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Topology Optimization of Membranes Made of Orthotropic Material

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Abstract

This paper presents a formulation for the topology optimization of continuum structures including orthotropic materials. 2D membranes are considered. As for isotropic materials, the density is considered but fibers orientations are also included in the problem in a simple extension of the SIMP material parameterization. Only weight and stiffness criteria are taken into account. The optimization problem is solved with sequential convex approximation methods. Optimal topologies are obtained for several approximations in order to illustrate their ability in solving such a problem.

Keywords: topology optimization, composite material.

1 Introduction

With their high stiffness to weight ratio and their interesting strength properties, fibers reinforced composite materials are widely used in automotive and aerospace industries. The composite structures are made of laminates that include several unidirectional plies. Those plies are stacked together (Figure 1) in a way that tends to maximize the efficiency of a material that can be customized to the application.

Given that a huge number of parameters are necessary to define such materials, the design of composite structures naturally calls for optimization methods. Although the optimal sizing problem of composite structures has been addressed for some years in an industrial context, more advanced techniques such as topology optimization are still under research.

Topology optimization aims at finding the optimal distribution of a material inside a prescribed design domain for a given amount of material. Amongst all the possible solutions, one looks for the one leading to the stiffest structure [2]. In the frame of FEM, a pseudo-density μ varying between 0 and 1 is assigned to each finite element, defining void or material, what leads to the definition of holes in the optimized structure. Usually, isotropic materials are distributed in the structure. Initially, some specific rank 2 laminated materials (including two densities and one orientation design variables) were used to solve the optimal topology problem. They were a mathematical tool allowing to determine theoretically optimal solutions, and the related orientation design variables were determined with a slow converging optimality criteria only dedicated to 2D membrane problems [13,10].

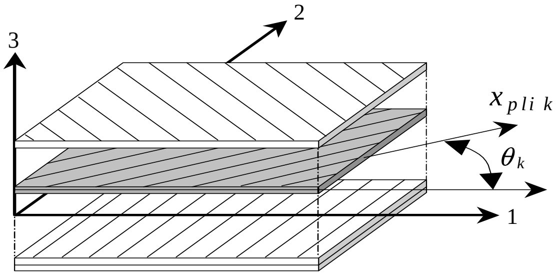

Figure 1: Plies stacked together to form a laminate

For more than 20 years, the Sequential Convex Programming approach, also called Approximation Concepts approach, proved to be efficient for solving structural optimization problems [15]. This technique has been successfully applied to optimal sizing problems [12], shape optimization [18], topology optimization [10], structural optimization with discrete variables [1] and composite structures optimal design [4]. For considering simultaneously thickness and orientation design variables in composite structures optimization, the initially monotonous approximation schemes were extended to mixed monotonous-non monotonous ones in the GBMMA-MMA approximation [5]. This scheme, called GCM in the Boss/Quattro (www.samcef.com) optimization tool box, allowed to solve industrial problems [8]. The dual solver of this approximation makes it efficient for treating topology optimization problems including a very large amount of design variables (of the order of 10^6) and a small number of design functions (about 20). Recently GCM was found reliable in solving composite wing structures optimization problems including more than 300000 constraints and 1000 design variables.

This optimization method and some of its variants are tested here for topology optimization including orthotropic materials. In Section 2, the topology optimization problem for structures made of an isotropic material is reminded. The topology optimization problem for composites is presented in Section 3, together with the limitations of the present study. The SIMP law and a proposed extension to composite materials are explained in Section 4. In Section 5, the optimization method is described. Finally, Section 6 provides some numerical results on classical benchmarks.

2 The topology optimization problem

Topology optimization is a very general tool from structural optimization techniques. It allows to determine automatically the layout of the structure, that is the optimal distribution of the mechanical properties in a prescribed design domain for a given amount of material, as illustrated in Figure 2.

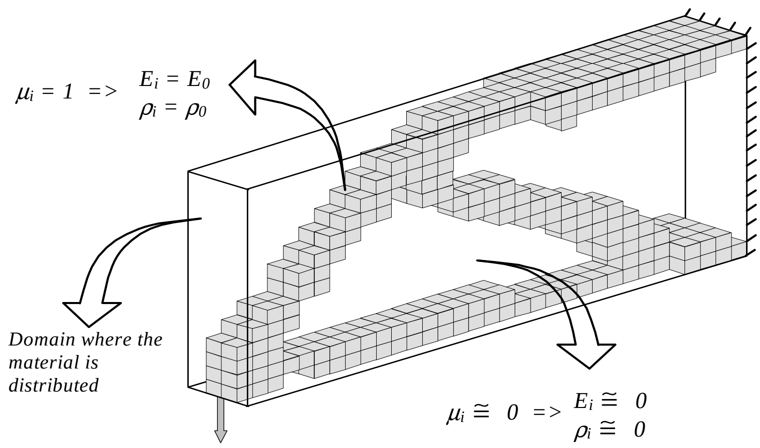

Figure 2. The topology optimization problem (after [4])

Compliance minimization, that results in the stiffest structure that can be exhibited for a given volume fraction of material at the solution, has been intensively studied in the literature [2]. The problem writes:

$$\min \mathbf{g}^{\mathbf{T}}\mathbf{q} \quad \text{s.t.} \quad V \leq \overline{V}$$

where **g** and **q** are the nodal forces and displacements, respectively. $\overline{V}$ is the available amount of material to be distributed in the design area.

Local stress constraints were also taken into account [11]. Since the related optimization problem is huge in terms of design variables and restrictions, those local (that is at the element level) restrictions were only considered in solving academic applications of limited sizes. Specific problems including body loads were investigated, as well [7]. This technology, applied to isotropic materials, has reached maturity and is now used in the aerospace industry (e.g. the TOPOL software – www.samcef.com).

3 Topology optimization with orthotropic materials

In the frame of composite structures, two topology optimization problems can be addressed. The first one is the transverse topology optimization. In this case, one looks for the optimal stacking sequence at several locations in the structure (Figure 3). In the second problem, the optimal topology at a given height of the non homogenous laminate is to be found. This is an in-plane topology optimization (Figure 4). A comprehensive topology optimization problem includes those two aspects, as well as inter-regional and ply drop-off constraints related to ply continuity and manufacturability considerations. This is a very difficult problem to be solved, and efficient formulations should first be derived separately for each of those two problems.

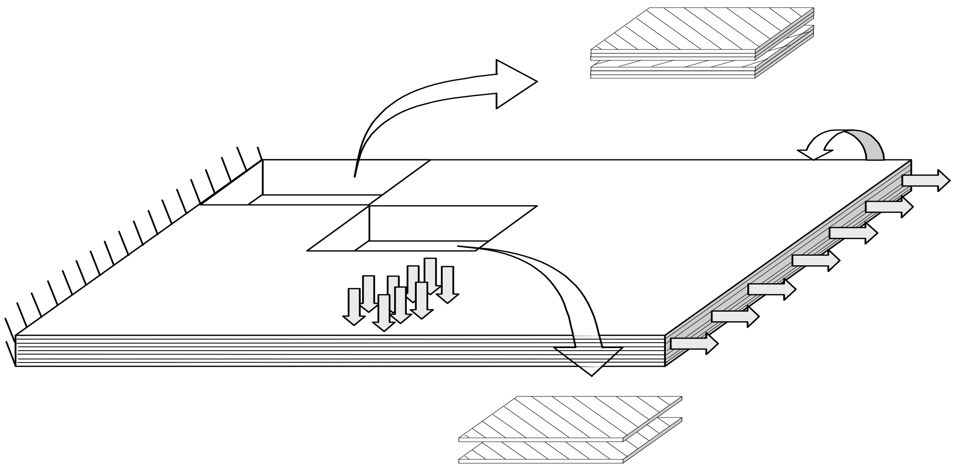

Figure 3. Optimal stacking sequence as a transverse topology optimization problem (after [4])

In this paper, we propose a simple solution for the problem of Figure 4, that is the in-plane topology optimization formulation. It is based on a direct extension of the SIMP parameterization used for isotropic materials, and is illustrated here in the frame of the 2D membrane structures. Even though 2D problems are considered, the method is directly applicable to 3D and shell problems.

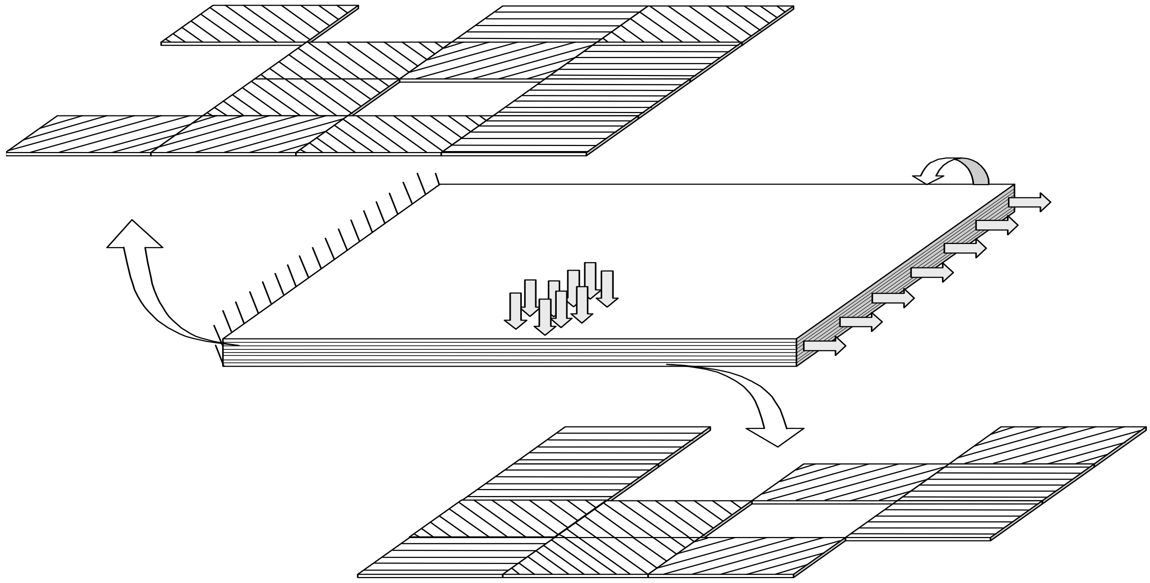

Figure 4. In-plane topology optimization problem (after [4])

4 The SIMP law for topology optimization

4.1 The case of isotropic materials

The design variable is the pseudo-density μ_i that varies between 0 and 1. Such a variable is defined for each finite element *i*. The SIMP material law takes the following form:

$$E_i = \mu_i^p E^0 \qquad \rho_i = \mu_i \rho^0 \tag{1}$$

where E^0 and ρ^0 are the Young modulus and the density of the base material (e.g. steel), E and ρ are the effective material properties, and p is the exponent of the SIMP law, chosen by the user ($1<p<4$).

In the case of plane stress, the stresses are linked to the deformations via the following relation:

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}$$

With the SIMP parameterization, it comes that:

$$\mathbf{C} = \mu_i^p \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} = \mu_i^p \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{yx} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} = \frac{\mu_i^p E^0}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

and the material stiffness matrix **C** depends on the density design variable μ:

$$\boldsymbol{\sigma} = \mathbf{C}(\mu)\boldsymbol{\varepsilon} \qquad (2)$$

4.2 Extension of the SIMP law to orthotropic materials

For orthotropic materials in a plane stress state, the stiffness in the material axes is given by the following expression:

$$\mathbf{C} = \begin{bmatrix} mE_x & m\nu_{yx}E_x & 0 \\ m\nu_{xy}E_y & mE_y & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{yx} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \qquad m = \frac{1}{1-\nu_{xy}\nu_{yx}}$$

where 4 material properties E_x, E_y, ν_{xy} and G_{xy} must be provided. For a material with orthotropic axes oriented at an angle θ with respect to the reference axes (Figure 5), the material stiffness is given by:

$$\mathbf{Q}_{(1,2,3)} = \begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{Bmatrix}_{(1,2,3)} = \begin{bmatrix} c^4 & s^4 & 2c^2s^2 & 4c^2s^2 \\ s^4 & c^4 & 2c^2s^2 & 4c^2s^2 \\ c^2s^2 & c^2s^2 & c^4+s^4 & -4c^2s^2 \\ c^2s^2 & c^2s^2 & -2c^2s^2 & (c^2-s^2)^2 \\ c^3s & -cs^3 & cs^3-c^3s & 2(cs^3-c^3s) \\ cs^3 & -c^3s & (c^3s-cs^3) & 2(c^3s-cs^3) \end{bmatrix} \begin{Bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{xy} \\ Q_{ss} \end{Bmatrix}_{(x,y,z)}$$

with $c = \cos\theta$ and $s = \sin\theta$.

As for isotropic materials, the SIMP parameterization can be used here, and the material law for topology optimization is now written as:

$$\mathbf{C} = \mu_i^p \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}$$

$$\boldsymbol{\sigma} = \mathbf{C}(\mu, \theta)\boldsymbol{\varepsilon} \tag{3}$$

The material stiffness now depends on both kinds of design variables, i.e. the material density and the fibers orientation.

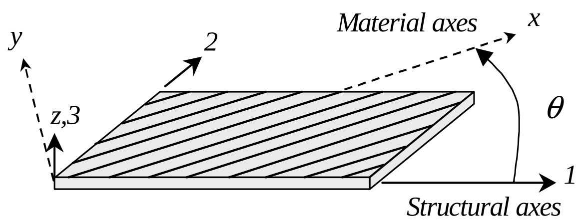

Figure 5. Material and structural axes

5 The optimization method

The optimization method used to solve topology optimization of membranes made of an orthotropic material is described in [8]. In the Sequential Convex Programming, the primary optimization problem (4) is replaced by the solution of successive convex approximated optimization problems (5).

$$\begin{aligned} &\min g_0(\mathbf{X}) \\ &g_j(\mathbf{X}) \le g_j^{\max} && j = 1,\dots,m \\ &\underline{x}_i \le x_i \le \overline{x}_i && i = 1,\dots,n \end{aligned} \tag{4}$$

$$\begin{aligned} &\min \tilde{g}_0^{(k)}(\mathbf{X}) \\ &\tilde{g}_j^{(k)}(\mathbf{X}) \le g_j^{\max} && j = 1,\dots,m \\ &\underline{x}_i^{(k)} \le x_i \le \overline{x}_i^{(k)} && i = 1,\dots,n \end{aligned} \tag{5}$$

The approach used here includes the monotonous MMA approximation [16] and the non-monotonous GBMMA approximations [3]. The selection of the kind of approximation, either monotonous or not, is done automatically for each design variable in each design function based on the derivatives at two successive iterations:

$$\frac{\partial g_j(\mathbf{X}^{(k)})}{\partial x_i} \times \frac{\partial g_j(\mathbf{X}^{(k-1)})}{\partial x_i} > 0 \qquad \Rightarrow \qquad \text{monotonous approximation}$$

$$\frac{\partial g_j(\mathbf{X}^{(k)})}{\partial x_i} \times \frac{\partial g_j(\mathbf{X}^{(k-1)})}{\partial x_i} < 0 \qquad \Rightarrow \qquad \text{non monotonous approximation}$$

The mixed monotonous – non monotonous approximation is called GBMMA-MMA. The GCMMA approximation is only non monotonous and doesn't use information from the previous design point [17]. The GBMMA1 approximation is built by fetching the derivatives at two successive iterations, while GBMMA2 is a second order approximation using a finite difference of the derivatives to compute the second order information. Local linear approximations can also be built. More details can be found in [6].

6 Numerical applications

The results presented in the sequel were obtained in [14].

6.1 First application

The problem is illustrated in Figure 6. It includes 1800 design variables (900 densities and 900 orientations). The optimal topologies obtained for several starting points are provided in Figures 7 and 8. Depending on the initial values for the orientations, different optimal topologies are obtained. This is explained by the non convex behavior of compliance with respect to the orientation variables, as depicted in Figure 9. Since the optimization method builds local approximations of the structural responses, the global optimum can not be reached.

The compliance (Figure 9) is calculated for an element located on the top of the structure. A local optimum can be reached around 50° whereas the global optimum is about 140° for the considered element. This conclusion is inverted if a bottom element is considered.

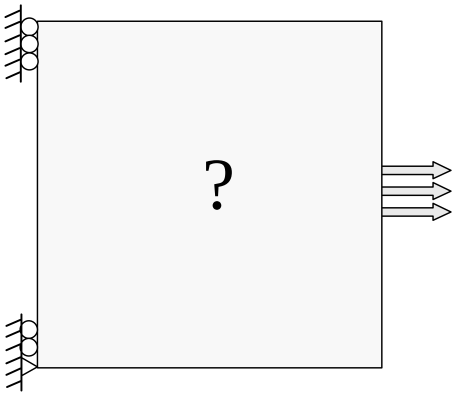

Figure 6. Definition of problem 1

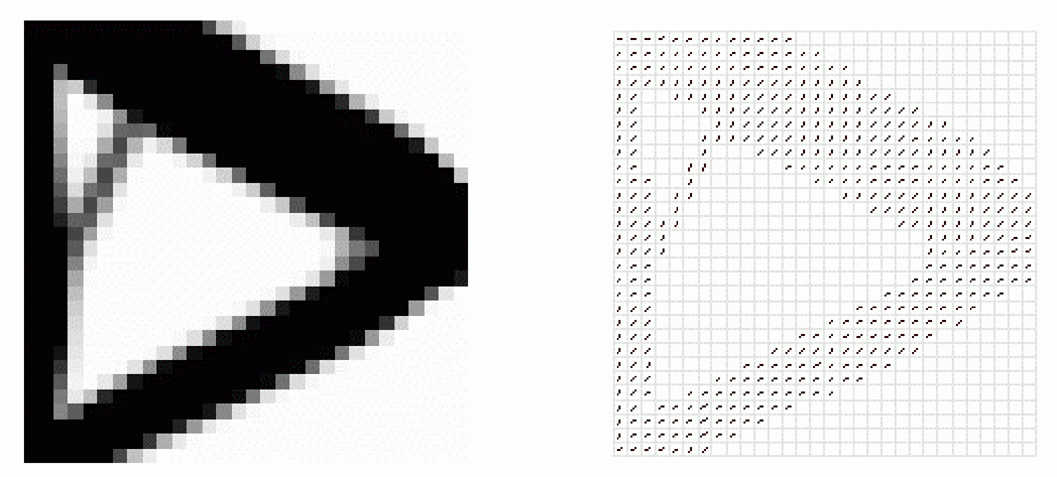

Figure 7. Solution for initial fibers oriented at 45°

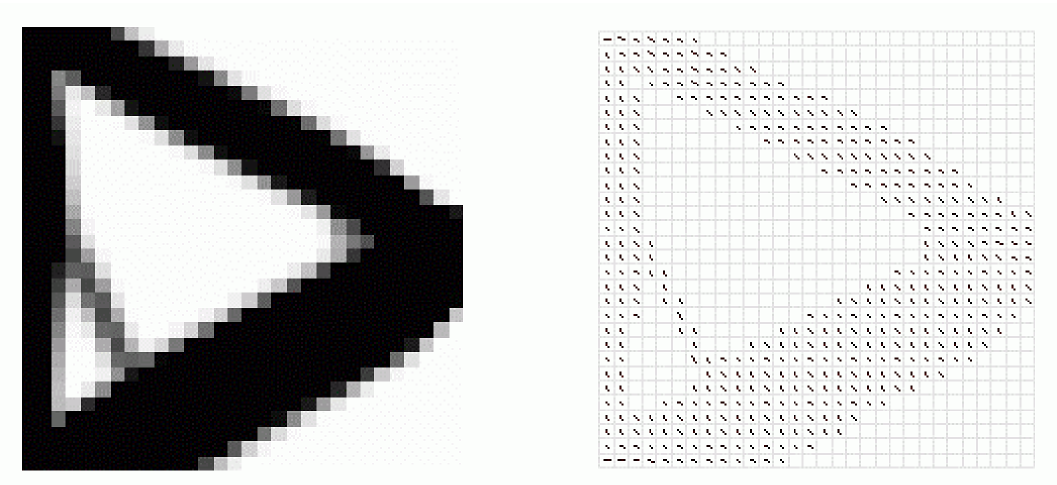

Figure 8. Solution for initial fibers oriented at -45°

Don’t lose sight that the solution provided by the optimizer corresponds to a local optimum which could be the global one. This is well highlighted in this first application, where a symmetrical solution is expected, but the optimizer reaches first a local optimum.

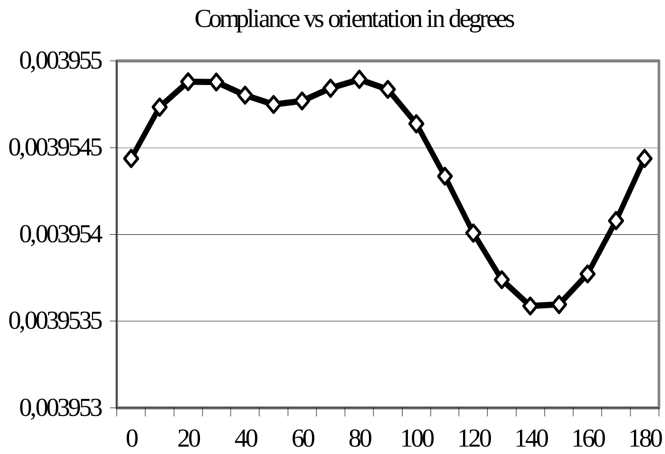

Figure 9. Evolution of the compliance with respect to the orientation in one element

6.2 Second application

The problem is defined in Figure 10. It includes 3750 design variables. The optimal topology and orientations obtained for an half of the structure are given in Figure 11. A comparison of the convergence speed for several approximations is provided in Figure 12.

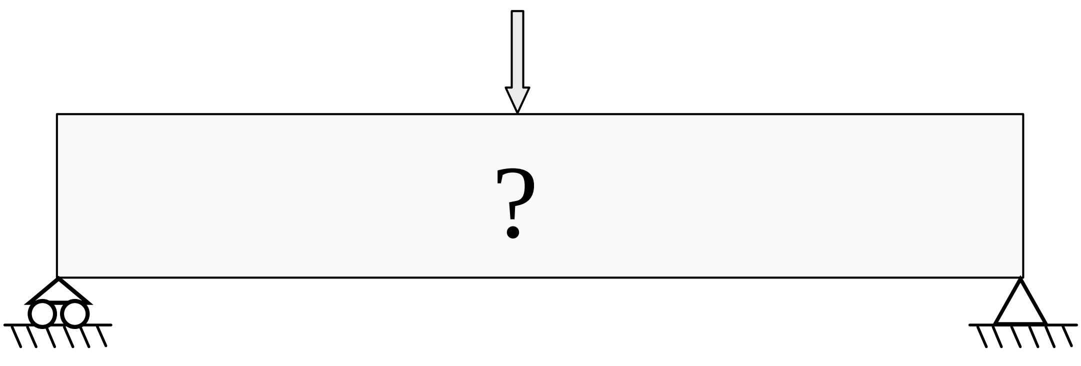

Figure 10. Definition of problem 2

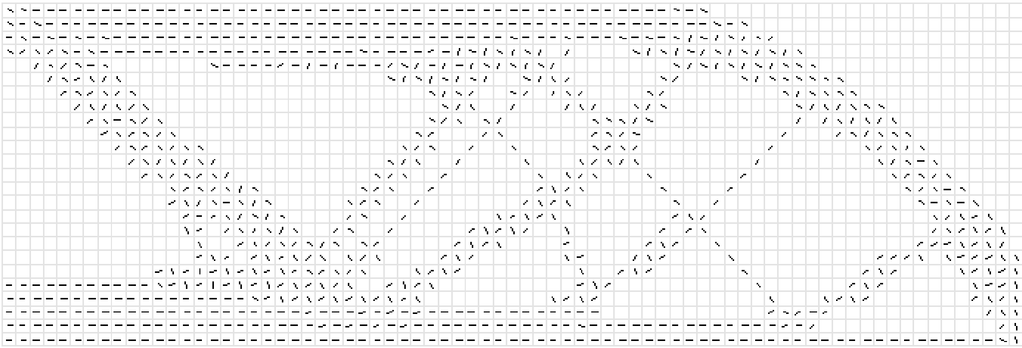

Figure 11. Optimal topology for problem 2

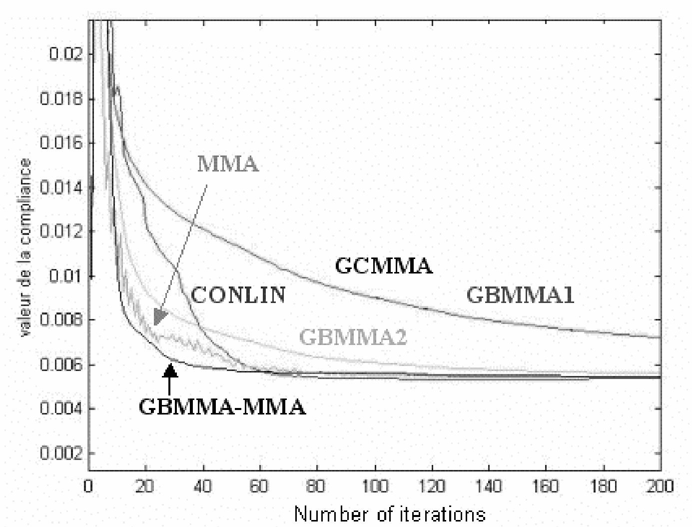

Figure 12. Comparison of the convergence speed of several approximations

6.3 Third application

The problem is defined in Figure 13. It includes 1200 design variables. Results are provided in Figures 14 and 15. The size and the boundary conditions are different from problem 2.

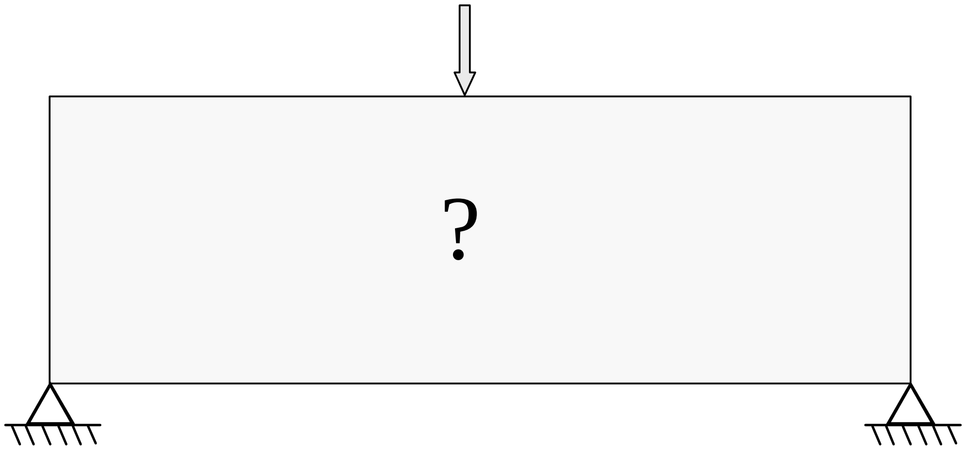

Figure 13. Definition of problem 3

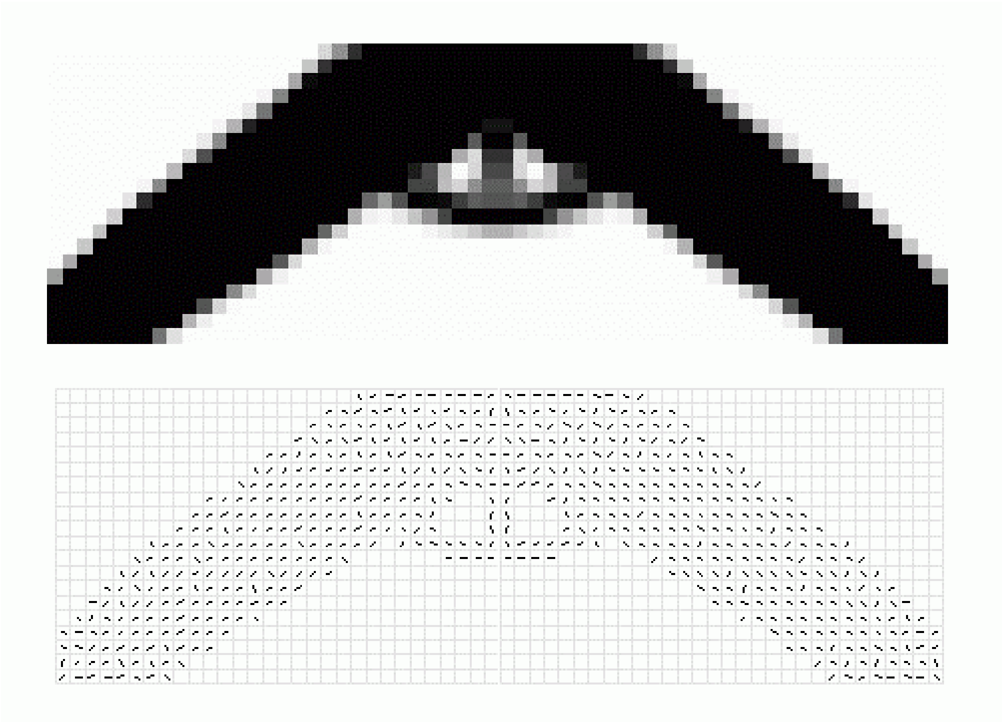

Figure 14. Optimal topology for problem 3

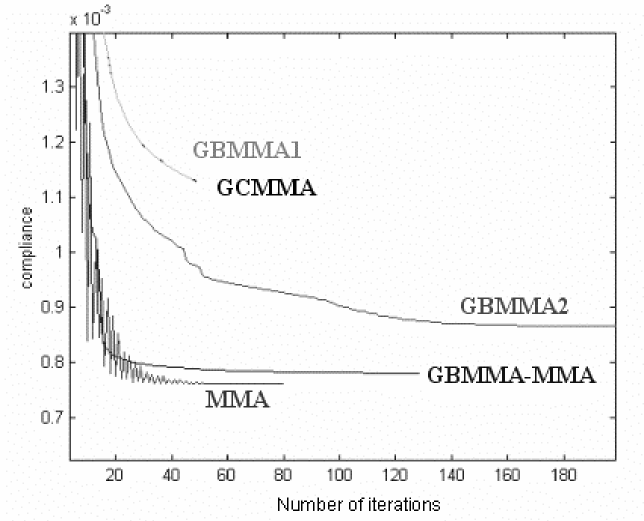

Figure 15. Comparison of the convergence speed of several approximations

7 Conclusions

The optimization procedure proved to be reliable in solving topology optimization problems including orthotropic materials. The mixed GBMMA-MMA approximation seems to be the most efficient scheme mainly due to the switch between monotonous and non-monotonous approximations. Truss-like structures are sometimes observed in the solutions: the use of a lower value for the exponent of the power law material should prevent this effect and lead to structures including intermediate densities.

Future work will study the influence of strength constraints (envelop of Tsai-Wu, Tsai-Hill,… criteria) on the optimal topology of composite structures. The related optimization problem will then include a lot of design function together with a high number of design variables. Finally, being able to consider fibers orientations as design variables will avoid the singularity of strength constraints in the optimization problem [9].

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