ASTEROSEISMIC INVERSIONS IN THE CONTEXT OF PLATO

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**Definition:**

Using seismic (and non-seismic) constraints to determine the optimal set of parameters describing the theoretical model of a real star.

**Typically:** Mass, age, $Y$, $Z$, $\alpha_{\text{MLT}}$, $\alpha_{\text{ov}}$, ... with a given physics (opacities, convection treatment, extra-mixing, ...)

- Limitations?
- Physically representative?

Bias on determinations of fundamental parameters (Mass, Age, Radius)
Improving the use of seismic information:

1. By using **seismic indicators** (glitches, fine analysis of frequency combinations,...).

2. By relating seismic information to structural corrections using a **less model-dependent approach**.

- Not subject to the limitations of forward modelling
- Sufficient number of frequencies required
Using the SOLA method (Pijpers & Thompson 1994), we determine integrated quantities.

**Illustration for the mean density** (Reese et al. 2012):

\[
\bar{\rho} = \int_0^R 4\pi \frac{\rho}{R^3} r^2 dr
\]  

(1)

\[
\frac{\delta \nu_i}{\nu_i} = \int_0^R K_{i,\rho,\Gamma_1} \frac{\delta \rho}{\rho} dr + \int_0^R K_{\Gamma_1,\rho} \frac{\delta \Gamma_1}{\Gamma_1} dr
\]  

(2)

\[
\frac{\delta \bar{\rho}_{inv}}{\bar{\rho}} = \sum_i c_i \frac{\delta \nu_i}{\nu_i} \approx \int_0^R 4\pi \frac{\rho}{R^3 \bar{\rho}} r^2 \frac{\delta \rho}{\rho} dr
\]  

(3)

Provided accurate results up to 0.5% and a way to assess its errors.
But we are not limited in the choice of the integrated quantity:

\[ A = \int_0^R f(s_1, s_2, r) \, dr \]

\[ \frac{\delta A}{A} = \int_0^R \mathcal{T}_A \frac{\delta s_1}{s_1} \, dr + \int_0^R \mathcal{T}_{A,\text{cross}} \frac{\delta s_2}{s_2} \, dr \]

With \( s_1, s_2 \) being \( \rho, c^2, \Gamma_1, u = \frac{P}{\rho}, Y, \ldots \) defining custom-made targets functions \( \mathcal{T}_A \) and \( \mathcal{T}_{A,\text{cross}} \).

**Combining approaches:**

- **Observational constraints**
- **Forward modelling**
- **Indicator inversion**
GENERAL APPROACH (2): NEW INDICATORS

\[ T_\rho = 4\pi x^2 \tilde{\rho} \]
\[ \tilde{\rho} = \int_0^R 4\pi r^2 \rho dr \propto (\Delta \nu)^2 \]

\[ \bar{\rho} = \int_0^R 4\pi r^2 \rho dr \propto (\Delta \nu)^2 \]

\[ T_\tau = \frac{1}{c} \]
\[ \tau = \int_0^R \frac{1}{c} dr \propto (\Delta \nu)^{-1} \]

\[ t = \int_0^R \frac{1}{\tilde{r}} \frac{dc}{dr} dr \propto \frac{\nu d\nu}{\Delta \nu} \]
\[ t_u = \int_0^R f(r) \left( \frac{du}{dr} \right)^2 dr \]

\[ T_t = \frac{1}{x} \frac{dc}{dx} \]

\[ t = \int_0^R \frac{1}{\tilde{r}} \frac{dc}{dr} dr \propto \frac{\nu d\nu}{\Delta \nu} \]
**Test strategy** (see Buldgen et al. 2014, arXiv:1411.2416)

- Build target including complex physics
- Seismic modelling using simple physics
- Carry out inversions for indicators

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1.0 $M_\odot$, 4.0 Gyr, with turbulent pressure ($<\Delta\nu> <\delta\nu>$ with Mass, Age)
RESULTS WITH MULTIPLE INDICATORS (1)

1.05 \, M_\odot, 1.5 \, \text{Gyr}, \, \alpha_{\text{MLT}} = 1.7 \quad (\langle \Delta \nu \rangle < \delta \nu > \text{ with Mass, Age and } \alpha_{\text{MLT}} = 1.522)

1.0 \, M_\odot, 3.0 \, \text{Gyr}, \, \text{non-adiabatic frequencies} \quad (\langle \Delta \nu \rangle < \delta \nu > \text{ with Mass, Age})

\Delta t = 0.2\% \\
\Delta \tau = 0.5\%

\Delta t = 0.08\% \\
\Delta \tau = 0.03\%

\Delta t = 1.8\% \\
\Delta \tau = 0.03\%

\Delta t = 4.3\% \\
\Delta \tau = 3.2\%
RESULTS WITH MULTIPLE INDICATORS (2)

0.9 \(M_\odot\), 3.08 Gyr, with diffusion \((\Delta \nu(\nu), \delta \nu(\nu))\) with \(\alpha_{MLT}, Y_0, Mass, Age\)

1.0 \(M_\odot\), 4.02 Gyr, with diffusion \((r_{02}(\nu), r_{01}(\nu), < \Delta \nu >)\) with \(Z_0, Y_0, Mass, Age\)
Objectives:

- Offer **new constraints** on stellar models and **overcome the current accuracy limitations** in ages, mass and radii determinations.

What we have:

1. **Accurate** (usually < 1%) determinations of $\tau$, $t$, $t_u$ and $\bar{\rho}$
2. Method tested for various **sets of modes** (50 freq, $\ell = 0, 1, 2, (3)$)
3. Method tested for various **constraints on forward modelling**

Prospects:

1. **New indicators** (helium, surface temperature gradient, ...)
2. Test on **new generation of models**
3. Apply to **observations** (Plato data is crucial)
Thank you for your attention!
General expression for any $A$:

$$
\frac{\delta A}{A} = \int_0^R \mathcal{T}_A \frac{\delta s_1}{s_1} dr + \int_0^R \mathcal{T}_{A,\text{cross}} \frac{\delta s_2}{s_2} dr
$$

Target functions:

$$
\mathcal{T}_\tau = \frac{1}{2cT} - \frac{Gm(r)}{r^2} \rho \left[ \int_0^x \frac{1}{2cT} P ds \right] - 4\pi Gr^2 \left[ \int_r^R \left( \frac{\rho}{s^2} \int_0^s \frac{1}{2cT} P dt \right) ds \right]
$$

$$
\mathcal{T}_{\bar{\rho}} = 4\pi \frac{\rho}{R^3 \bar{\rho}} r^2
$$

$$
\mathcal{T}_t = \frac{1}{r} \frac{dc}{dr} \frac{1}{\int_0^R r \frac{dc}{dr} dr}
$$

$$
\mathcal{T}_{t_u} = -\frac{2u_0}{t_u} \frac{d}{dr} \left( f(r) \frac{du}{dr} \right)
$$
Obtaining new kernels: From Masters et al. 1979:

\[ \int_0^R K_{s_1,s_2}^{n,l} \frac{\delta s_1}{s_1} \, dr + \int_0^R K_{s_2,s_1}^{n,l} \frac{\delta s_2}{s_2} \, dr = \int_0^R K_{s_3,s_4}^{n,l} \frac{\delta s_3}{s_3} \, dr + \int_0^R K_{s_4,s_3}^{n,l} \frac{\delta s_4}{s_4} \, dr \]

Relate \((s_1, s_2)\) to \((s_3, s_4)\) (e.g. \((\rho, \Gamma_1)\) to \((u_0, \Gamma_1)\) in the simplest cases)

⇒ new kernels by solving a differential equation.

Ex: \((u_0, \Gamma_1)\), \((u_0, Y)\), \((c^2, \Gamma_1)\), \((c_2, Y)\), \((\Gamma_1, Y)\), \((P_0, \Gamma_1)\), \((P_0, Y)\), \((N_2, c^2)\),...
**Table : Set of modes used in the accuracy tests**

<table>
<thead>
<tr>
<th></th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
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<td>$\ell = 0$</td>
<td>$n = 5 - 24$</td>
<td>$n = 9 - 28$</td>
<td>$n = 5 - 27$</td>
<td>$n = 9 - 34$</td>
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<tr>
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<td>$n = 5 - 27$</td>
<td>$n = 9 - 34$</td>
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<tr>
<td>$\ell = 3$</td>
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<td>$n = 12 - 22$</td>
<td>$n = 15 - 24$</td>
<td>$n = 15 - 21$</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Set 5</th>
<th>Set 6</th>
<th>Set 7</th>
<th>Cyg 16A</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>$n = 15 - 24$</td>
<td>$n = 15 - 21$</td>
</tr>
</tbody>
</table>

| | Cyg 16B | Set 33 |
|---|---|
| $\ell = 0$ | $n = 13 - 26$ | $15 - 25$ |
| $\ell = 1$ | $n = 13 - 26$ | $15 - 25$ |
| $\ell = 2$ | $n = 12 - 25$ | $15 - 25$ |
| $\ell = 3$ | $n = 17 - 24$ | – |
Target properties:

- Total number: 57
- Mass range: 0.9 − 1.1 $M_{\odot}$
- Age range: 1.1 − 8.1 $Gyr$
- $Z_{0}$ range: 0.01 − 0.02
- $X_{0}$ range: 0.73 − 0.68
- No convective cores!
- Including various mismatches: $Y_{0}$, $Z_{0}$, diffusion, heavy elements mixing.

Observational constraints:

$\Delta \nu(\nu)$, $\delta \nu(\nu)$, $r_{02}(\nu)$, $r_{01}(\nu)$, $< \Delta \nu >$, $< \delta \nu >$.

Free parameters:

$\alpha_{MLT}$, $Z_{0}$, $Y_{0}$, Age, Mass.