Computers and Geotechnics 9 (1990) 59–72

NUMERICAL MODELLISATION OF CONTACT WITH FRICTION PHENOMENA BY THE FINITE ELEMENT METHOD

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ABSTRACT

Finite element modelling contact with friction in two-dimensional, axisymmetric and three-dimensional cases are proposed. They take in account large displacements and rotations between a strained body and a so-called tool or between two strained bodies. The interface behaviour is based on a penalty method and on the COULOMB dry friction law, and is developed using the elasto-plastic formalisms. Due to the algorithm modularity it seems to be easy to introduce other interface behaviours as instance including dilatancy. The time integration of the generalised contact stresses and the objectivity treatment are discussed. At the end an application is proposed which could serve as comparison test between algorithms modelling same problems.

INTRODUCTION

This paper is dedicated to the numerical simulation of contact with friction phenomena by the finite element method. Bodies to be studied are solids in two-dimensional, axisymmetric, or three-dimensional state. They are subjected to very large strains, large displacements and large rotations. These peculiarities are implying a special development of the contact model with regards to small displacements contact models because the contact condition are quickly evolving during a loading process.

Typical applications of the developed finite element code are rolling and forging modelling but it seems to be able to model for example piling or other large strains soil mechanics problems.

Computers and Geotechnics 0266-352X/90/$03·50 © 1990 Elsevier Science Publishers Ltd, England. Printed in Great Britain

THE FINITE ELEMENT CODE LAGAMINE

The hereafter presented contact modelling was implemented in the finite element code LAGAMINE which has been developed for 7 years by the M.S.M. department of the Université de Liège [1,2]. The solid finite elements are isoparametric, two-dimensional, axisymmetric or three-dimensional elements. The edges are of first degree (two nodes by edge) or second degree (three nodes by edges). The virtual work is numerically integrated by the Gaussian scheme in order to give the nodal forces energetically equivalent to the stresses.

A large number of constitutive laws has been implemented: elastic, elastoplastic and elastoviscoplastic VON MISES laws for metals; DRUCKER-PRAGER non-associated law and CAM-CLAY like law for soils or ductile rocks; fractured rock model; GURSON law for strain localisation in metals, etc.. Each one uses the CAUCHY stress tensor and the classical JAUMAN objective derivatives. The numerical integration of stresses is performed with division of the time step into sub-intervals. In each sub-interval a two step implicit and second order accurate scheme is used.

CONTACT WITH FRICTION FINITE ELEMENT

The easiest way to implement contact model in a classical finite element code is the development of a special "contact finite element" as an interface element. This element must be compatible with the solid elements. Therefore it is an isoparametric element with first or second degree edges (figure 1).

Contact implies two bodies. For a large number of applications one of them is quite rigid with regards to the other (for example tool or rolling cylinder in forming processes, pile in piling processes, ...). This case will be first developed. Contact between two strained bodies will be discussed later. The second and stiffer body will be named hereafter the "tool". Its boundary is described segment by segment in a special table (this is the most important change in the program skeleton for the modelling of the contact phenomena). Each segment (figure 1.) is defined by two or three nodes and a shape code (designating linear, parabolic or circular segment or triangle for three-dimensional case). Thanks to the use of nodes we can shift the tool by shifting its nodes what is very easy in a finite element code. If the tool is strained and is meshed by finite elements the boundary adaptation is automatically done.

Because we are now considering only rigid tools all computations will be made only on one face of the interface element which is of course the strained one.

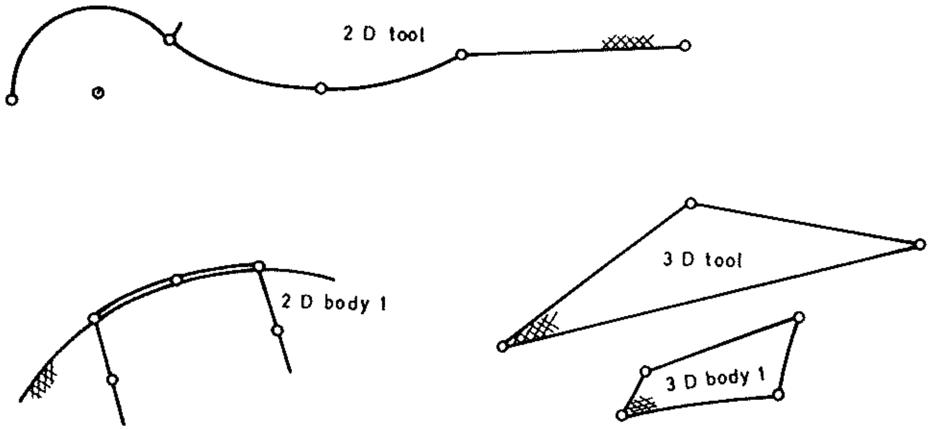

FIGURE 1. Contact element

STRESSES, STRAINS AND VIRTUAL POWER

Solid equilibrium is expressed in the current state by the virtual power principal [1] for solids as well as for contact element and uses the same kind of formulation. The stress tensor becomes the stress vector and is expressed in a local triad whose unit vectors are normal and tangent to the element i.e. to the strained body boundary (figure 2.)

$$\sigma = < p, \tau_R, \tau_S >$$

So defined in a corotational triad the stress vector σ is objective. It is not modified by a rigid rotation.

The stress rate is associated to a strain rate $\dot{\varepsilon}$ defined in the same triad. It is the time derivative of the distance between the two elements faces.

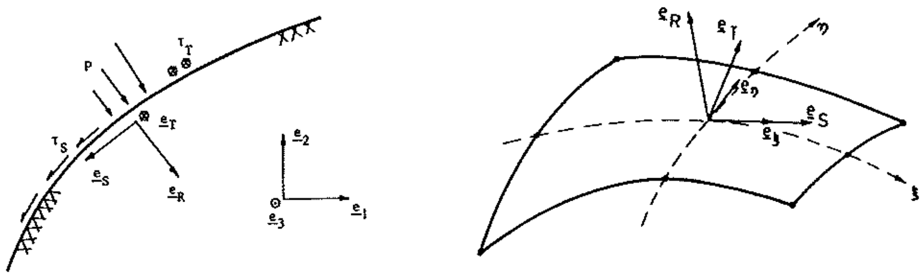

FIGURE 2. Local triad and stress vector.

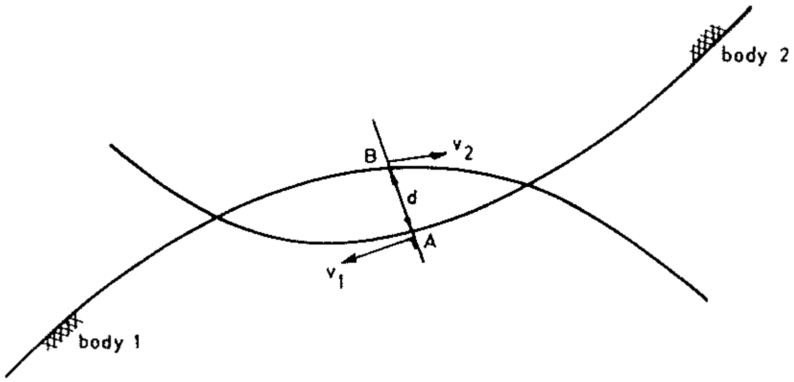

FIGURE 3. Distance between element faces.

If **A** is a point of the solid boundary and **B** is the point of intersection between the tool boundary and the solid normal $\mathbf{e}_R$ at **A** the distance **u** is expressed in the local triad by

$$\boldsymbol{u} = \boldsymbol{R}(\boldsymbol{x}^B - \boldsymbol{x}^A)$$

It is objective i.e. independent of a rigid rotation of the two bodies *together*. The strain rate vector is then

$$\dot{\varepsilon} = \frac{d\boldsymbol{u}}{dt} = \boldsymbol{R}(\frac{d\boldsymbol{x}^B}{dt} - \frac{d\boldsymbol{x}^A}{dt} + \frac{d\boldsymbol{R}}{dt}(\boldsymbol{x}^B - \boldsymbol{x}^A)$$

where the second term is an objectivity correction due to the rigid rotationR Finally the virtual power developed on the solid boundary is

$$\delta W = \int_s \sigma^T \delta\varepsilon ds$$

with

$$\delta\varepsilon = -\boldsymbol{R}\delta\boldsymbol{x}^A$$

CONSTITUTIVE LAW OF CONTACT WITH FRICTION

The relation between the generalised stress rate $\dot{\sigma}$ and the generalised strain rate $\dot{\varepsilon}$ is depending on the interface rheology: surface state, special material in the interface, special behaviour of one body along the interface,... This point was studied by a large number of authors: DIETERICH [5,6], GHABOUSSI[7], SHAFER[8], CHANDRA and MUKHERJE[9], AL

KHATTAT[10], NAGTEGAAL AND REBELO[11]... For metals in contact, rheology is depending on cleanliness and finish of surface and on lubricant properties (GODET[12]). For soils and rocks the rheological problem is perhaps more complex as shown by BOULON[13] and NOVA[14].

We are supposing here the interface to be thin with regard to the finite elements and body size as it is done in plasticity rheology when one supposes the metal to be homogeneous and not a crystal assembly. Therefore the constitutive law is a "mean" law, what is the opposite of the local concept proposed by ODEN and PIRES [15,16,17].

We will develop hereafter the constitutive law following the elastoplastic formalisms as done by CURNIER [18,19] and thermodynamically justified by SIDOROFF[20]: slipping is an irreversible mechanical phenomenon described as plasticity in metals.

A gap between the two bodies implies null contact stresses :

$$\sigma = < 0, 0, 0 >$$

In the stress space, we are supposing an elastic field (which has essentially a numerical meaning) bounded by a plasticity surface. Elasticity implies sticking contact. Real rigid and sticking contact is difficult to introduce in a classical finite element code. The elastic formalisms is in fact a penalty of the rigid contact and sticking condition. From a geometrical point of view elasticity of contact implies a interpenetration of the two bodies in contact. The elastic constitutive law is a diagonal one :

$$\dot{\sigma} = \begin{pmatrix} K_p & 0 & 0 \\ 0 & K_t & 0 \\ 0 & 0 & K_t \end{pmatrix} \dot{\epsilon}$$

K_p et K_t are the penalty coefficients; they must be as large as possible in order to reduce the penetration of the bodies, but there is a limitation due to the decrease of the numerical convergence. The code user unfortunately is not completely free of choosing these coefficients. Therefore it seems unprofitable to search a physical meaning to these parameters.

At the present time we have only implemented a COULOMB yield surface (figure 4.):

$$f = \sqrt{(\tau_S^2 + \tau_T^2)} - \phi p$$

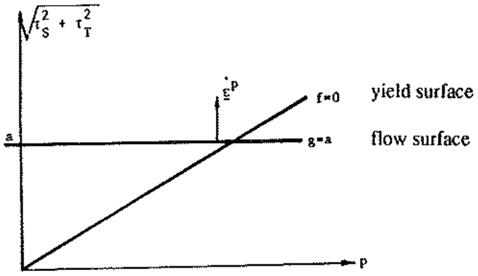

FIGURE 4. COULOMB yield surface.

It seems to be easy to introduce in the code an other yield surface as proposed by NOVA[14]. For example we could use a CAM-CLAY like yield surface:

$$f = (\tau_S^2 + \tau_T^2) - \phi p(p + a)$$

Irreversible strains are the slipping displacements. For metals there isn't any normal irreversible displacement, i.e. no dilatancy occurs. Therefore one can not use the classical associated normality rule but has to define a flow surface different from the yield one. In order to exclude dilatancy we use the following flow surface:

$$g = (\tau_S^2 + \tau_T^2)$$

$$\dot{\varepsilon} = \frac{\dot{\lambda}\partial g}{\partial \sigma}$$

If one want to model soil or rock interfaces dilatancy must be introduced. When using the CAM-CLAY like yield surface and an associated flow rule, dilatancy is varying in connection with the contact pressure level.

The last parameter to be defined is the hardening one. But hardening is not easy to experiment and seems to be very small for contact between metallic bodies in a forming processes. It must of course be related to the wear evolution, but for forming the wear is really small during each process. For more details the reader could report to GODET[12] and DIETERICH[5,6]. Hereafter we are neglecting hardening because of the lack of data, the lack of interest for large displacement slipping problems and the simplicity of report.

Using the classical consistency condition,

$$\dot{f} = 0$$

one obtains easily the value of the scale coefficient $\dot{\lambda}$ and by a well known development the tangential constitutive C tensor relating the stress and strain rates :

$$\dot{\sigma} = C\dot{\varepsilon}$$

$$\dot{\lambda} = \frac{\frac{\partial f}{\partial \sigma_i} C^e_{ij} \dot{\varepsilon}_j}{\frac{\partial f}{\partial \sigma_a} C^e_{ab} \frac{\partial g}{\partial \sigma_b}}$$

$$\dot{\sigma}_i = C^e_{ij}\dot{\varepsilon}_j - \dot{\lambda} C^e_{ij} \frac{\partial g}{\partial \sigma_j}$$

STRESS TIME INTEGRATION

For the two-dimensional and axisymmetric cases, the yield surface is reduced to a straight line. The constitutive relation is therefore always linear as for elasticity as well for elastoplasticity. The plasticity time integration scheme are all equivalent because the yield surface normal is sole. At the opposite for the three-dimensional case the yield surface is conical and a refined integration scheme is needed. We have implemented a division of the time step in some sub-increments and in each sub-increment we are performing a two step generalised mid-point integration scheme. This scheme is second order accurate (RICE and TRACEY[21], MARQUES[22], CHARLIER[1])

An other point to be discussed in the incremental objectivity: for a finite time step what is the reply of the time integration scheme under a rigid body rotation (implying of course the two bodies) ? As one can see on the figure 5 if constant velocity of the nodal points is supposed during the step, some strains will appear in the interface for a rigid body rotation. More precisely no tangential displacement will occurs but some normal displacement. For undilatant interface (metal contact) this normal displacement will produce reversible stresses returning at the end of the step to their initial value. But for dilatant material, this could induce some irreversible behaviour. Therefore it seems to be better to use a mean normal strain. This mean strain is simply the variation of the normal distance from the beginning to the end of the step.

$$\dot{\varepsilon}_n = \frac{(d^B - d^A)}{dt}$$

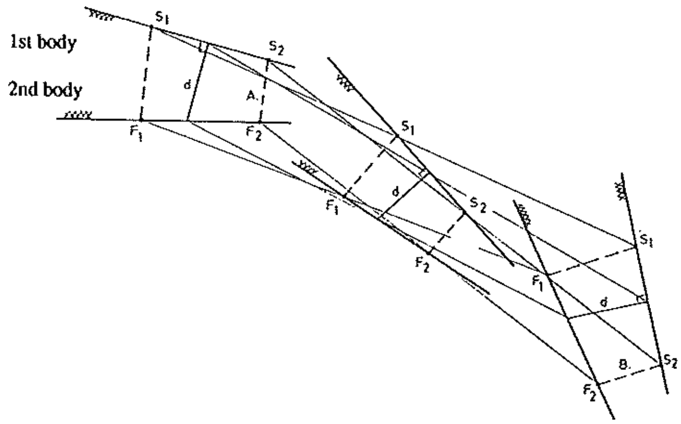

FIGURE 5. Incremental objectivity problem.

CONTACT FINITE ELEMENT

The contact finite elements will be located on the boundaries of strained body. They are surfaces for three-dimensional problems and lines for two-dimensional ones. For the kinematical compatibility these elements are of isoparametric kind with first or second degree interpolation (figure 6) and the numerical integration of the virtual power uses the Gaussian scheme of first, second or third order. Therefore everything (stress, strain-rate,..) is computed only at the integration points. Especially the local triad need to be defined only at these points. Its axes are normal and tangent to the boundary of the strained body but not to the rigid one because the penalty technique induces some discrepancy between the contact surfaces.

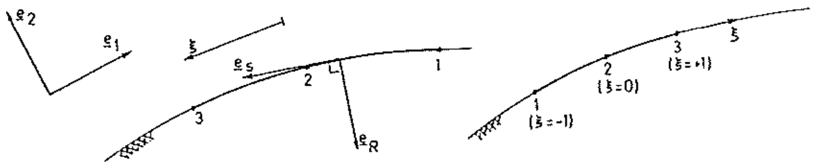

FIGURE 6. Contact finite element geometry.

The main advantages of the triad expression at the integration points is in addition to the formulation elegance its uniqueness on the contrary to the nodal triad definition.

At each integration point, after computing the local triad one must determine if there is any contact and which tool segment is affected. Especially numerical experiments have shown that some *false contact* can occur. For algorithms robustness it is important to exclude these false contacts. The figure 7 shows two examples of false contacts (at points B) which are avoided by the code. First the normal to the element intersects the tool segment out of its active range i.e. outside of the extreme nodes. The second described case is more complex. We want to describe the whole tool as one alone boundary. Therefore each integration point candidate to contact has very large choice of tool segments and its normal could sometimes intersect the boundary at the opposite of the body.

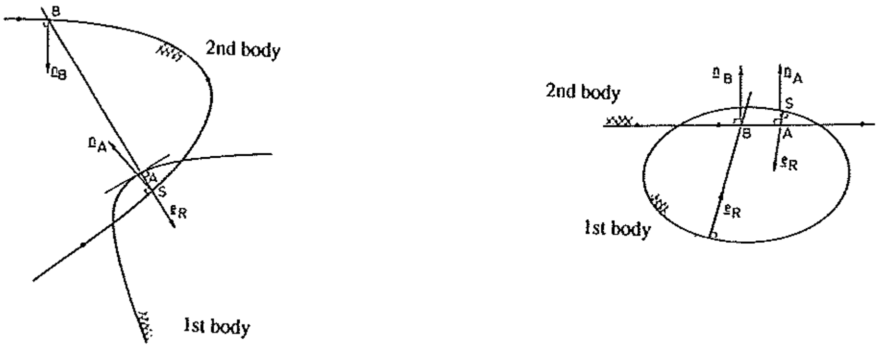

FIGURE 7. False contact cases.

These false contacts are detected by comparing the two internal normals (one for each body): their scalar product is negative for true contact:

$$\text{if} \quad e_R^T \cdot n_A > 0 \quad \text{then contact exist.}$$

$$\text{if} \quad e_R^T . n_A < 0 \quad \text{then contact do not exist.}$$

After determination of a true contact the strain rate $\dot{\varepsilon}$ is easily computed using the classical analytical geometry The shape functions describe the element shape as well as the tool shape. For more details the reader could refer to [1,4].

The nodal forces energetically equivalent to the stresses are for the two dimensional case:

$$F_{L1} = \sum_{PI}(-p\frac{dx_2}{d\xi} + \tau_S\frac{dx_1}{d\xi}).N_L.W$$

$$F_{L2} = \sum_{PI}(+p\frac{dx_1}{d\xi} + \tau_S\frac{dx_2}{d\xi}).N_L.W$$

where W are the integration weights, N_L are the shape functions and ξ is the isoparametric coordinate. For the three-dimensional case one obtains:

$$\begin{pmatrix} F_{L1} \\ F_{L2} \\ F_{L3} \end{pmatrix} = \sum_{PI} \boldsymbol{R}^T \sigma\ N_L \cdot |e_\xi \wedge e_\eta| \cdot W$$

ITERATION MATRIX

For non-linear problems the NEWTON-RAPHSON iteration technique generally offers the best convergence rate but not always the lowest CPU time. For the LAGAMINE code a pure NEWTON-RAPHSON method and a released one are possible. This last possibilty consists in using the same iteration matrix for some iterations or steps. However it is necessary to be able to compute the best tangent iteration matrix in the NEWTON-RAPHSON sense which is defined as the nodal forces derivative:

$$FHE_K^{(i+1)} = FHE_K^{(i)} + \frac{\partial FHE_K^{(i)}}{\partial u_L}\Delta u_L$$

where FHE represents the out-of-balance forces (i.e. the difference between nodal forces F and imposed forces).

As shown by the complex expression of the nodal forces it is very difficult to compute analyticaly this matrix. Indeed it would be necessary to take into account the geometrical effect for the element (what is classical but unsymmetric) and of the tool (what is more complex especially if the contact occurs obliquely with a parabolic or circular segment). For the constitutive law one must especially take into account the time integration scheme which uses sub-incrementation and a two step scheme in each sub-increment. These last effects can not be computed analytically. CESCOTTO and the authors [23,1] have proposed therefore to compute an exact tangent iteration matrix by a numerical perturbation technique. The implied cost is high for one element but generaly the number of active contact elements is small with regards to the number of solid elements and the winning of CPU time and of convergence can be important for highly non-linear problems.

CONTACT BETWEEN TWO STRAINED BODIES

Contact between two strained bodies including very large displacements and rotations is difficult to model numerically. Most authors studying this contact have only interest in small displacements problems and represent the interface by a third body which must be meshed. One point contacts always the same zone of the other body during the whole loading process. When large displacements are occurring this assumption is not any more valid. We have modelled this kind of problems (for example the rooting up of sheet-pile's claws) simply by considering that each boundary is candidate to contact the other, ignoring its strains and using the preceding algorithm. Therefore the contact stresses are computed two times (one on each side). The only modification to include in our element is the taking into account of that double computation of the same variable: the virtual work is done in two parts, one on each side, the stresses are equal on each side, and one divides the strain rate by a factor two. So the virtual work is exactly computed. The equality of the stresses on each face is perfectly realised except the penalty precision. But we don't introduce the coupling between the faces in the iteration matrix. Each element matrix is related only to one side of the interface, and in this way the matrix is not any more tangent in the NEWTON-RAPHSON sense.

APPLICATIONS

It is difficult to develop a application of only contact and friction including large displacements but not plasticity or other special features for the solid. We are only presenting one such problem in this paper: displacement of a axisymmetric tube on a rigid tool with a variation of diameter (figure 8). The strained body is supposed to be elastic (E=210000 N/mm^2 , ν=0.3). The loading is done by imposing displacements. Two cases are presented, the first do not include friction (ϕ=0.0), and in the second the friction coefficient is quite normal for steel (ϕ=0.2). Some results are presented at the figure 9. The contact stress vectors are represented on the deformed

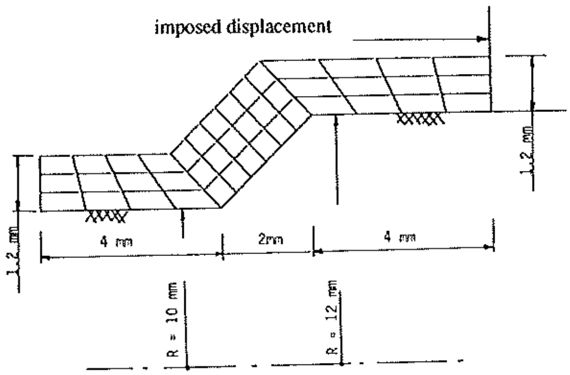

FIGURE 8. Contact test including large displacements.

mesh for three load step. Their inclination on the boundary is depending on the friction coefficient. A stress concentration appears clearly at the ends of the contact line, especially when slip is free. Some penetration appears between the strained body and the tool. This is the result of penalty method and it could be reduced by increasing the penalty coefficients.

CONCLUSIONS

Finite elements specialised in the modelling of contact with friction phenomena have been developed. Their mean features are the isoparametric formulation and the integration of generalised stresses at the Gaussian integration points, the ability of taking in account very large displacements and rotations between a strained body and a tool or between two strained bodies ... They have been implemented in the finite element code LAGAMINE which has been applied to many industrial forming processes. An axisymmetric example was presented which shows some of these properties and could be the base of comparison between different contact algorithms.

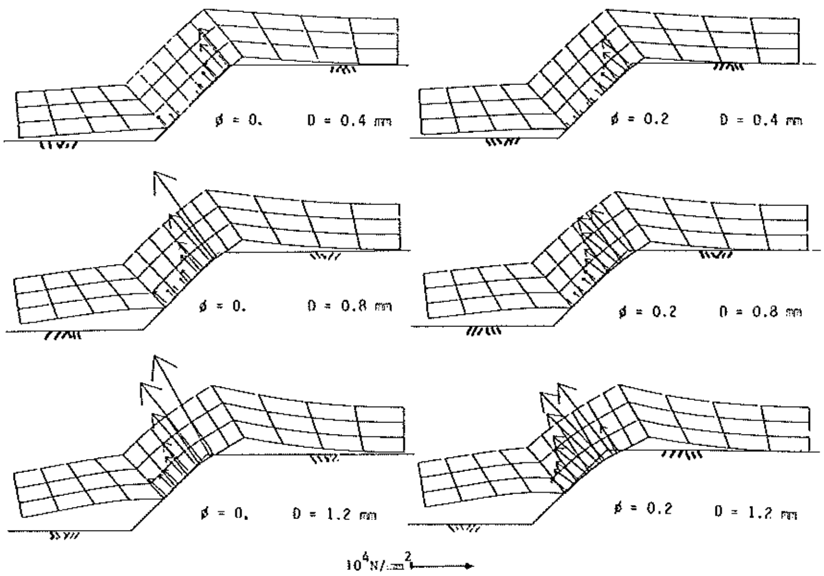

FIGURE 9. Deformed mesh and stress vectors

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