A wavelet-based mode decomposition compared to the EMD

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Abstract

We introduce a new method based on wavelets for decomposing a signal into quasi-periodic oscillating components with smooth time-varying amplitudes. This method is inspired by both the "classic" wavelet-based decomposition and the empirical mode decomposition (EMD). We compare the efficiency of the method with the well-established EMD on toys examples and the ENSO climate index.

1. Method

a) Perform the continuous wavelet transform of the signal f:

$$Wf(a,t) = \int f(t)\bar{\psi}\left(\frac{x-t}{a}\right)\frac{dx}{a}$$

where ψ is a Morlet-like wavelet, $\bar{\psi}$ is the complex conjugate of ψ, t stands for the time, and $a > 0$ is the scale parameter.

b) Compute the wavelet spectrum Λ associated to f:

$$\Lambda(a) = E\,|Wf(a,.)|$$

where E denotes the mean over time and look for the scale a^* for which Λ reaches its global maximum.

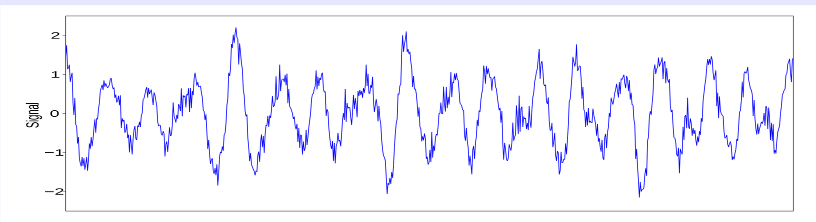
c) Extract the component associated to a^*:

$$|Wf(a^*,t)|\cos(\arg Wf(a^*,t)).$$

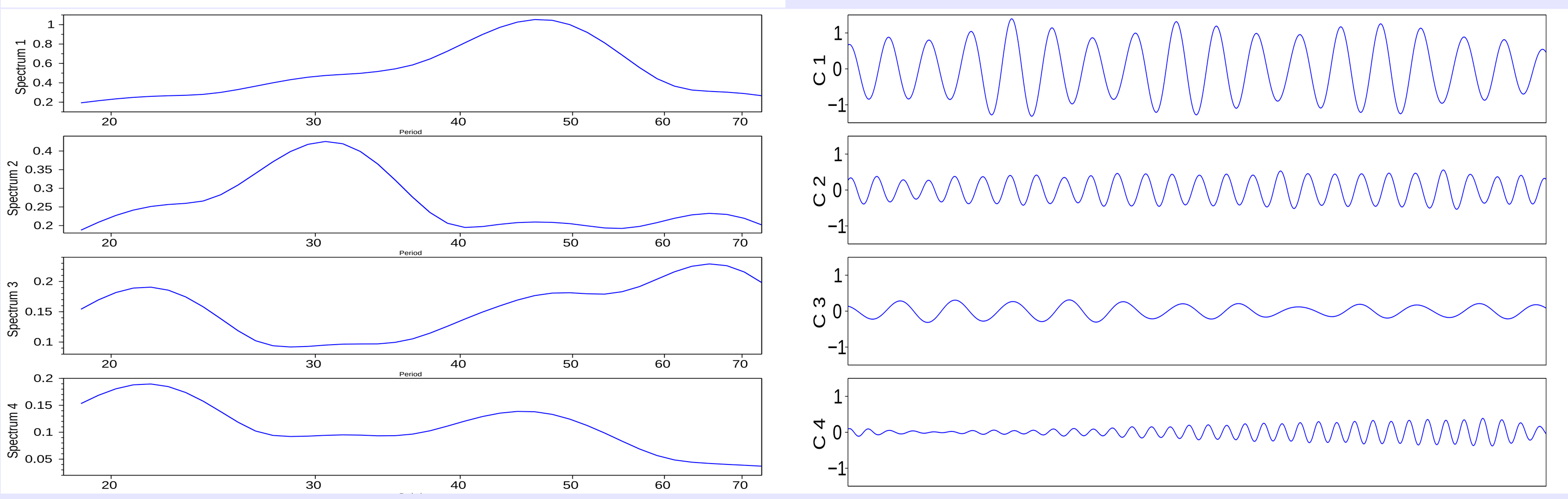
d) Subtract this component from f and repeat steps (a) to (d).

e) For now, we stop the process when the extracted components are not relevant anymore. A more adequate stopping criterion may still be found. The sum of the components successively extracted is an accurate reconstruction of f.

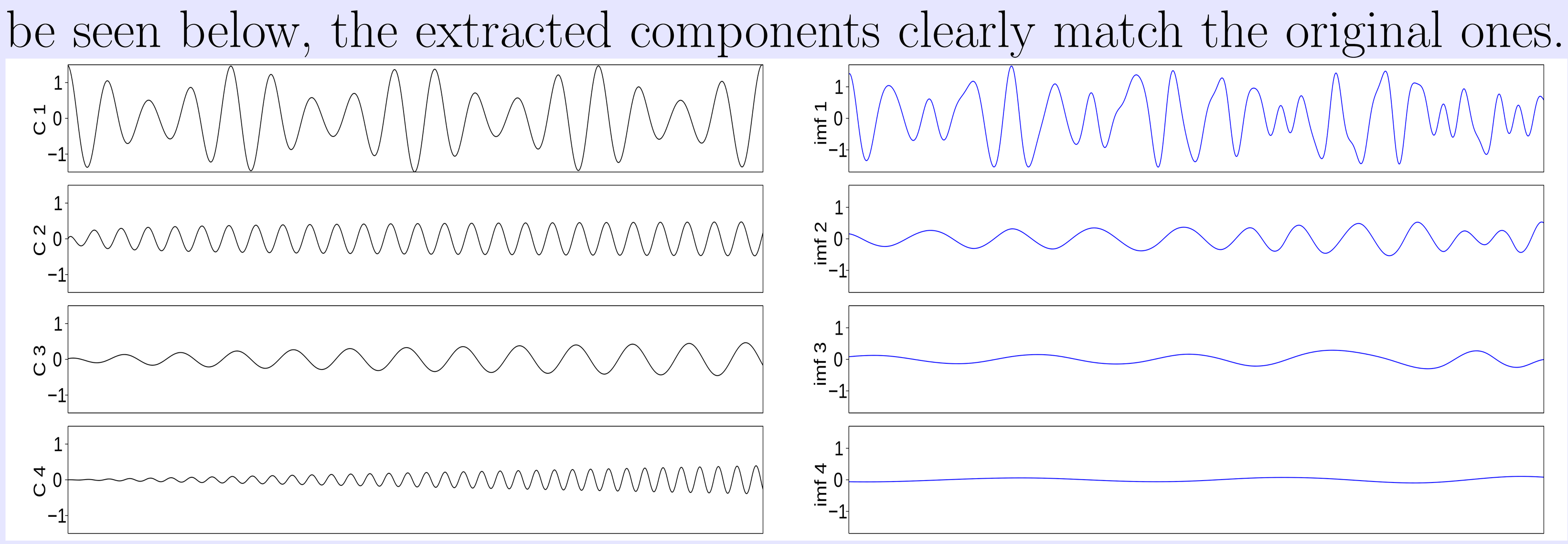
3. Period detection



A signal made of four components with modulated amplitudes and periods (of about 23, 31, 47, 65 units).

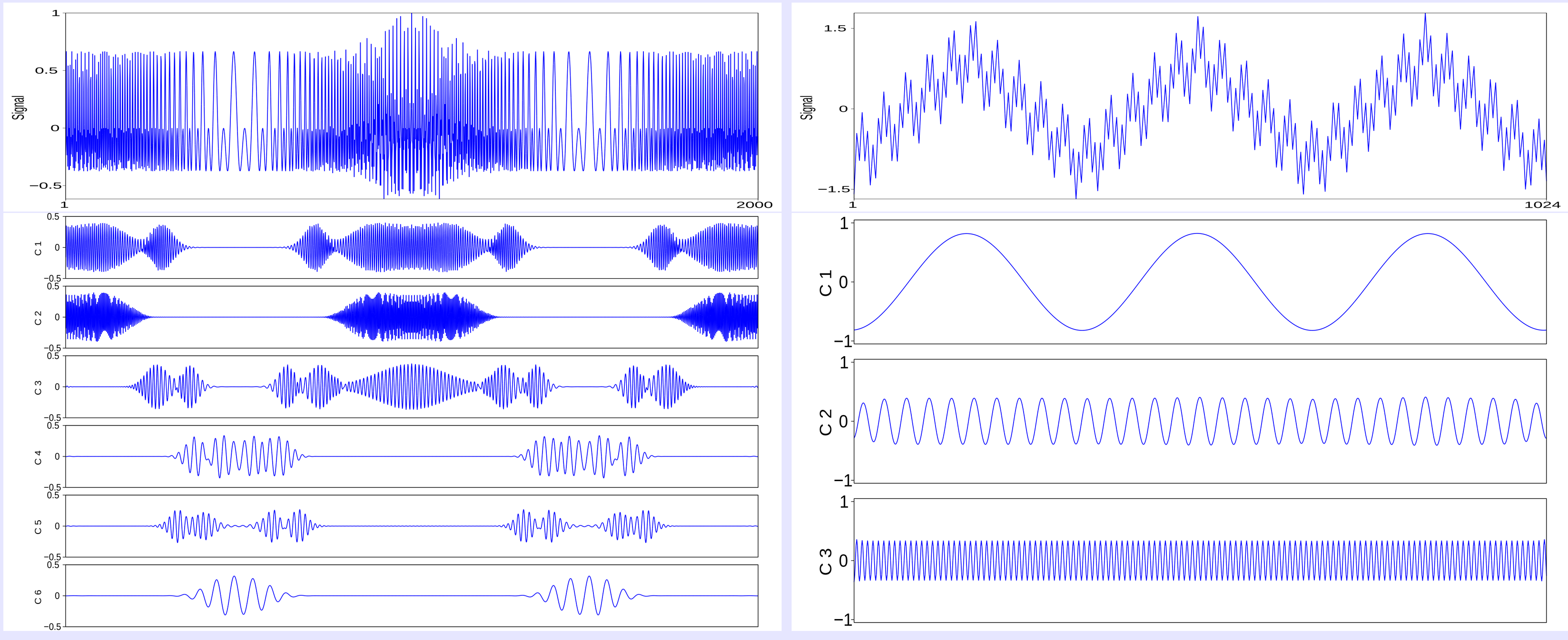


Left: the spectra obtained from the successive wavelet transforms. Right: the corresponding extracted components. The periods detected by the method are 21.6, 30.6, 46.4 and 65.6. The RMSE between the reconstructed signal and the original is 0.069 and the correlation is 0.996. As it can be seen below, the extracted components clearly match the original ones.



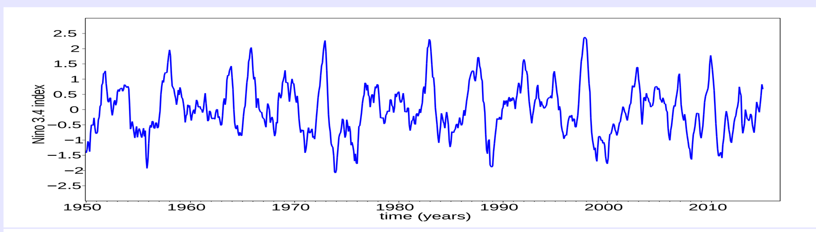
Left: the original components. Right: the IMF's extracted from the EMD. One can see that they do not match the original ones as good as those above. Even though the RMSE and correlation are slightly better in this case (resp. 0.068 and 0.998), the periods extracted from the Hilbert-Huang transform (41, 75, 165, 284) are far from the expected ones. Besides, the EMD is more noise-sensitive for the period detection than the wavelet-based method (not shown).

2. Classic examples of the EMD

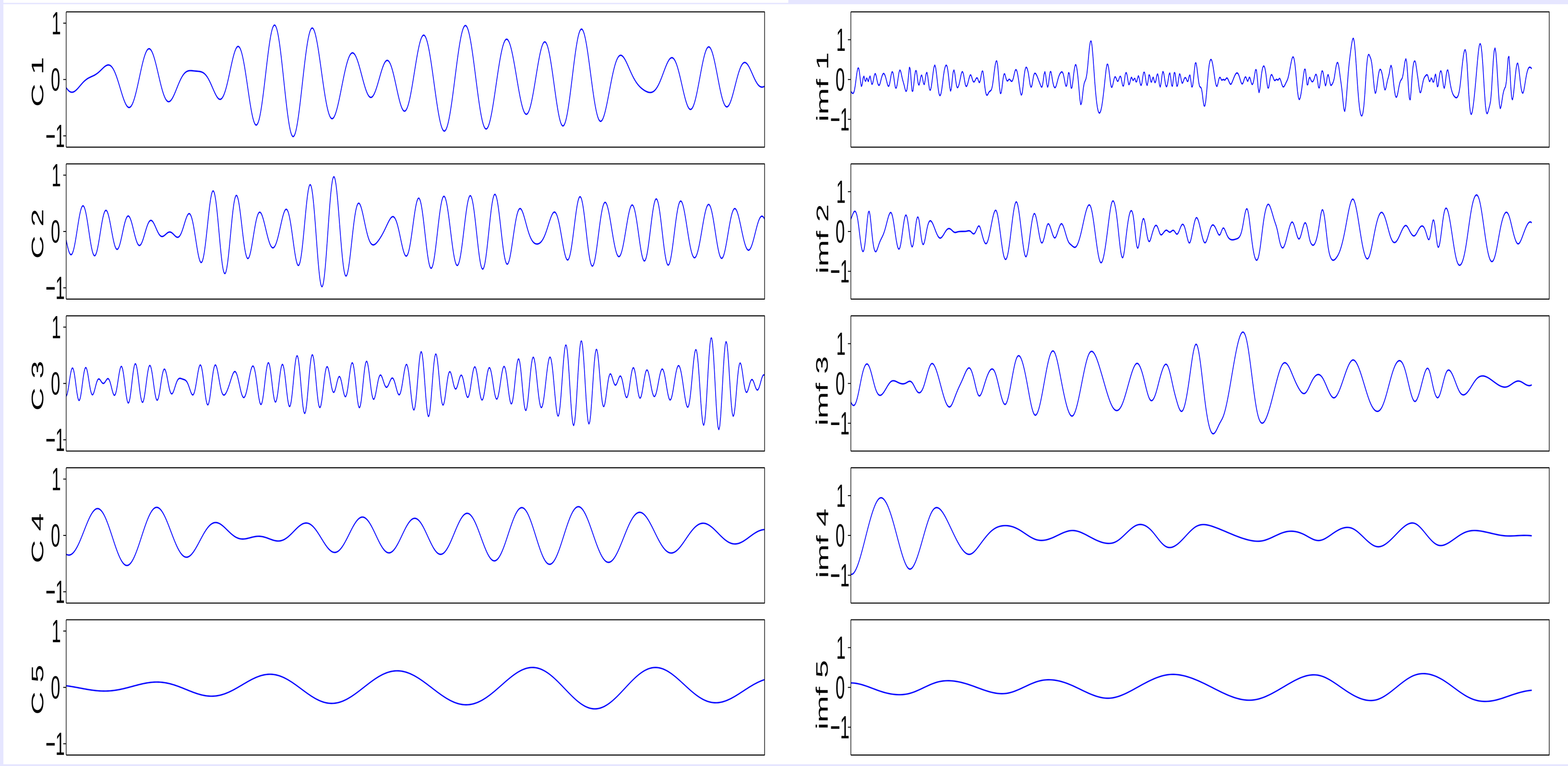


Top: two signals analyzed with the EMD in [8]. Then, the components extracted from the wavelet-based mode decomposition. The RMSE's between the reconstructed signals and the original ones are respectively 0.086 and 0.085 and the correlations are 0.968 and 0.992. The EMD gives similar results (RMSE's equal 0.07 (left) with the first two IMF's and 0.023 (right) and correlations are 0.979 and 0.999).

4. Real data: the ENSO index



The ENSO index, i.e. monthly sea surface temperature anomalies in the Equatorial Pacific Ocean ([5]).



Left: the components extracted from our method. Right: the IMF's given by the EMD. The periods we detect are of 44.8, 28.6, 17, 65.6, 140.6 months and are 9.8, 21, 38.6, 75.9, 138.4 for the EMD. The reconstruction is slightly better with the EMD (RMSE of 0.193 vs 0.277 and correlation of 0.973 vs 0.941) but the 9.8-months component is somewhat weird, noisy and not convenient from a practical point of view. Without it, the RMSE rises to 0.355 and the correlation drops to 0.903. Also, the periods detected by our method seems more in agreement with some previous studies (e.g. [5, 7]).

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