
Numerical modelling of a Municipal Waste Disposal as a Bio- Chemo- Thermo- Hydro-Mechanical problem

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This paper considers the Municipal Solid Waste (MSW) as a multi-physics porous medium, where Bio-Chemo-Thermo-Hydro-Mechanical phenomena have a dominant effect on the long term behavior. Considering MSW in a bioreactor landfill provides a perfect application for coupled and multiphysics phenomena. A two-stage anaerobic biochemical model based on McDougall's formulation is considered accounting for the progressive degradation of the organic matter. In presence of water, this latter decomposition is an exothermic reaction leading to an increase of the temperature, a generation of by-products as gases and chemical species, and finally compaction of the waste. The proposed model couples McDougall's formulation with an unsaturated flow model, a thermal model including a source term for heat generation from the biodegradation of organic matter and finally a mechanical model. As proposed by [Hue97], the constitutive law is a modified Camclay model allowing biochemical hardening/softening. The fully coupled model is implemented into the LAGAMINE multi-physics finite element code. Numerical simulations are performed to study the couplings between all the phenomena and to propose a prediction for the long-term settlement of a bioreactor landfill. The first part of this paper introduces the main features of the BCHTM model. The second part deals with its application to the fully coupled modelling of a 1D column of waste. Each physical phenomenon is introduced sequentially in order to understand its effect on the evolution of the waste column. Analytical solutions are provided for each simplified physical problem in order to validate the numerical results and to isolate the influence of the main parameters.

1 Introduction

The need for more efficient landfill management has led to the development of new generation "bioreactor landfills" [RMT02]. They are characterized by water injection or recirculation of generated leachate to accelerate the biodegradation of the organic fraction [KCM11]. It leads to rapid stabilization of waste, better control over biogas production, gain of landfill space and overall shorter and cheaper monitoring and maintenance [RAY96, Bea00, YWSM01]. One of the keys to properly operate bioreactor landfills is the ability to accurately predict the settlements. It is a great challenge because of the inherent complexity of the landfill system. The whole process is governed by the biodegradation of organic waste coupled with thermal, hydraulic and mechanical phenomena [McD07, CZL14]. The first models trying to predict the settlement were empirical time dependent models with or without taking into account waste degradation such as [GL61, Sow75, ER90, Oli03]. They were useful due to their simplicity but lacked the ability to accurately predict long term settlements. More recently, the focus has been brought on the necessity of integrated analysis which would include biodegradation model into "classical" mechanical and hydraulic models used in geomechanics [HMTH07, McD07, RRW11, CXZ12, WB13, CZL14]. This would allow to capture every fundamental phenomenon and merge them into multi-physics models able to better estimate long term behavior. It is, then, an absolute necessity to understand the main biochemical reactions occurring during biodegradation [PK00, AJ00, Rod05, BBH⁺10] and to translate them into usable mathematical models. There are mostly two types of biodegradation models : multi-phase degradation using Monod kinetic [EFFL96, HHO⁺01, WRR03] and two stage anaerobic digestion model [McD07, RRW11, GED08, CGLZ15]. The latter became increasingly popular because of its ability to represent the whole process in a simplified manner whereas the former requires lots of parameters whose values are not always easily determined. Most integrated models do not take into account the thermal effects on biological reactions despite important temperature variation during the decomposition of organic content. Although some [KCM11, GED08, EFFL96] consider the thermal aspect of the process, their thermo-hydro-biological models are unable to calculate landfill settlements. Also, few models [McD07] use sophisticated constitutive models within elasto-plastic framework to describe the chemo-mechanical behavior of solid waste.

The objective of this work is to present an example of multiphysical couplings in geomechanics. It aims at describing in a logical sequence how to manage each physical problem separately and how to couple it with the other phenomena. The biochemo-thermo-hydro-mechanical (BCTHM) modelling of a column of waste is a perfect case study for that purpose.

The two-stage anaerobic biodegradation model adopted by [McD07] is implemented into a fully coupled thermo-hydro-mechanical framework for unsaturated porous media, which has been developed in the LAGAMINE code over the last three decades [Cha07, Col03]. Details of the implementation as well as the main parameters can be found in [HLC15]. The biodegradation model is linked to both the governing mass balance equations (for VFA and methanogen biomass) and the energy balance equa-

tion through source term. The mechanical model adopted is a simplified version of the chemo-hydro-mechanical (CHM) model presented by [LBL⁺05]. The physical phenomena are considered sequentially, from the simple flow model to the fully coupled BC-THM model. Numerical results are presented and compared with a closed-form solution of each independent problem. This aims at both validating the results and emphasizing the importance of the main parameters. This procedure provides a better understanding of the physical phenomena and their couplings.

2 BC-THM behavior of Municipal Waste

Municipal wastes are porous media, where highly coupled multiphysical phenomena take place. In addition to the hydromechanical behavior classically observed in geomaterials, bio-chemical processes responsible for the organic matters degradation make the coupled behavior of the waste much more complex. In the following, the model formulation is described for the different physical phenomena.

2.1 Bio-chemical model

The microbiological activity within the landfills is responsible for the mineralization of the organic content and the production of biogas. This process modifies the hydromechanical properties of the waste and has to be considered in a detailed analysis of the MSW long term behavior. The biodegradation can be split into two main stages [Rod05], which are briefly described in the following sections.

Aerobic stage The aerobic phase is the first step of the biodegradation and begins just after the wastes are landfilled. It lasts at most a few weeks since the deposit of subsequent layers of waste will deprive the previous ones of any oxygen and will cut the aerobic stage short. During this process, the organic content (*Org*) is degraded into macromolecules by bacteria. It is a very exothermal reaction leading to an important temperature raise sometime reaching over 60 °C.

Anaerobic stage The anaerobic stage begins as soon as the aerobic one ends. It can last up to 40 or 50 years. This stage consists of the four reactions defined below [Rod05]

- Hydrolysis: the macromolecules are decomposed by hydrolytic bacteria into smaller molecules (lipid into fatty acid ; polysaccharide into monosaccharide ; protein into amino acid).
- Acidogenesis: the products of the hydrolysis are transformed into ethanol, organic acid and Volatile Fatty Acid (*VFA*).

- Acetogenesis: the products of the hydrolysis are consumed and transformed into acetyl acid and hydrogen.
- Methanogenesis: during this last step, the acetyl acid is consumed to produce carbon dioxide and the hydrogen is consumed to produce first methanogen biomass in a liquid phase that will transform into methanogen biogas and carbon dioxide in a second step.

The two-stage biochemical model described by [McD07] is adopted here to describe the hydrolysis/acidogenesis and methanogenesis reactions. McDougall's formulation neglects the aerobic stage but since the aerobic waste decomposition represents a minor part of the landfill lifetime it is, therefore, less significant than anaerobic decomposition [ZB04]. This formulation is used to determine the growth/decay term for the internal variables characterizing the biodegradation,

- Org [$kg.m^{-3}$] the organic content,
- c [$g.m^{-3}$] the VFA concentration in water,
- m [$g.m^{-3}$] the methanogen biomass in water.

McDougall's model provides the formulation of the biochemical model and the governing mass balance equations for these three chemical species, presented in the following sections. These equations describe the reaction rate and are expressed in ($g.m_{aqueous}^{-3}.s^{-1}$). This unit highlights the dependency of the reaction on the moisture content.

2.1.1 Hydrolysis and acidogenesis

Hydrolysis and acidogenesis is the first stage of the biodegradation, which represents the depletion of the organic content and its transformation into VFA . These latter intermediate products serve as a precursor for methanogen biomass. However, high VFA concentration has inhibitory effects on those reactions [GED08], which is also taken into account in the model through an inhibitor factor.

The modified enzymatic hydrolysis equation is proposed by [McD07] and mathematically describes the reaction rate

$$r_g = b\theta_e\phi P, \quad (1)$$

where four governing factors are taken into account

- $\theta_e = (\theta - \theta_{res})/(\theta_{sat} - \theta_{res})$ [-] is the effective water content (in volume, $\theta = n.S_{r,w}$), θ_{res} [-] is the residual water content and θ_{sat} [-] is the water content at saturation.
- b [$g.m_{aqueous}^{-3}.s^{-1}$] is the maximum VFA growth rate under the most favorable environmental conditions, which normally occurs at the early stage of hydrolysis reaction.

- $\phi = 1 - (1 - Org/Org_0)^\xi$ [-] is the relative digestibility decreasing with the organic matter depletion, where Org_0 [$kg.m^{-3}$] is the initial organic content and ξ [-] is a parameter.
- $P = \exp(-k_{VFA}.c)$ [-] is the inhibition factor accounting for the inhibitory effect of high VFA concentration, in which k_{VFA} [$g.m^{-3}$] $^{-1}$] is an inhibition constant.

2.1.2 Acetogenesis and methanogenesis

The second stage of the biochemical reactions occurring in the MSW transforms the VFA generated from the hydrolysis/acidogenesis reactions to methanogen biomass. Note that the biogas is not taken into account in current model for the sake of simplicity. The methanogen biomass production rate r_j is calculated through a Monod kinetic equation [BC10] and the VFA consumption rate r_h is directly linked to the methanogen biomass accumulation through a substrate yield coefficient Y ,

$$r_j = \frac{k_0.c/\theta}{k_{MC} + c/\theta} \cdot \frac{m}{\theta}, \quad (2)$$

$$r_h = r_j/Y, \quad (3)$$

where k_0 [s^{-1}] is the maximum specific growth rate, k_{MC} [$g.m_{aqueous}^{-3}$] is the half saturation constant.

2.1.3 Methanogen decay

The methanogen biomass decay over time r_k is given by the first order decay equation as follow:

$$r_k = k_2 \cdot \frac{m}{\theta}, \quad (4)$$

where k_2 [s^{-1}] is the methanogen death coefficient.

2.1.4 Governing balance equations

The degradation rate of organic matter and accumulation rate of VFA and methanogen biomass can be incorporated as sink or source terms into classical advection-dispersion equations. They lead to advection-dispersion-reactive transport models, respectively written per m^3 of waste, for VFA (c), methanogen biomass (m) and organic matter (Org),

$$\text{div}(\underline{u}.c) - \text{div}\left(\underline{\underline{D}}_h \cdot \nabla c\right) + [r_g - r_h].\theta = \frac{\partial c}{\partial t}, \quad (5)$$

$$\text{div}(\underline{u}.m) - \text{div}\left(\underline{\underline{D}}_h \cdot \nabla m\right) + [r_j - r_k] \cdot \theta = \frac{\partial m}{\partial t}, \quad (6)$$

$$-Z.r_g \cdot \theta = \frac{\partial Org}{\partial t}, \quad (7)$$

where $\underline{u} = \underline{q}_l / (S_{r,w} \cdot n_e)$ is the actual average velocity of the liquid flow, $S_{r,w}$ [-] is the water saturation degree, n_e [-] is the effective porosity, \underline{q}_l is the water Darcy's velocity and Z [-] a substrate yield coefficient.

On the left side of equations (5) and (6), the first terms represent advective flux, linked to the actual average velocity of liquid flow \underline{u} . The second terms describe diffusive flux, which combines mechanical dispersion and molecular diffusion and the thirds are the source terms describing the generation or degradation of *VFA* or methanogen biomass. The terms on the right side are the storage term of *VFA* and methanogen biomass, respectively. Mass balance equation for organic matter (7) does not include any transport terms because the organic matter is considered as a part of solid skeleton and no erosion of organic matter occurs during the leachate recirculation or water injection.

It is worth noticing that the above reactive transport models were derived with no consideration of immobile water phase. These models are more suitable for the MSW with low organic matter content while not for the MSW with high organics matter content.

2.2 Hydraulic model

In municipal waste disposal, the materials are under unsaturated conditions and the temperature increases generated by the waste decomposition induce production of water vapour. Each fluid phase of the medium (liquid and gaseous) constitutes a mixture of two components, which are dry air and water vapour for the gas phase and liquid water and dissolved air for the liquid phase.

The variables chosen for the description of the flow problem are liquid water pressure, gas pressure and temperature. As a first approach and for sake of simplicity, the gas pressure variations are not considered in the following. That is the reason why the gas mass balance equation will not be expressed.

2.2.1 Water mass balance equation

The compositional approach [PC89, OCGA94, LS87] is used here to write balance equations, *i.e.* we assume that the conservation mass of each chemical species (water and air). The phase exchange term is cancelled in balance equations,

$$\underbrace{\frac{\partial}{\partial t} (\rho_w \cdot n \cdot S_{r,w}) + \text{div} (\underline{f}_w)}_{\text{Liquid water}} + \underbrace{\frac{\partial}{\partial t} (\rho_v \cdot n \cdot S_{r,g}) + \text{div} (\underline{f}_v)}_{\text{Water vapour}} - Q_w = 0 \quad (8)$$

where n is the porosity of the medium, ρ_w is the liquid water density, ρ_v is the water vapour density, \underline{f}_w et \underline{f}_v are the mass flows respectively for water and water vapour, $S_{r,g} = 1 - S_{r,w}$ is the gas saturation degree and Q_w is water source term.

The mass flows have two contributions, taking into account the advection of each phase (Darcian flow) and the diffusion of the different components within the phase (Fickian flow). Given the small amount of dissolved gas, liquid water diffusion within the liquid phase will be neglected.

The mass flows used in equation (8) are expressed as

$$\underline{f}_w = \rho_w \cdot \underline{q}_l, \quad (9)$$

$$\underline{f}_v = \rho_v \cdot \underline{q}_g + \underline{i}_v, \quad (10)$$

where \underline{q}_l et \underline{q}_g are the advection flow of the liquid and gas phases, \underline{i}_v is the diffusive flow of water vapour within the gaseous phase. The expression of each terms are described in the following.

2.2.2 Advection of the liquid phase

In unsaturated conditions, Darcy's law remains valid provided that the permeability is modified as a function of the saturation degree. The water permeability is usually expressed as the product of the intrinsic permeability K_{int}^{sat} (measured in saturated conditions) by a relative permeability coefficient $k_{r,w}$ depending on the water saturation degree:

$$\underline{K}_w(S_{r,w}) = \underline{K}_{int}^{sat} \cdot k_{r,w}(S_{r,w}). \quad (11)$$

The generalized Darcy's law becomes

$$\underline{q}_l = \frac{-\underline{K}_{int}^{sat} \cdot k_{r,w}(S_{r,w})}{\mu_w} [\underline{grad}(p_w) + g \cdot \rho_w \cdot \underline{grad}(z)], \quad (12)$$

where $\underline{K}_{int}^{sat}$ is the intrinsic permeability tensor (independent from the nature of the fluid), $k_{r,w}$, the water relative permeability and μ_w , the water dynamic viscosity.

It is worth noting that in equation (12), the fluid properties would have been those of the liquid phase (mixture of liquid water and dissolved air), and not those of liquid water. However, given the small amount of dissolved gas, its influence on the liquid viscosity and the liquid density is neglected.

2.2.3 Diffusion within the gaseous phase

The gas phase is a mixture of water vapour and dry air. Even if gas phase flows are neglected, a binary diffusion of each component within the phase is possible. Thus, the diffusive flows of water vapour and dry air in the gas phase are given by the Fick's law following

$$\dot{i}_v = -n \cdot (1 - S_{r,w}) \cdot \tau \cdot D_{v/a} \cdot \rho_a \quad (13)$$

where $D_{v/a}$ is the diffusion coefficient of water vapour in dry air.

2.3 Thermal Model

Due to the exothermal nature of the early stage of biodegradation reactions (so-called aerobic reactions) and the general heat transfer associated with boundary conditions, a significant temperature increase normally takes place in bioreactor landfills [BBLR07]. In order to simulate the temperature evolution within the landfill, a source term related to the heat generation due to the biodegradation reaction is introduced into a classic governing energy balance equation, leading to the following heat transport equation

$$\frac{\partial S_T}{\partial t} + \text{div}(V_T) - Q_T = 0, \quad (14)$$

where S_T is the heat storage, V_T is the heat flux and Q_T is the heat production term.

In equation (14), the heat storage term is given by the sum of each components contribution, as follows

$$\begin{aligned} S_T = & n \cdot S_{r,w} \cdot \rho_w c_{p,w} \cdot (T - T_0) + n \cdot S_{r,g} \cdot \rho_a c_{p,a} (T - T_0) \\ & + (1 - n) \rho_s c_{p,s} (T - T_0) + n \cdot S_{r,g} \cdot \rho_v c_{p,v} (T - T_0) \\ & + n \cdot S_{r,g} \cdot \rho_v \cdot L \end{aligned} \quad (15)$$

in which $c_{p,i}$ [$J.kg^{-1}.K^{-1}$] is specific heat of the component i , ρ_s [$kg.m^{-3}$] is the solid waste density, ρ_a [$kg.m^{-3}$] is the dry air density, L [$J.kg^{-1}$] is the latent heat of water vaporization, T_0 is the initial temperature and T is the temperature.

The heat flux consists of a conduction term proportional to the thermal conductivity of the MSW and a convective term related to the heat transported by water flows

$$\begin{aligned} V_T = & -\Gamma \nabla T + c_{p,w} \rho_w \underline{q}_l (T - T_0) + c_{p,v} \left(\rho_v \cdot \underline{q}_g + \dot{i}_v \right) (T - T_0) \\ & + \left(\rho_v \cdot \underline{q}_g + \dot{i}_v \right) L \end{aligned} \quad (16)$$

where Γ [$W.kg^{-1}.K^{-1}$] is the thermal conductivity of the waste. The thermal conductivity of the MSW is estimated by summing the thermal conductivities of different phase components of the MSW, including the water, air and the solid phase.

The heat production term (Q_T) is derived by empirical consideration of the energy release from exothermal biochemical reactions occurring in the MSW, which is similar to the formulation proposed by [EFFL96],

$$Q_T = \frac{\partial Org(t)}{\partial t} H_m, \quad (17)$$

where H_m [J/kg] is the quantity of heat produced by the degradation of one kilogram of organic matter.

2.4 Mechanical model

McDougall's work suggests that the total settlement can be calculated based on three contributions: elastic and plastic load induced strain, creep and biodegradation strain. It is proposed to implement these contributions (except creep) within a simplified version of the chemo-hydro-mechanical (CHM) model presented by [LBL⁺05] which is based on previous works of [Hue92, Hue97, HLG02].

The CHM model has been originally developed to simulate the behavior of unsaturated clay in presence of chemicals in the pore fluid. A concentration parameter (Ω [-]) is included in the model to express the modification of properties with chemicals as well as computing the (elastic or plastic) strains induced by concentration changes. In this study, the concentration parameter is related to organic matter content such that

$$\Omega = 1 - \frac{Org}{Org_0}. \quad (18)$$

The lowest the organic matter content, the lowest the strength of the waste.

The constitutive model is thus written in terms of effective stress tensor and the concentration parameter. Bishop's effective stress has been chosen to describe the stress-strain relation

$$\sigma'_{ij} = \sigma_{ij} - p_g \delta_{ij} + S_{r,w} (p_g - p_w) \delta_{ij}, \quad (19)$$

where σ'_{ij} is the effective stress tensor, σ_{ij} is the total stress tensor, p_g is the gas pressure (constant and equal to the atmospheric pressure) and δ_{ij} is Kronecker's tensor.

2.4.1 Constitutive equations

The equations relate the strain to the stress and the organic content - through the concentration factor, Ω . The strain rate is the sum of an elastic reversible part and a plastic irreversible part. The elastic part is also decomposed into mechanical and chemical components.

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p = \dot{\epsilon}_{ij}^{(e,m)} + \dot{\epsilon}_{ij}^{(e,\Omega)} + \dot{\epsilon}_{ij}^{(p,m)} \quad (20)$$

The three main strain components are presented as follows

1. The elastic strain-stress law is a classical Hooke's law;
2. The chemical elastic strain is defined according to the formulation in [Hue97]

$$\dot{\epsilon}_{ij}^{(e,c)} = -1/3\beta\dot{\Omega}\delta_{ij}, \quad (21)$$

where β [-] is a parameter depending on the waste and the concentration;

3. The plastic strain rate is defined within a classical elastoplastic framework.

Three plastic yielding mechanisms are implemented into the chemo-hydro-mechanical (CHM) model by [LBL⁺05]: pore collapse, frictional-cohesive failure and tensile failure. They are represented respectively by the following equations

$$f_1 \equiv q^2 + M^2(p + p_s)(p - p_0) = 0, p \geq (p_0 - p_s)/2 \quad (22)$$

$$f_2 \equiv q - M(p - p_s) = 0, \sigma_t < p \leq (p_0 - p_s)/2 \quad (23)$$

$$f_3 \equiv p + \sigma_t = 0, \quad (24)$$

where p_0 [Pa] is the pre-consolidation pressure, p_s [Pa] is a parameter related to the cohesion, M [-] is a parameter defining the slope of the frictional cohesive failure in the deviatoric plane, σ_t [Pa] is the limit tensile strength, $p = \sigma_{kk}/3$ and $q = \sqrt{3/2}s_{ij}s_{ij}$ are the stress invariants and s_{ij} is the deviatoric part of the stress tensor.

2.4.2 Chemical hardening/softening

The large deformation due to the biodegradation of organic matter fraction of MSW is considered in our proposed constitutive model by introducing chemical softening, controlled by the concentration parameter (Ω). This parameter varies from zero to one with degradation of the organic matter from initial content to zero.

This concentration parameter (Ω) influences both the pre-consolidation pressure and the cohesion parameter in order to describe the biodegradation effect on pore collapse mechanism and frictional-cohesive failure mechanism, respectively.

1. Effect on pore collapse mechanism: the preconsolidation pressure is a decreasing function of the concentration parameter,

$$p_0(\Omega) = p_0^*S(\Omega), \quad (25)$$

where p_0^* is the pre-consolidation pressure for initial organic content ($\Omega = 0$) and $S(\Omega) = \exp(-a\Omega)$ is the chemical softening function, where a [-] stands for a constant governing the decrease of the pre-consolidation pressure with the increase in concentration parameter.

2. Effect on frictional cohesive failure mechanism: the parameter p_s [Pa], controlling the cohesion, is assumed to vary as a linear function of the concentration parameter (Ω)

$$p_s = p_s^* + k_\Omega\Omega \quad (26)$$

where p_s^* [Pa] is the value of the parameter for initial organic content ($\Omega=0$) and k_Ω is a model constant.

3 Application of the BC-THM model to a municipal waste disposal

The multiphysical processes occurring during the lifetime of a municipal waste disposal are simplified into a one-dimensional problem. Initial and boundary conditions of a 30 meter high column are depicted in Figure 1. A one meter thick drain is located at the base of the column.

In the sequel, the different physical processes are introduced progressively, highlighting their impact and respective couplings. At each step, a simplified analytical solution is proposed in order to emphasize the main parameters describing the physical phenomena. This solution is compared with the fully coupled numerical computations obtained from the implementation of the model previously described in LAGAMINE [CLRC02, HLC15].

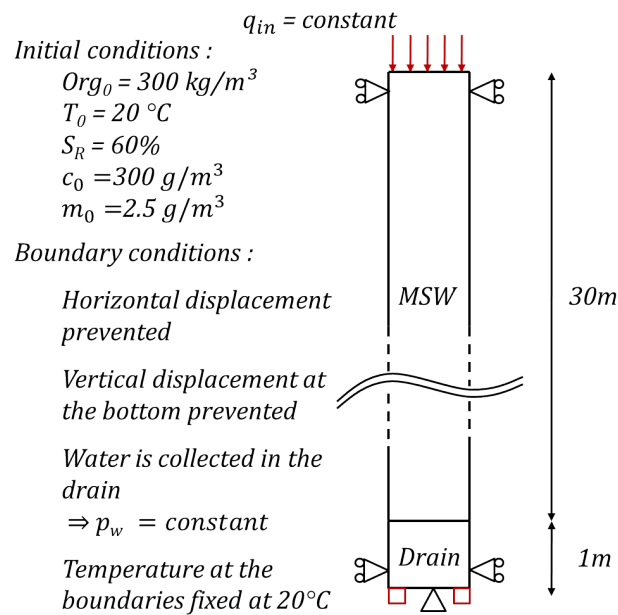


Figure 1: Initial and boundary conditions

3.1 Flow model

During the exploitation period of the disposal, a water influx q_{in} is imposed at the top boundary (negative in case of injection). This flow is either a rainfall infiltration rate

or a water injection rate in case of active management of the disposal as a bioreactor. For the sake of simplicity, the retention curve is defined as

$$S_{r,w} = \exp\left(\frac{-p_c}{4A}\right) \leq 1, \quad (27)$$

where $p_c = p_g - p_w$ is the capillary pressure. In turn the relative permeability is defined as

$$k_{r,w} = (S_{r,w})^4. \quad (28)$$

3.1.1 Hydraulic analytical approach

It is proposed to provide a closed-form solution of the water mass balance equation (8) for steady-state conditions. Considering an incompressible fluid and a uniform temperature of 20 °C (the water vapour contribution can be neglected), this equation is rewritten in one dimension as

$$\frac{\partial}{\partial z} (q_{l,z} \rho_w) = 0. \quad (29)$$

The obtained solution imposes that $q_{l,z} \rho_w = q_{in}$, which is constant all along the waste column since the considered solution is stationary.

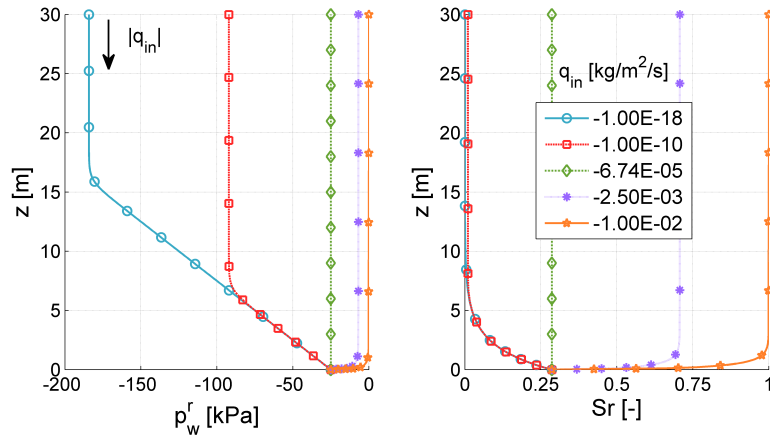


Figure 2: Profile of relative water pressure (a) and saturation (b) as a function of incoming water flow $|q_{in}|$ (Analytical solution).

Injecting equations (12) and (28) into equation (29) leads to

$$\frac{\partial p_w^r}{\partial z} + \frac{q_{in} \mu_w}{\rho_w K_{int}^{sat}} \exp\left(-\frac{p_w^r}{A}\right) = -\rho_w g, \quad (30)$$

where $p_w^r = -p_c$ is the relative water pressure. Substituting the expression $j = \exp(-p_w^r/A)$ into equation (30) reads

$$\frac{\partial j}{\partial z} = \beta j^2 + \gamma j \quad (31)$$

where $\beta = (q_{in} \mu_w)/(\rho_w K_{int}^{sat} A)$ and $\gamma = \rho_w g/A$ which has the form of the Verhulst equation. It is solved by the substitution $u = 1/j$ and finally provides the profile of relative water pressure

$$p_w^r(z) = A \ln \left[C_1 \exp(-\gamma z) - \frac{\beta}{\gamma} \right], \quad (32)$$

where C_1 is an integration constant determined from the boundary condition, p_0^r in $z = 0$, yielding to

$$C_1 = \frac{\beta}{\gamma} + \exp\left(\frac{p_0^r}{A}\right). \quad (33)$$

The profile of relative water pressure and saturation degree along the soil column are provided in Figure 2. It illustrates the influence of water injection at the top of the column.

3.1.2 Hydraulic numerical approach

The numerical model described in section 2.2 considers the transient response of the waste, as well as the contribution of the water vapour. Figure 3 shows that the steady-state is actually reached after 30 days and corresponds to the analytical solution. Therefore it can be concluded that the water vapour phase is negligible at 20°C.

3.2 Bio-Chemo-Hydraulic model

The evolution of the VFA (c), the methanogen biomass (m) and the organic matter (Org) are linked to the flow problem since the water content directly controls the biochemical reactions. This introduces a first coupling and the column waste disposal is considered as a bio-reactor described in section 2.1.

3.2.1 BC-H Analytical approach

The analytical approach focuses only on the degradation of the organic matter since it is coupled with the thermal and mechanical problems. The equation of the decomposition of the organic content reads

$$\frac{\partial org}{\partial t} = -\theta Z b \theta_e \exp(-k_{vfa} c) \left[1 - \left(\frac{org_0 - org}{org_0} \right)^\xi \right], \quad (34)$$

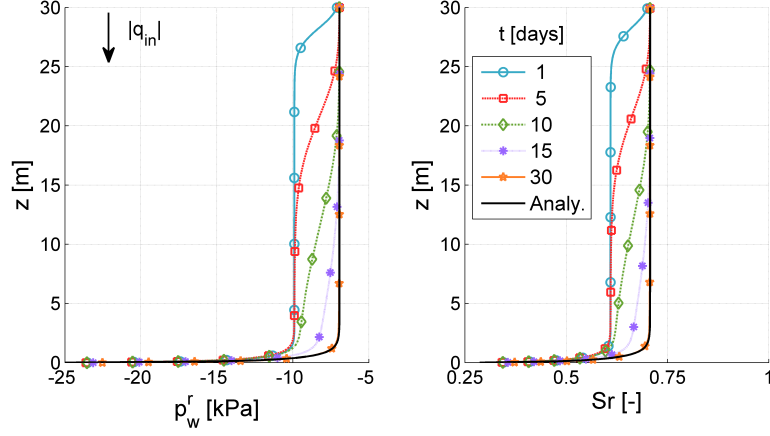


Figure 3: Profile of water pressure (a), p_w , and saturation degree (b) at different time steps (Numerical results).

where variable c is constant and θ is equal to its stationary value. The equation (34) is recast as a function of the concentration variable Ω

$$\frac{\partial \Omega}{\partial t} = C (1 - \Omega^\xi), \quad (35)$$

where $C = \theta Z b \theta_e \exp(-k_{vfa} c) / \text{org}_0$ is a time constant. This equation is solved using Mathematica leading to the approximated series solution

$$t = \frac{\Omega}{C} \sum_{n=1}^{\infty} \frac{(1)_n \left(\frac{1}{\xi}\right)_n \Omega^{n\xi}}{\left(1 + \frac{1}{\xi}\right)_n n!}, \quad (36)$$

where t is the time variable and

$$(x)_n = x(x+1)(x+2)\dots(x+n-1). \quad (37)$$

Figure 4(b) illustrates the influence of the C constant on the evolution of Ω . This parameter is mainly a function of the water content and maximal degradation rate b . It explains why the degradation can spend over several dozen of years.

3.2.2 BC-H Numerical approach

Numerical results take into account the full bio-chemical couplings equations (5)-(7) and a transport term linked to the water flow. Figure 4(a) depicts the evolution of

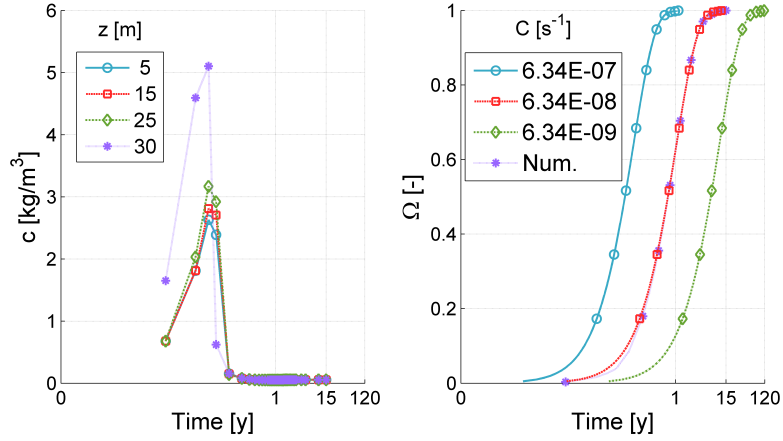


Figure 4: (a) Time evolution of the *VFA* at different depth (Numerical results) and (b) concentration variable Ω for different C values (Analytical results).

the *VFA* concentration with time. The *VFA* is a product of the first stage reaction and is simultaneously consumed by the second stage reaction generating methanogen biomass. At first, the *VFA* concentration increases quite a lot because of the injected water inducing important organic matter degradation and thus *VFA* production. Then, the *VFA* consumption dominates its production due to the high value of *VFA* concentration reached, leading to an important inhibitory effect on the depletion of organic matter and in turn of the *VFA* production. Consequently, it quickly decreases to an equilibrium state, where rates on *VFA* production and consumption are equal, as shown by [McD07]. This is confirmed by the profile of the *VFA* concentration at different times depicted in Figure 5(a).

Figure 5(b) exhibits an almost uniform organic matter degradation. It is slightly slower near the drain layer, where the water saturation degree is lower. The analytical solution well captures the numerical results, as shown in Figure 4(b). Indeed the flow and *VFA* concentration are stationary during the main lifetime of the waste disposal. Therefore approximating a constant *VFA* concentration and water content is a good approximation.

3.3 Bio-Chemo-Thermo-Hydraulic model

The first coupling is due to the exothermic nature of the bio-chemical reactions. Therefore the temperature within the waste disposal varies with the bio-degradation processes. The second coupling arises from the dependency on the bio-chemical reactions to the water saturation degree. Finally the heat produced is transferred either by thermal conduction or flow convection. One coupling effect has been neglected in our

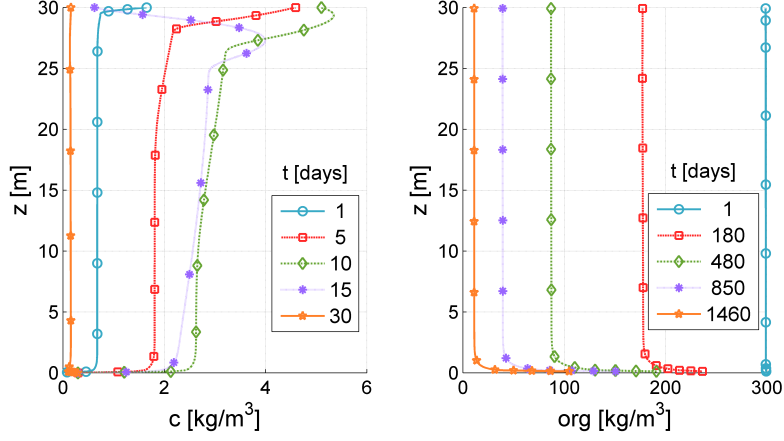


Figure 5: Profile of the *VFA* concentration and the organic matter content (Numerical results).

formulation: the BC reaction rates do not depend on the temperature.

3.3.1 BC-TH Analytical approach

The general heat balance equation (14) is simplified, discarding the convection terms and the water vapour contribution. The one-dimensional equations is then written as

$$\frac{\partial T(z, t)}{\partial t} - \alpha \frac{\partial^2 T(z, t)}{\partial z^2} = Q(z, t), \quad (38)$$

where $T(z, t)$ is the temperature, α [m^2/s] is the thermal diffusivity and $Q(z, t)$ is a source term.

Following equation (17), the heat production is related to the degradation rate of the organic matter. Assuming a time decreasing exponential function of the organic matter, ($\Omega = \exp(-\zeta t)$), the source term is expressed as follows

$$Q(z, t) = \underbrace{\frac{H_m \zeta}{\rho c}}_{\delta} \exp(-\zeta t). \quad (39)$$

Let us assumed a column of soil at the initial temperature T_0 of 20 degrees, the final solution reads

$$T(z, t) = T_0 + \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L} z\right) \int_0^t B_n(s) \exp(-\alpha \lambda_n (t - s)) ds, \quad (40)$$

where $\lambda_n = (n\pi/L)^2$ and

$$B_n(s) = \frac{2}{L} \int_0^L \delta \exp(-\zeta s) \sin\left(\frac{n\pi}{L}z\right) dz \quad (41)$$

$$= \begin{cases} \frac{4\delta}{n\pi} \exp(-\zeta s) & n \text{ odd} \\ 0 & n \text{ even} \end{cases} \quad (42)$$

Introducing equation (42) into (40) leads to

$$T(z, t) = T_0 + \sum_{n=1,2}^{\infty} \frac{4\delta}{n\pi} \frac{1}{-\zeta + \alpha\lambda_n} \sin\left(\frac{n\pi}{L}z\right) [\exp(-\zeta t) - \exp(-\alpha\lambda_n t)] \quad (43)$$

Observed profiles of temperature result from the competition of two distinct effects: heat generation and heat diffusion. It is clear in Figure 6 that the heating is fast with respect to the heat diffusivity. Therefore the temperature evolves almost constantly over most part of the waste column. Afterwards, heat is progressively dissipated through the upper and lower boundaries.

3.3.2 BC-TH Numerical approach

Numerical model for heat transfer enables us to study the influence of different effects: heat generation, heat diffusion and heat convection. In order to evidence their influence, three numerical simulations are performed. The water vapour is shown to have no influence on the results and is not considered. In the following, *Case1* corresponds to the modelling without convection (equivalent to the analytical solution). *Case2* and *Case3* refer to the solution of the problem taking into account heat convection for two distinct water flows, respectively q_{in} and $q_{in}/100$.

Figure 6 depicts that the analytical solution well captures numerical results for mean and long term predictions. The observed short-term discrepancies are related to the assumption we took on the evolution of the organic matter as a decreasing exponential function.

Case2 and *Case3* exhibit the influence of the convection. In the first case the temperature does not increase significantly because the heat loss due to convection is high at the bottom of the column. In the second case, the temperature profile becomes progressively non symmetric (and non-parabolic) due to convection.

3.4 Bio-Chemo-Thermo-Hydraulic-Mechanical model

The biodegradation reactions degrade the properties of the waste: the apparent pre-consolidation pressure is a decreasing function of the concentration variable Ω . The

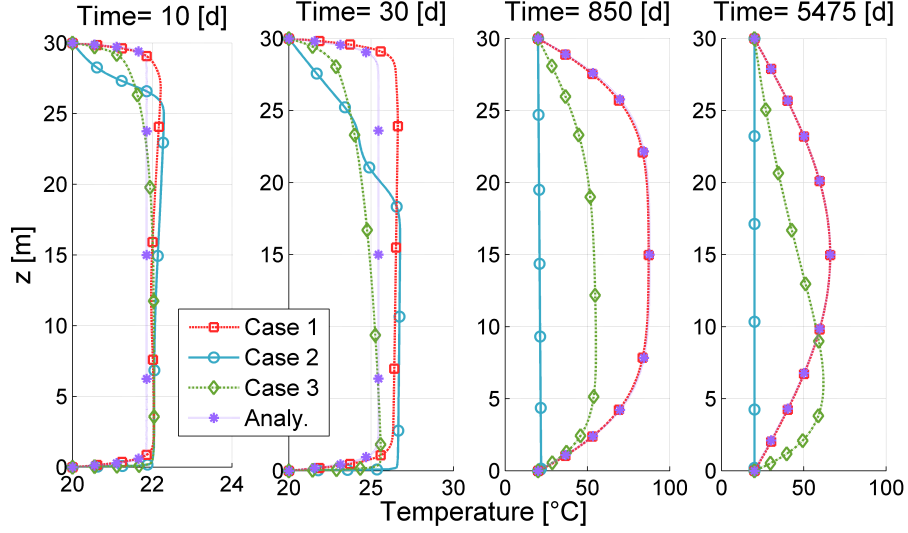


Figure 6: Profile of temperature for different time steps (Analytical and numerical results).

biochemical reactions firstly lead to a chemical softening of the waste and secondly to a mechanical hardening, as far as the stress state remains constant. Therefore the increase of the concentration variable induces a plastic compaction of the waste column.

3.4.1 BC-THM Analytical approach

First, we only consider the plastic volumetric deformation due to the bio-chemical couplings, in the frame of the constitutive model described in section 2.4. Under the assumption of a constant effective stress state, the consistency condition reads

$$\frac{\partial f}{\partial \Omega} . d\Omega + \frac{\partial f}{\partial p_0^*} dp_0^* = 0. \quad (44)$$

In addition, the hardening function in a CamClay model is classically defined as

$$dp_0^* = \frac{1 + e_0}{\lambda - \kappa} p_0^* d\epsilon_v^p. \quad (45)$$

Combining these two latter equations with equation (22), the relation between the plastic volumetric strain and the variation of the concentration variable is written as

$$d\epsilon_v^p = \frac{\lambda - \kappa}{1 + e_0} (-a) d\Omega \quad (46)$$

where a is the constant governing the decrease of the pre-consolidation pressure with the decomposition.

3.4.2 BC-THM Numerical approach

Figure 7 presents first the evolution of the concentration variable at different locations. It is observed that the degradation is actually proportional to the water saturation degree. Indeed, the shift of the results in this figure is due to the difference of saturation with depth near the drain (low depth). The plastic volumetric strains follow the evolution of the degradation. Figure 7 shows that the analytical solution provides an upper-bound estimation of the numerical response.

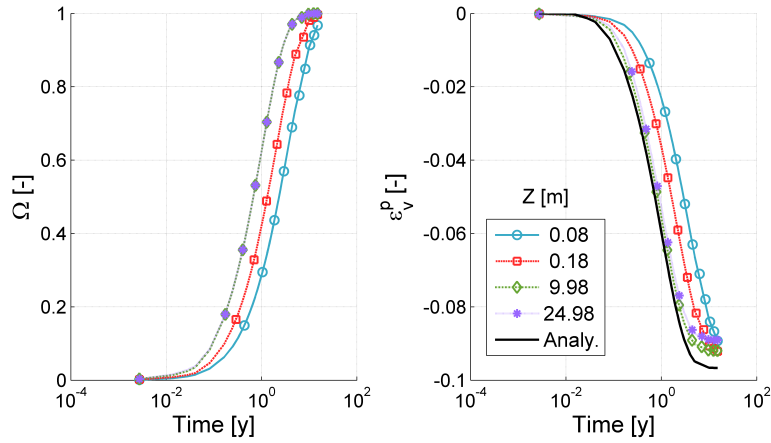


Figure 7: Profile of concentration content (a) and plastic volumetric strain (b) for different time steps (Analytical and numerical results).

4 Conclusions

This paper presents the formulation of a biochemo-thermo-hydro-mechanical model for the study and analysis of long term behavior of bioreactor landfill. The particularity of the BC-THM model formulation is the multi-physics coupling accounting for the inherent complexity of the bioreactor landfill system. The two stage anaerobic biodegradation model proposed by McDougall [McD07] is adopted as biochemical sub-model to reproduce the biodegradation of organic matter. It is incorporated into the thermo-hydro-mechanical framework of the LAGAMINE code. Furthermore, the effect of the biodegradation on mechanical behavior is taken into account using a simplified version of the chemo-mechanical model developed by [Hue97]. The hydraulic

model is based on Darcy's law for water flow in unsaturated soils. Finally the thermal model is a classical energy balance equation with a source term taking into account the heat generated by the degradation of organic matter.

Then, an application of the BC-THM model on a one-dimensional problem is presented and the different physical processes are introduced progressively, in order to highlight the impact of each process and their respective couplings. For each step, an analytical solution is proposed in order to explain the physical phenomenon.

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