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Computational Modelling of Free and Moving Boundary Problems

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Water Table Aquifers and Finite Element Method: Analysis and Presentation of a Case Study

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ABSTRACT

In groundwater modelling of water table aquifers the finite element codes searching for the free (or water table) surface and adapting the mesh at each time step can be considered now as obsolete. Different ways of modelling the free surfaces in transient conditions and with a fixed mesh have been found and used. The main concepts of the methods, based on the introduction in the constitutive laws of a storage or permeability variation law, are recalled and discussed. Non-linear codes are required but CPU time is spared because the geometry of the mesh has not to be adapted in function of the new position of the water table. One of the methods developed at the University of Liege and based on a storage variation law has been used to simulate the water table alluvial aquifer of the River Meuse located downstream the city of Liege (Belgium). In this simulation a very detailed 3D mesh of 8-nodes brick finite elements represents the discretized aquifer and a very accurate calibration has been completed fitting the model results on more than 200 measurement piezometers.

INTRODUCTION

Classically, the simulation of groundwater problems by the finite element method is realized using linear codes. However, in the case of water table aquifers, the free surface corresponds to an unknown seepage boundary and the mesh must be adjusted after each time step. As the discretization is changed, the solution of a new equation system has to be found, based on the new boundary conditions. If the mesh adjustments are made by an automatic meshing procedure, some distorted finite element could appear in the neighbourhood of the new free surface boundary.

New methods, based mainly on the works of Bathe and Khoshgoftaar [1], have been developed using non-linear codes. Despite the fact that normally, non-linear codes consumes more CPU time than linear ones, for this particular application, it is certainly not the case considering the mesh adjustment procedure which takes a very long CPU time and increases the risk of instability or non-convergence by introduction of uncontrolled distorted elements.

FLOW EQUATION IN CONFINED AND UNCONFINED AQUIFERS

The usual form of the equation describing the flow in a porous medium is written :

$$\frac{\partial}{\partial x_i}\left(K_{ij}\,\frac{\partial h}{\partial x_j}\right) - Q = S_s \cdot \frac{\partial f}{\partial t} \qquad (1)$$

where general index notations are adopted, x_j are the coordinates, K_{ij} is the permeability tensor, h is the piezometric head, Q is the external sink/source term and S_s is the specific storage coefficient.

For confined aquifers, S_s is deduced (De Marsily [2]) from :

$$S_s = \rho . g . m_v \qquad (2)$$

where the water compressibility and the solid grain compressibility are neglected in regard to the compressibility of the porous medium (m_v), ρ is the water volumic mass.

The storage coefficient of the confined aquifer is written :

$$S = \int_0^d S_s . dz \qquad (3)$$

where d is the thickness of the confined aquifer and z the vertical coordinate.

For unconfined or water table aquifers, the storage coefficient, noted S, is usually approximated by the effective porosity (n_e) of the geological formation, corresponding to the quantity of water that could be obtained by the total drainage of the porous medium.

In some cases of intensive water withdrawal, a confined aquifer may become a free or water table aquifer by lowering the piezometric head under the top of the upperlying aquitard. In such particular conditions, the storage coefficient passes from its value in equation (3) to the value of the effective porosity (n_e), i.e. there is a discontinuity.

METHOD USING THE NON-LINEARITY OF THE PERMEABILITY

The permeability coefficient is introduced in the non-linear code, as a function of the pore pressure (figure 1). Different relationships between K and the pore pressure (p) can be chosen depending on the nature of the studied porous medium.

In the non-saturated part ($p \leq 0$), this approach can take into account the fact that the medium situated above the water table is not completely impervious. As K is not equal to zero, a (reduced) unsaturated flow is simulated and, of course, it can influence the flow in the saturated zone.

As mentioned by Dysli and Rybisar [3], from an hydrogeological point of view, the unsaturated zone can be characterized by a relationship linking the permeability coefficient to the suction pressure (figure 2). This one corresponds to the negative pressure energetically equivalent to the pressure which is applied to the medium in order to mobilize the capillarity water. The evolution of water content of the porous formation in function of the suction pressure varies also from a geological medium to another (figure 2).

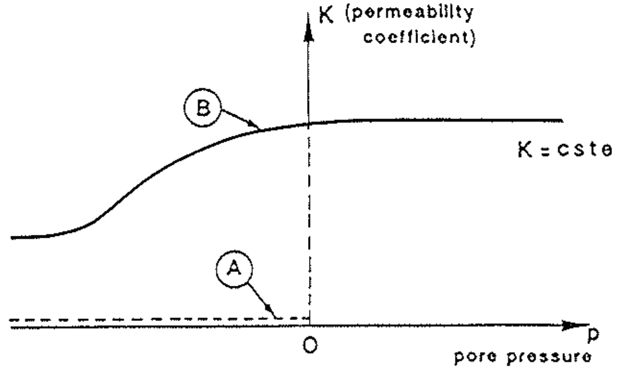

Fig.1. Permeability coefficient in function of the pore pressure :
A) theoretical relationship used in common FEM codes with adjustment of the mesh to the water table
B) relationship used in the non-linear code

(after [3])

In flow-compaction problems linked to the lowering of a water table aquifer, a coupling relation linking the permeability coefficient to the porosity or the void-ratio has to be introduced in a second finite element code. The coupling could be achieved by an interprocessor (Dysli and Rybisar [3]), based on a relationship between the variation of pore pressure and the variation of effective stress i.e. the Terzaghi [4] principle.

In flow-compaction problems induced by lowering of the pore pressure in confined aquifers (most cases of land subsidence are of this type), a second coupling affecting the storage coefficient is necessary as expressed by equation (2).

METHOD USING THE NON-LINEARITY OF THE STORAGE COEFFICIENT

The storage coefficient is introduced in the non-linear code, as a function of the pore pressure (Dassargues, Radu and Charlier [5]). The method has been imagined from the modelling by "enthalpic technique" of the phase changes in heat conduction problems : the phase changes occur at constant temperature, with heat storage. In our case, the geological porous medium passes from unsaturated to saturated state at a constant zero pressure with storage of water. This storage corresponds mainly to the effective porosity of the medium. The variation of the storage in function of the pore pressure is shown for a sandy aquifer on figure 3.

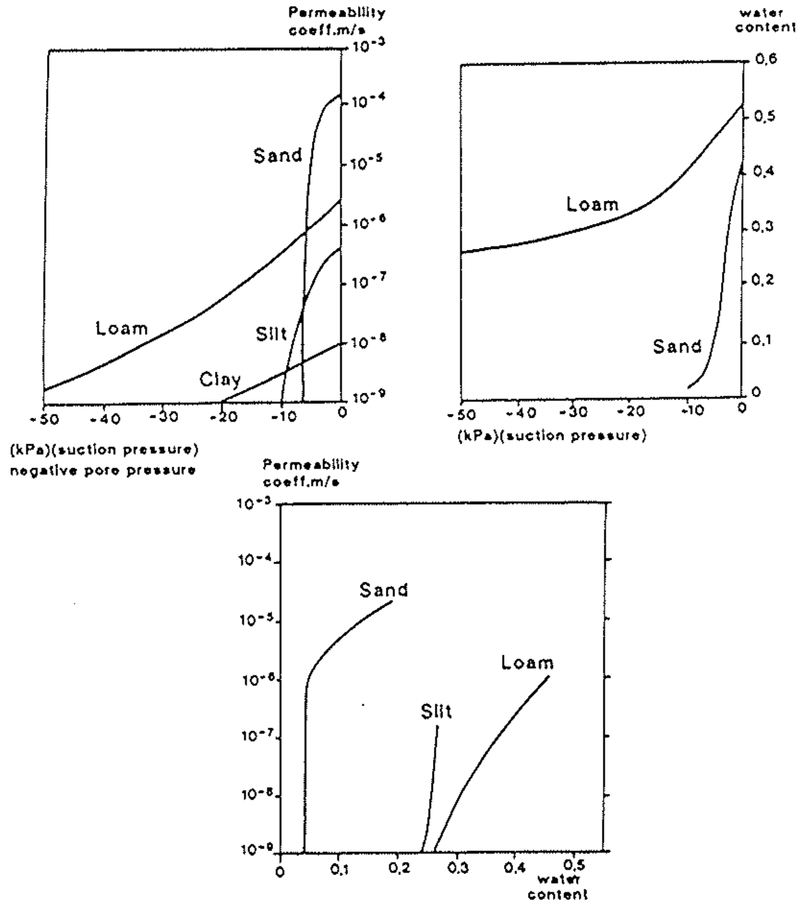

Fig.2. Evolution of the permeability and the water content in function of the suction pressure, for different porous media.

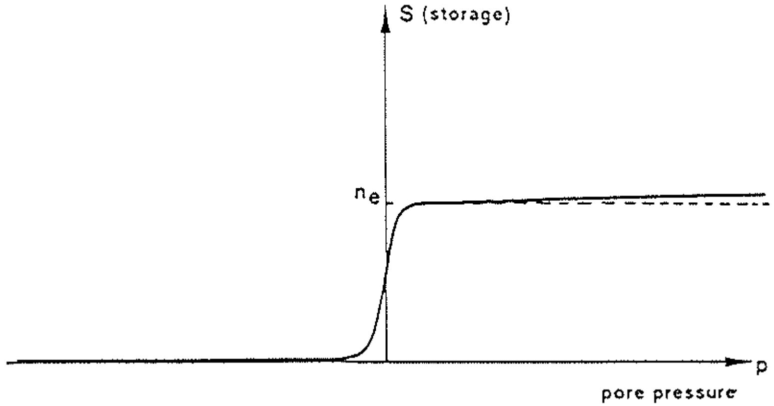

Fig.3. The real storage law.

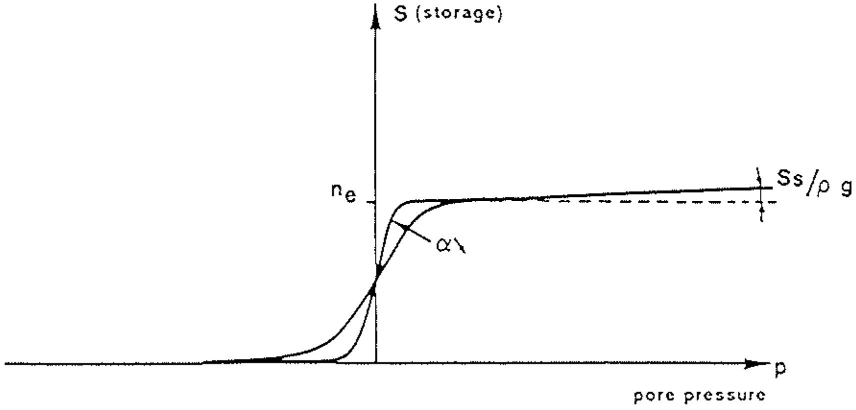

Fig.4. The storage law introduced in the non-linear code LAGAMINE.

In the non-linear code, the storage law which approximates the real variation is an arc-tangent function (figure 4) (Charlier, Radu and Dassargues [6]) :

$$S = n_e \left(\frac{1}{\pi} \operatorname{arctg} \left(\frac{p}{\alpha} \right) + \frac{1}{2} \right) + \frac{S_s}{\rho . g} \; . \; \langle p \rangle \qquad (4)$$

where S_s = specific storage coefficient
$\langle p \rangle = p$ if $p > 0$
$\langle p \rangle = 0$ if $p \le 0$
α = coefficient influencing the shape of the function (figure 4).

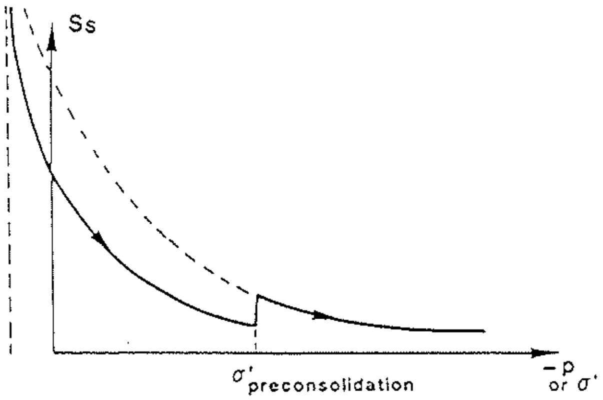

Fig.5. Evolution of the S_s value in function of the pore pressure (or minus the effective stress, by the Terzaghi principle and with constant total stress).

In flow-compaction problems linked to the lowering of a water table, any coupling relation linking the permeability coefficient to the porosity or void-ratio can be chosen without problem. This new non-linearity can be introduced in the same FEM code.

In flow-compaction problems induced by the lowering of the pore pressure in confined aquifers, the variation of S_s (figure 5) is introduced by the following coupling relation (Dassargues [7] :

$$f^V = S_s \cdot \dot{p} / \rho . g \qquad (5)$$

where f^V is the total quantity of water expelled during the compaction and $\dot{p}$ is the time derivation of the pore pressure.

Assuming the Terzaghi principle with a constant total stress, this equation can be written :

$$- f^V = S_s . \dot{\sigma}' / \rho . g = -\dot{\varepsilon} \qquad (6)$$

where $\dot{\varepsilon}$ is the time derivation of the strain.

APPLICATION

Our non-linear code,called LAGAMINE (Charlier [8]), has been already used to model different kinds of problems :

- . model of the Ekofisk oil-field subsidence (Schroeder *et al*. [9])
- . groundwater model of a regional water table aquifer,called "Hesbaye aquifer", in Belgium (Bolly *et al*. [10])
- . model of the subsidence of the central area of Shanghai (Dassargues *et al*. [11] and Dassargues & Li [12]).

The case study that will be illustrated hereafter consists of the modelling of a regional water table aquifer: the alluvial aquifer in the valley of the River Meuse, downstream to the city of Liege (Belgium). The LAGAMINE code has been applied with the non-linear storage law in order to simulate the water table conditions.

A very detailed model of the alluvial plain has been built, including a complex spatial distribution of the permeability (more than 25 different values are used). Additionally, all the geometrical features and constraints are physically represented in the model : uniform recharge, leakage from a canal, infiltration from the hills feeding the alluvial plain, uplift infiltration from a confined aquifer in karstified limestones underlying the alluvial plain, impervious banks of the River Meuse in some places, pumping and recharge. In such a complex case study, the non-linear code with implementation of the storage law for modelling the water table, is very useful avoiding the need for an automatic meshing procedure on a regional domain with highly heterogeneous conditions and more than 2350 elements disposed in four layers.

The calibration with such a detailed spatial distribution of the permeability was particularly uneasy, but the final results are fitted with measured piezometric maps very accurately (less than 15 cm of mean error on the whole domain). One of the computed piezometric map is shown on figure 6. On basis of these calibrated results, fluxes maps have been computed and drawn (figure 7) showing the main trends in the alluvial aquifer.

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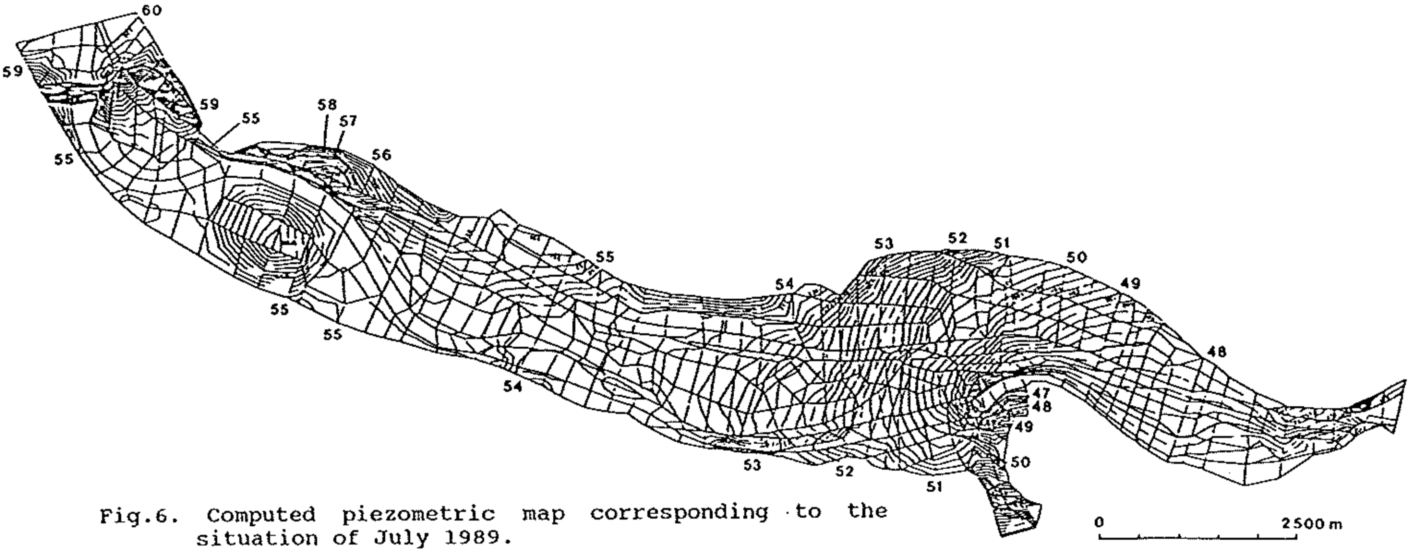

Fig.6. Computed piezometric map corresponding to the situation of July 1989.

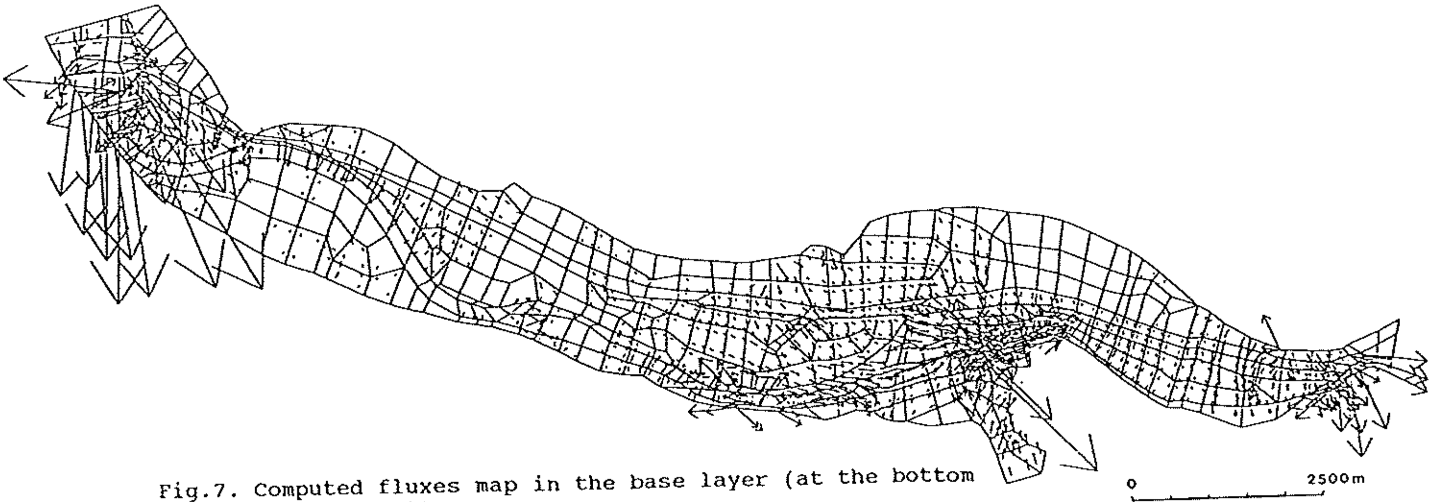

Fig.7. Computed fluxes map in the base layer (at the bottom of the model).

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