapour in dry air
- \( \tau \ [-] \) is the tortuosity
Retention properties

Diversity of the retention curves:

![Graph showing diversity of retention curves.](image-url)
Analytical solution

In order to explain the pore pressure evolution, an analytical approach is proposed for a simplified problem.

Hypotheses

- Constant gas pressure \( p_g \)
- Liquid properties = liquid water properties
- Water is incompressible
- Stationary problem
- No water vapour

The water mass balance equation becomes:

\[
\frac{\partial}{\partial t} (\rho_w n S_{rw}) + \text{div} (f_w) + \frac{\partial}{\partial t} (\rho_v n S_{rg}) + \text{div} (f_v) - Q_w = 0
\]
In order to explain the pore pressure evolution, an analytical approach is proposed for a simplified problem.

Hypotheses

- Constant gas pressure $p_g$
- Liquid properties = liquid water properties
- Water is incompressible
- Stationary problem
- No water vapour

The water mass balance equation becomes:

$$\text{div}\left(f_w\right) = 0$$
Analytical solution

Mass balance equation for 1D problem:

\[
\frac{\partial}{\partial z} (q_{lz} \, \rho_w) = 0 \Rightarrow q_{lz} \, \rho_w = q_{in}
\]  

Liquid advection flow

\[
q_{lz} = - \frac{k_{int} \, k_{rw}}{\mu_w} \left( \frac{\partial p_w}{\partial z} + g \, \rho_w \right)
\]
Hydraulic model

Analytical solution

Retention curve

\[ S_{rw} = \exp \left( -\frac{p_c}{4A} \right) \leq 1 \]  \hspace{1cm} (3)

- \( p_c = p_g - p_w \) [Pa] capillary pressure
- A [Pa\(^{-1}\)] material parameter

Relative permeability curve

\[ k_{rw} = (S_{rw})^4 \]  \hspace{1cm} (4)
Hydraulic model

Analytical solution

Retention curve

Relative permeability curve
Analytical solution

Introducing Eqs. (2)(3)(4) into (1)

\[
\frac{\partial p_w^r}{\partial z} + \frac{q_{in} \mu_w}{\rho_w k_{int}^{sat}} \exp \left( - \frac{p_w^r}{A} \right) = -\rho_w \ g
\]

and \( p_w^r = -p_c \) is the relative water pressure (unknown).
Analytical solution

First change of variable \( j = \exp \left( -\frac{p_0}{A} \right) \)

\[
\frac{\partial j}{\partial z} = \beta j^2 + \gamma j
\]

where

- \( \beta = \frac{q_{in} \mu_w}{\rho_w k_{sat} A} \)
- \( \gamma = \frac{\rho wg}{A} \)

⇒ Classical Verhulst equation
Analytical solution

Second change of variable $u = 1/j$

\[ \frac{\partial u}{\partial z} = -\beta - \gamma u \]

\[ \Rightarrow u = C_1 \exp(-\gamma z) - \frac{\beta}{\gamma} \]

\[ \Rightarrow p_w^r (z) = A \ln \left[ C_1 \exp(-\gamma z) - \frac{\beta}{\gamma} \right] \]

Relative water pressure $p_w^r$ imposed in $z = 0$

\[ C_1 = \frac{\beta}{\gamma} + \exp \left( \frac{p_w^r}{A} \right) \]
**Statement of the simplified problem**

- Imposed water flux at the top
- Imposed pressure under the drain (85 kPa)

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>0.5</td>
<td>[-]</td>
</tr>
<tr>
<td>$A$</td>
<td>5</td>
<td>[kPa]</td>
</tr>
<tr>
<td>$k_{sat}$</td>
<td>$10^{-12}$</td>
<td>[m²]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.1</td>
<td>[-]</td>
</tr>
</tbody>
</table>

**Initial values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{rw}$</td>
<td>0.6</td>
<td>[-]</td>
</tr>
<tr>
<td>$p_w$</td>
<td>90.1</td>
<td>[kPa]</td>
</tr>
</tbody>
</table>
Hydraulic model

Analytical solution

Variations of incoming flux $|q_{in}|$

![Graph showing variations of incoming flux $|q_{in}|$ with $p^r_w$ and $Sr$.](image)
The mass balance equation is solved numerically

\[ \frac{\partial}{\partial t} (\rho_w n S_{rw}) + \text{div} \left( f_w \right) + \frac{\partial}{\partial t} (\rho_v n S_{rg}) + \text{div} \left( f_v \right) - Q_w = 0 \]

The Finite element code Lagamine is used:

- 300 elements (8-noded isoparametric elements)
- Modeling time: 15 years
- Constant injection flow
Hydraulic model

Numerical solution

Transient solutions $|q_{in}| = 2.5 \cdot 10^{-3} \text{kg/m}^2/\text{s}$
Table of contents

1. Municipal Waste Disposal context
2. Hydraulic model
5. Bio-Chemo-Thermo-Hydraulic Mechanical model
Landfills are bio-reactors where organic matter is degraded by microorganisms.

- Temperature:
  - Psychrophile Bacteria (T°<20°C)
  - Mesophile Bacteria (20°C<T°<44°C)
  - Thermophile Bacteria (T°>44°C)

- pH: almost neutral, when acidity increase -> biological activity decrease

- Water content: Mandatory for bacteria activity
  Minimal water content: 25 - 35%
Landfills are bio-reactors where organic matter is degraded by microorganisms.

Landfill gas production pattern phases:

- **I**: Anaerobic phase with high CO₂ production
- **II**: Transition period
- **III**: Aerobic phase with high O₂ consumption
- **IV**: Post-degradation phase

Diagram adapted from Farquhar and Rovers (1973).
Landfills are bio-reactors where organic matter is degraded by microorganisms.

**Bio-Chemical model**

**Evolution of the landfill**

- **Hydrolysis**
  - Complex organic matter → soluble organic compounds

- **Acidogenesis**
  - Soluble organic compounds → volatile fatty acids + alcohols
  - Sugars, fatty acids, amino acids

- **Acetogenesis**
  - Volatile fatty acids + alcohols → acetic acid
  - Acetic acid → CO₂, H₂

- **Methanogenesis**
  - Acetic acid → CH₄, CO₂
Landfills are bio-reactors where organic matter is degraded by microorganisms

(figure showing the evolution of the landfill gas production pattern and phases)

(after Farquhar and Rovers, 1973)
Two-stage bio-chemical model (McDougal, 2007)

- Aerobic stage neglected
- Three internal variables
- All the reaction rates are defined per water volume

![Bio-Chemical model diagram]

- **Organic Matter** Org $[g/m^3]$
- **VFA** $c [g/m^3]$
- **Methanogen Biomass** m $[g/m^3]$

**Inhibition of Hydrolisis**

- **Hydrolisis & Acidogenesis**
  - Degradation of org
  - Generation of c
- **Acetogenesis & Methanogenesis**
  - Degradation of c
  - Generation of m
- Transformation of m
Bio-Chemical model

**Hydrolisis & Acidogenesis**
- Degradation of org
- **Generation rate of** c : \( r_g \) [kg/m\(^3\)_w/s]

\[
r_g = b \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}} \left( 1 - \Omega^\xi \right) \exp(-k_{VFA} c)
\]

- \( b \) [kg/m\(^3\)_w/s], maximum VFA growth rate
- \( \theta \) [-], water content (in volume)
  - \( \theta_{res} \) [-] residual water content
  - \( \theta_{sat} \) [-] saturated water content
**Bio-Chemical model**

![Diagram](image)

Hydrolisis & Acidogenesis
- Degradation of org
- **Generation rate of** \( c \) : \( r_g \) \([\text{kg/m}^3\text{w/s}]\)

\[
r_g = b \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}} \left(1 - \Omega^\xi\right) \exp(-k_{\text{VFA}} c)
\]

- \( \Omega \) [-], organic matter ratio
  \[
  \Omega = 1 - \frac{\text{org}}{\text{org}_0} = \begin{cases} 
  0 & (t = 0) \\ 
  1 & (t = \infty)
  \end{cases}
  \]

- \( \text{org} \) \([\text{g/m}^3]\), organic content

- \( \xi \) [-], parameter
Hydrolisis & Acidogenesis

- Degradation of org
- **Generation rate of** $c : r_g \ [\text{kg/m}^3_{w}/s]$

$$r_g = b \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}} \left(1 - \Omega^\xi\right) \exp(-k_{VFA} \ c)$$

- $k_{VFA} \ [-]$, parameter
- $c \ [\text{g/m}^3]$, the VFA concentration in water
### Bio-Chemical model

**Organic Matter**

- Org [g/m³]

**VFA**

- c [g/m³]

---

**Hydrolisis & Acidogenesis**

- **Degradation of org**
- Generation rate of c : \( r_g [\text{kg/m}^3_{\text{w}}/\text{s}] \)

\[
Z \, r_g
\]

- **Z [-]**, substrate yield coefficient
### Bio-Chemical model

#### Acetogenesis & Methanogeneseis

- **Generation rate of** $m$ $r_j$ [kg/m³/s]
- **Degradation of** $c$

$$r_j = \frac{k_0}{k_{MC} \theta^2/c + \theta} m$$

- $\theta$ [-], water content
- $m$ [g/m³], methanogen biomass in water
- $k_0$ [s⁻¹], the specific growth rate
- $k_{MC}$ [g/m³], the half saturation constant

---

**VFA** $c$ [g/m³]  
**Methanogen Biomass** $m$ [g/m³]
Bio-Chemical model

VFA $c \text{[g/m}^3\text{]}$, the VFA concentration in water

Methanogen Biomass $m \text{[g/m}^3\text{]}$

Acetogenesis & Methanogenesis
- **Generation rate of** $m \ r_j \text{[kg/m}^3\text{/s]}$
- **Degradation of** $c$

$$r_j = \frac{k_0}{k_M c \theta^2 / c + \theta} \ m$$

- $c \text{[g/m}^3\text{]}$, the VFA concentration in water
Bio-Chemical model

- **VFA** $c$ [g/m³]
- **Methanogen Biomass** $m$ [g/m³]

### Acetogenesis & Methanogenesis

- **Generation of** $m$
- **Degradation rate of** $c$ $r_h$ [kg/m³/s]

$$r_h = \frac{r_j}{Y}$$

- $Y$ [-], substrate yield coefficient
Bio-Chemical model

Transformation of $m$

- **Degradation rate of $m$** $r_k [\text{kg/m}^3/\text{s}]$

$$r_k = k_2 \frac{m}{\theta}$$

- $k_2 [\text{s}^{-1}]$, methanogen death coefficient
Bio-Chemical model

Balance equations

VFA concentration

\[
\text{div}(u \ c) - \text{div}(D_h \cdot \nabla c) + (r_g - r_h) \theta = \frac{\partial c}{\partial t}
\]

- **Advective flux**, where

\[
u = \frac{q_l}{(S_{rw} \ n_e)}
\]

- \(q_l\) [m/s], water Darcy velocity
- \(S_{rw}\) [-] water saturation degree
- \(n_e\) [-], effective porosity

- **Diffusion flux** (mechanical dispersion and molecular diffusion)

- \(D_h\) [m²/s], apparent diffusion coefficient

- **Reaction term**
**Bio-Chemical model**

## Balance equations

**Methanogen concentration**

\[
\text{div}(\textbf{u} m) - \text{div}(D_h \cdot \nabla m) + (r_j - r_k) \theta = \frac{\partial m}{\partial t}
\]

- **Advective flux**, where
  
  \[
  \textbf{u} = \frac{q_l}{(S_{rw} n_e)}
  \]

  - \(q_l\) [m/s], water Darcy velocity
  - \(S_{rw}\) [-] water saturation degree
  - \(n_e\) [-], effective porosity

- **Diffusion flux** (mechanical dispersion and molecular diffusion)
  
  - \(D_h\) [m²/s], apparent diffusion coefficient

- **Reaction term**
Balance equations

Organic content

\[-Z r_g \theta = \frac{\partial \text{org}}{\partial t}\]

- \(Z\) [-] substrate yield coefficient
- \(r_g\) [kg/m³/s] generation rate of \(c\)
- \(\theta\) [-] water content
Imposed VFA concentration at the bottom of the drain ($c = 0$)

### Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>$2.9 \times 10^{-5}$</td>
<td>[kg/m³/s]</td>
</tr>
<tr>
<td>$k_0$</td>
<td>$5.7 \times 10^{-6}$</td>
<td>[s⁻¹]</td>
</tr>
<tr>
<td>$k_{MC}$</td>
<td>$4.2$</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>$k_2$</td>
<td>$2.3 \times 10^{-7}$</td>
<td>[s⁻¹]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$0.36$</td>
<td>[-]</td>
</tr>
<tr>
<td>$k_{VFA}$</td>
<td>$0.2$</td>
<td>[m³/kg]</td>
</tr>
<tr>
<td>$Y$</td>
<td>$0.08$</td>
<td>[-]</td>
</tr>
<tr>
<td>$Z$</td>
<td>$2.7$</td>
<td>[-]</td>
</tr>
</tbody>
</table>

### Initial values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$org$</td>
<td>$300$</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>$c$</td>
<td>$0.3$</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>$m$</td>
<td>$0.0025$</td>
<td>[kg/m³]</td>
</tr>
</tbody>
</table>
Bio-Chemical model

Internal variable couplings

- Single point bioreactor
- Constant saturation
- No advection
- No diffusion
Analytical solution

Degradation of organic matter

\[
\frac{\partial \text{org}}{\partial t} + \theta \theta_e Z b \exp(-k_{VFA} c) \left[ 1 - \left( 1 - \frac{\text{org}}{\text{org}_0} \right)^\xi \right] = 0
\]

Change of variable \( \Omega = 1 - \frac{\text{org}}{\text{org}_0} \)

\[
\frac{\partial \Omega}{\partial t} = C_2 \left( 1 - \Omega^\xi \right)
\]

Where \( C_2 = \theta \theta_e Z b \exp(-k_{VFA} c)/\text{org}_0 \), solved by Mathematica

\[
t = \frac{\Omega}{C_2} \sum_{n=1}^{\infty} \frac{(1)_n \left( \frac{1}{\xi} \right)_n}{\left( 1 + \frac{1}{\xi} \right)_n} \frac{\Omega^{n\xi}}{n!}
\]

where \((x)_n = x (x + 1)(x + 2) ... (x + n - 1)\)
Bio-chemical model

Analytical solution

Influence of the time constant $C_2 = \theta \theta_e Z b \exp(-k_{VFA} c)/or g_0$
Bio-chemical model

Numerical solution

Non linear system of equations to be solved:

\[
\frac{\partial}{\partial t} (\rho_w n S_{rw}) + \text{div} \left( f_w \right) + \frac{\partial}{\partial t} (\rho_v n S_{rg}) + \text{div} \left( f_v \right) - Q_w = 0
\]

\[
\text{div}(u c) - \text{div}(D_h \cdot \nabla c) + (r_g - r_h) \theta = \frac{\partial c}{\partial t}
\]

\[
\text{div}(u m) - \text{div}(D_h \cdot \nabla m) + (r_j - r_k) \theta = \frac{\partial m}{\partial t}
\]

\[-Z r_g \theta = \frac{\partial org}{\partial t}\]

2 nodal unknowns (\( p_w \) and c) and two internal variables (m and org)
Numerical solution

- Sub-stepping procedure for the time integration of the reaction rate.

\[
\text{div}(u \ c) - \text{div}(D_h \cdot \nabla c) + (r_g - r_h) \theta = \frac{\partial c}{\partial t}
\]

As proposed by Wessling et al., 2008, the calculation of VFA accumulation can be split into a transport-only part \((V)\) and a production-only part \((Q)\).
- Sub-stepping procedure for the time integration of the reaction rate.

The reaction rate is highly non-linear:

\[ \Delta t_{ch} = 0.1 \frac{1}{VFA \text{ production rate}} \]
Bio-chemical model

Numerical solution

Evolution of $\Omega$ almost uniform over the waste column
Bio-chemical model

Numerical solution

Results for the analytical solution

\[ \text{Sr} = 0.7071 \quad [-] \quad \text{c} = 0.055 \quad [\text{kg/m}^3] \]
Table of contents

1. Municipal Waste Disposal context
2. Hydraulic model
5. Bio-Chemo-Thermo-Hydraulic Mechanical model
Energy balance equation

\[ \frac{\partial S_T}{\partial t} + \text{div}(V_T) - Q_T = 0 \]

- Heat storage
- Heat flux
- Heat production
Heat storage

\[
\frac{\partial S_T}{\partial t} + \text{div}(V_T) - Q_T = 0
\]

\[
S_T = n S_{rw} \rho_w c_{pw} (T - T_0) + n S_{rg} \rho_a c_{pa} (T - T_0) \\
+ (1 - n) \rho_s c_{ps} (T - T_0) + n S_{rg} \rho_v c_{pv} (T - T_0) \\
+ n S_{rg} \rho_v L
\]

- \( c_{pi} \) [J/kg/K] specific heat of the component
  - Liquid water: w
  - Water vapour: v
  - Dry air: a
  - Solid waste: s

- \( \rho_i \) [kg/m³] density

- \( L \) [J/kg] latent heat of water vaporisation

- T & T₀ [°K] Temperature and initial temperature
Thermal model

Heat flux

\[
\frac{\partial S_T}{\partial t} + \text{div}(V_T) - Q_T = 0
\]

\[
V_T = -\Gamma \nabla T + c_{pw} \rho_w q_l (T - T_0) + c_{pv} (\rho_v q_g + i_v) (T - T_0) \\
+ \left( \rho_v q_g + i_v \right) L + c_{pa} (\rho_a q_g + i_a) (T - T_0)
\]

- Conduction
  - \( \Gamma \) [W/m/K] thermal conductivity of waste (including contributions from different phases)

- Advection
Thermal model

Heat source

\[ \frac{\partial S_T}{\partial t} + \text{div}(V_T) - Q_T = 0 \]

\[ Q_T = \frac{\partial \text{org}(t)}{\partial t} H_m \]

- Energy release from exothermal biochemical reactions
- \( H_m [J/kg] \) heat produced by the degradation of 1kg of waste
1D heat balance equation (no advection/water vapour)

\[ \rho c_p \frac{\partial T(z, t)}{\partial t} - \Gamma \frac{\partial^2 T(z, t)}{\partial z^2} - Q_T(t) = 0 \]

Dividing by \( \rho c_p \)

\[ \frac{\partial T(z, t)}{\partial t} - \alpha \frac{\partial^2 T(z, t)}{\partial z^2} - Q^*(t) = 0 \]

Where \( \alpha = \frac{\Gamma}{\rho c_p} \) and \( Q^*(t) = \frac{Q_T(t)}{\rho c_p} \)
Analytical solution

Assuming the following evolution of the organic content:

\[ \Omega(t) = 1 - \exp(-\zeta t) \]

where \( \zeta = 2.8 \times 10^{-8} \text{ [s}^{-1}] \) is fitted from previous results

Heat source expression is given by:

\[ Q^*(t) = \frac{H_m}{\rho c_p} \frac{\partial \text{org}(t)}{\partial t} \]

\[ Q^*(t) = -\frac{H_m \zeta}{\rho c_p} \text{org}_0 \exp(-\zeta t) \]
Analytical solution

Classical solution of non homogeneous heat equation

\[ T(z, t) = T(z, 0) + \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{H} z\right) \int_{0}^{t} B_n(s) \exp\left(-\alpha \lambda_n (t - s)\right) ds \]

Initial and boundary conditions

\[ T(z, t = 0) = 20^\circ & \quad T(z = 0, t) = T(z = 30m, t) = 20^\circ \]

where \[ \lambda_n = \left(\frac{n\pi}{H}\right)^2 \]

\[ B_n(s) = \frac{2}{H} \int_{0}^{H} Q^*(s) \sin\left(\frac{n\pi}{H} z\right) dz \]

\[ = \begin{cases} \frac{H_{m} \zeta}{\rho c_p} & \text{or} \quad g_0 \frac{4}{n\pi} \exp(-\zeta s) \quad n \text{ odd} \\ 0 & \text{n even} \end{cases} \]
Analytical solution

Classical resolution of non homogeneous heat equation

\[ T(z, t) = T(z, 0) + \sum_{n=1,2}^{\infty} \frac{4}{n\pi} \frac{H_m\zeta}{\rho c_p} \cdot \frac{1}{\eta - \zeta + \alpha\lambda_n} \sin \left( \frac{n\pi}{H} z \right) \left[ \exp(-\zeta t) - \exp(-\alpha\lambda_n t) \right] \]
Imposed temperature at the top

Imposed temperature at the bottom

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value 1</th>
<th>Unit 1</th>
<th>Value 2</th>
<th>Unit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_s$</td>
<td>1939</td>
<td>[J/kg/K]</td>
<td>$\rho_s$</td>
<td>1000</td>
</tr>
<tr>
<td>$c_w$</td>
<td>4185</td>
<td>[J/kg/K]</td>
<td>$\rho_w$</td>
<td>1000</td>
</tr>
<tr>
<td>$c_a$</td>
<td>1004</td>
<td>[J/kg/K]</td>
<td>$\rho_a$</td>
<td>1.2</td>
</tr>
<tr>
<td>$\Gamma_s$</td>
<td>0.35</td>
<td>[W/m/K]</td>
<td>$\Gamma_w$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\Gamma_a$</td>
<td>0.025</td>
<td>[W/m/K]</td>
<td>$Q_M$</td>
<td>632</td>
</tr>
</tbody>
</table>
Numerical solution

Case 1: no convection
Bio-chemo-hydro-thermal model

Numerical solution

Non linear system of equations to be solved:

\[
\frac{\partial}{\partial t} (\rho_w n S_{rw}) + \text{div} (f_w) + \frac{\partial}{\partial t} (\rho_v n S_{rg}) + \text{div} (f_v) - Q_w = 0
\]

\[
\text{div}(u \, c) - \text{div} (D_h \cdot \nabla c) + (r_g - r_h) \theta = \frac{\partial c}{\partial t}
\]

\[
\frac{\partial S_T}{\partial t} + \text{div}(V_T) - Q_T = 0
\]

\[
(r_j - r_k) \theta = \frac{\partial m}{\partial t} \text{ and } -Z \, r_g \, \theta = \frac{\partial \text{org}}{\partial t}
\]

3 nodal unknowns ($p_w$, $T$ and $c$) and two internal variables ($m$ and org)
Case 1: no convection
Case 2: convection ($|q_{\text{in}}|$)
Case 3: convection ($|q_{\text{in}}|/100$)
# Table of contents

1. Municipal Waste Disposal context  
2. Hydraulic model  
5. Bio-Chemo-Thermo-Hydraulic Mechanical model
Chemo-mechanical behaviour: experiments

The wastes are highly heterogeneous media

- Large experimental set-up (around 1 m³) (Olivier 2003, Gandolla et al 1992, Jessberger 1993, …)
- The settlement evolves for several tenth of years
- The biodegradation of the waste concerns mainly the organic component of the wastes (paper, wood …)

Bio-chemo-mechanical behaviour:

- Reactive process (Hueckel, 2009)
- Time-dependent phenomena
- Complex processes
Chemo-mechanical behaviour: experiments

Experimental set-up

- **Choice of the material:**
  - Material with a high organic content: **wood**
  - Good biodegradability: **leafy tree (sawdust)**

- **Choice of a tree species:** beech
  \[ \rho_s = 742 \text{ kg/m}^3, n = 0.77 \]
Chemo-mechanical behaviour: experiments

Experimental set-up

- Parameters
  - Bacteria activity
  - Water content
  - Temperature
- Oedometer cell at controlled temperature
Chemo-mechanical behaviour: experiments

- First series (without leachates)

  - Sterilization in the autoclave
  - Water content between 100% and 400%
  - Precompaction at 20 kPa (or 40 kPa)
  - Temperature at 36°C
Chemo-mechanical behaviour: experiments

- First series (without leachates)
First series (without leachates)

Observations

- No influence of the water content
- Clear influence of the preconsolidation pressure
- Compaction higher than the one observed in the landfill.

\[ C_c = 1.0 - 1.1 \]
\[ C_s = 0.2 \text{ (0.13 for } p_0=40 \text{ kPa)} \]
Mechanical model

Chemo-mechanical behaviour: experiments

- Second series (with leachates)
  - Sterilisation in the autoclave
  - Leachate content at 100% and 300% (under Natrium flux)
  - Precompaction at 20 kPa (under Natrium flux)
  - Temperature at 36°C
Mechanical model

Chemo-mechanical behaviour: experiments

- Second series (with leachates)
Chemo-mechanical behaviour: experiments

- Second series (with leachates)

Observations

- Influence of the biodegradation: additional compaction
- No influence of the water content
- Initial biodegradation seems to have an influence
- Improve the reproducibility
Chemo-mechanical behaviour: constitutive model

- Features of the material

When loaded, a additional compaction is observed for degraded material

This behaviour is similar to unsaturated soil under wetting path (pore collapse)

The framework proposed by Hueckel is adopted in the following.
Deformation rate decomposition

\[ \dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^p = \dot{\varepsilon}_{ij}^{em} + \dot{\varepsilon}_{ij}^{e\Omega} + \dot{\varepsilon}_{ij}^{pm} \]

Elastic components

- \( \dot{\varepsilon}_{ij}^{em} \) mechanical, classical Hooke’s law
- \( \dot{\varepsilon}_{ij}^{e\Omega} \) chemical

\[ \varepsilon_{ij}^{e\Omega} = -\frac{1}{3} \beta \dot{\Omega} \delta_{ij} \]

\[ \beta = F_0 \beta_0 \exp(\beta_0[1 - \Omega + \ln \Omega]) \left( \frac{1}{\Omega} - 1 \right) \]

where \( F_0 \) [-] and \( \beta_0 \) [-] are material parameters

Plastic component

- \( \dot{\varepsilon}_{ij}^{pm} \) mechanical
### Yield surface

**Bishop’s stress definition**

\[
\sigma'_{ij} = \sigma_{ij} - p_g \delta_{ij} + S_{rw} (p_g - p_w) \delta_{ij}
\]

**Three plastic mechanisms**

- Pore collapse \((f_1)\)
- Frictional-cohesive \((f_2)\)
- Tensile \((f_3)\)

For \(f_1\):

\[
f_1 \equiv q^2 + M^2 (p + p_s)(p - p_0) = 0 \quad p \geq \frac{p_0 - p_s}{2}
\]

For \(f_2\):

\[
f_2 \equiv q - M(p - p_s) = 0 \quad \sigma_t < p \leq \frac{p_0 - p_s}{2}
\]

For \(f_3\):

\[
f_3 \equiv p + \sigma_t = 0
\]
Chemical hardening/softening

Evolution of plastic internal variables with mechanical loading and biodegradation $\Omega \in [0,1]$ for oedometric stress path

- Classical hardening of $p_0^*$ for the CamClay model
  \[
  dp_0^* = \frac{1 + e_0}{\lambda - \kappa} \ p_0^* \ d\epsilon_v^p
  \]

- Effect of biodegradation on pore collapse mechanism
  \[
  p_0(\Omega) = p_0^* \exp(-\alpha \Omega)
  \]
  - $p_0^*$ [Pa] preconsolidation pressure for initial organic content
  - $\alpha$ [-] material parameter
Analytical solution

Hypotheses

- Constant stress state $\dot{\sigma}_{ij} = 0$
- Biodegradation of plastic deformation only
- Pore collapse mechanism ($f_1$) only

Consistency condition

$$\frac{\partial f}{\partial \Omega} \ d\Omega + \frac{\partial f}{\partial p_0^*} \ dp_0^* = 0$$

$$\frac{\partial f}{\partial \Omega} = \frac{\partial f}{\partial p_0} \ \frac{\partial p_0}{\partial \Omega} \quad \frac{\partial p_0}{\partial \Omega} = -p_0^* \alpha \exp(-\alpha \Omega) \quad \frac{\partial f}{\partial p_0^*} = \frac{\partial f}{\partial p_0} \ \frac{\partial p_0}{\partial p_0^*} \quad \frac{\partial p_0}{\partial p_0^*} = \exp(-\alpha \Omega)$$

$$\Rightarrow dp_0^* = d\Omega \ \frac{\partial p_0}{\partial p_0^*} = -p_0^* \alpha d\Omega$$
Analytical solution

\[ dp_0^* = -p_0^* \alpha d\Omega \]

Classical hardening of \( p_0^* \) for the CamClay model

\[ dp_0^* = \frac{1 + e_0}{\lambda - \kappa} \ p_0^* \ d\varepsilon^p_v \]

- \( e_0 [-] \) initial void ratio
- \( \lambda [-] \) material parameter
- \( \kappa [-] \) material parameter

Relation between biodegradation and plastic deformation

\[ d\varepsilon^p_v = -\frac{\lambda - \kappa}{1 + e_0} \alpha \ d\Omega \]
Numerical solution

Non linear system of equations to be solved:

\[
\frac{\partial}{\partial t}(\rho_w \ n \ S_{rw}) + \text{div}(f_w) + \frac{\partial}{\partial t}(\rho_v \ n \ S_{rg}) + \text{div}(f_v) - Q_w = 0
\]

\[
\text{div}(u \ c) - \text{div}(D_h \cdot \nabla c) + (r_g - r_h) \theta = \frac{\partial c}{\partial t}
\]

\[
\frac{\partial S_T}{\partial t} + \text{div}(V_T) - Q_T = 0
\]

\[
\text{div}(\sigma) - \rho g = 0
\]

\[
(r_j - r_k) \theta = \frac{\partial m}{\partial t} \text{ and } -Z \ r_g \ \theta = \frac{\partial \text{org}}{\partial t}
\]

4 nodal unknowns (z, p_w, T and c) and two internal variables (m and org)
Statement of the simplified problem

- Fixed displacements at the base
- No lateral displacements on the sides

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.0648</td>
<td>[-]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>3.45</td>
<td>[-]</td>
</tr>
<tr>
<td>$c$</td>
<td>20</td>
<td>[kPa]</td>
</tr>
<tr>
<td>$k_2$</td>
<td>2.3 $10^{-7}$</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.00792</td>
<td>[-]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>35</td>
<td>[$^\circ$]</td>
</tr>
</tbody>
</table>

**Initial values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$OCR$</td>
<td>1.01</td>
<td>[-]</td>
</tr>
<tr>
<td>$e_0$</td>
<td>1.0</td>
<td>[-]</td>
</tr>
</tbody>
</table>
Bio-chemical model

Numerical solution

![Graphs showing numerical solution](image)
Bio-chemical model

Numerical solution

- Problems related to the waste settlement
  - Differential settlement of the capping system
  - Increase the shear stress on the confining system along the slope
  - Additional friction stress on the well tubing (leachate and biogas collection)

- Advantages
  - Optimization of the final waste height (maximization of the landfill capacity)
Table of contents

1. Municipal Waste Disposal context
2. Hydraulic model
5. Bio-Chemo-Thermo-Hydraulic Mechanical model
Bio-chemical-Thermo-Hydro-mechanical model

Finite element formulation

- 8-noded isoparametric finite element

- 6 d.o.f: x, y, \(p_w\), \(p_g\), T, c (4 nodes)
Finite element formulation

- 8-noded isoparametric finite element

\[
F_{L,1}^{\text{Int}} = \sum_{IP} \left( \sigma_{11} \frac{\partial N_L}{\partial x_j} + \sigma_{12} \frac{\partial N_L}{\partial x_j} \right) h |J| W_{IP}
\]

\[
F_{L,2}^{\text{Int}} = \sum_{IP} \left( \sigma_{12} \frac{\partial N_L}{\partial x_j} + \sigma_{22} \frac{\partial N_L}{\partial x_j} \right) h |J| W_{IP}
\]

\[
F_{L,pw}^{\text{Int}} = \sum_{IP} S_w^i N_L - \left( V_{w1} \frac{\partial N_L}{\partial x_1} + V_{w2} \frac{\partial N_L}{\partial x_2} \right) h |J| W_{IP}
\]

\[
F_{L,T}^{\text{Int}} = \sum_{IP} S_T^i N_L - \left( V_{T1} \frac{\partial N_L}{\partial x_1} + V_{T2} \frac{\partial N_L}{\partial x_2} \right) h |J| W_{IP}
\]

\[
F_{L,c}^{\text{Int}} = \sum_{IP} S_c^i N_L - \left( V_{c1} \frac{\partial N_L}{\partial x_1} + V_{c2} \frac{\partial N_L}{\partial x_2} \right) h |J| W_{IP}
\]
Bio-chemical-Thermo-Hydro-mechanical model

Finite element formulation

- 8-noded isoparametric finite element

\[
\begin{bmatrix}
K_{MM} & K_{WM} & K_{TM} & K_{CM} \\
K_{MW} & K_{WW} & K_{TW} & K_{CW} \\
K_{MT} & K_{WT} & K_{TT} & K_{CT} \\
K_{MC} & K_{WC} & K_{TC} & K_{CC}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial p_w} & \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial c} \\
\frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial p_w} & \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial c} \\
\frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial p_w} & \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial c} \\
\frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial p_w} & \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial c} \\
\frac{\partial F_p}{\partial x_1} & \frac{\partial F_p}{\partial x_2} & \frac{\partial F_p}{\partial p_w} & \frac{\partial F_p}{\partial T} & \frac{\partial F_p}{\partial c} \\
\frac{\partial F_p}{\partial x_1} & \frac{\partial F_p}{\partial x_2} & \frac{\partial F_p}{\partial p_w} & \frac{\partial F_p}{\partial T} & \frac{\partial F_p}{\partial c} \\
\frac{\partial F_c}{\partial x_1} & \frac{\partial F_c}{\partial x_2} & \frac{\partial F_c}{\partial p_w} & \frac{\partial F_c}{\partial T} & \frac{\partial F_c}{\partial c} \\
\frac{\partial F_c}{\partial x_1} & \frac{\partial F_c}{\partial x_2} & \frac{\partial F_c}{\partial p_w} & \frac{\partial F_c}{\partial T} & \frac{\partial F_c}{\partial c}
\end{bmatrix}
\]
Conclusions

- Each individual phenomenon has been studied
- Concerning the coupling:
  
  ✓ Biodegradation controlled by the water saturation
  ✓ Temperature increase induced by the biodegradation
  ✓ Mechanical compaction related to biodegradation

- But some additional coupling are not taken into account
  
  ✓ The permeability is reduced by the compaction
  ✓ The biodegradation depends on the temperature