Struct Multidisc Optim (2008) 35:181–186
DOI 10.1007/s00158-007-0129-z

BRIEF NOTE

Discussion on some convergence problems in buckling optimisation

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Received: 2 January 2007 / Revised: 16 March 2007 / Accepted: 19 March 2007 / Published online: 3 May 2007

Abstract This note reports and briefly discusses some of the numerous reasons for bad convergence in linear buckling optimisation. Above all, it highlights that erratic convergence history can be avoided when the design optimisation problem includes enough buckling modes (and not only the first ones as it is the usual case), which keep the whole structure sensitive to the design restriction. This strategy is illustrated with an example and shows that possible significant improvement in the convergence speed can also be achieved by simply considering a large number of buckling modes in the optimisation problem. The selection of a suitable approximation scheme is also discussed.

Keywords Buckling optimisation · Approximation schemes

1 Introduction

By definition, the first buckling load λ_1 is of interest when designing a structure to withstand instability. In an optimisation problem dealing with linear stability, a small set of the lowest buckling loads (e.g. the first 12) is usually computed. They become restrictions in the design process:

$$\begin{aligned} \min \quad & Weight \\ RF_j \geq 1 \quad & \quad j = 1, \ldots, m \end{aligned} \tag{1}$$

where RF_j is the jth reserve factor, that is the jth buckling load λ_j divided by a safety factor (e.g. 1.2). Slow convergence with an erratic behaviour often occurs when solving (1), especially for large scale structures. It is explained in this note that this is partly because of an incomplete definition of the optimisation problem, which simply does not include enough representative buckling modes.

Besides buckling optimisation, difficulties of convergence may appear when vibrations or vibration-type instabilities such as flutter are studied (Haftka and Starnes 1974; Seyranian 1982; Guo et al. 2005). Such problems are mentioned for instance by Grandhi (1993) and Xie and Steven (1996). Instead of controlling the values of a set of the first frequencies as in problem (1), the frequencies of some specific (initial) modes may become constrained as well. In this case, a mode-tracking technique must be used. Solutions to the switching of vibration modes are proposed by Eldred et al. (1995) who reported that, when an ordered frequency control is addressed, which is the case in (1), a mode-tracking procedure is not mandatory so long as an appropriate problem formulation is used. The definition of a suitable formulation of the buckling optimisation problem is precisely discussed in the present paper.

2 Some reasons for poor convergence in buckling optimisation

Anyone who has carried out optimal sizing with a buckling criterion has experienced an undesirable effect of very slow convergence speed and possibly large variations of the design functions during the iteration history. Such a poor convergence can appear even when reliable optimisation methods are used, such as the convex linearization method (Conlin) of Fleury and Braibant (1986), the method of moving asymptotes (MMA) of Svanberg (1987) or the mixed monotonous/non-monotonous approximation gradi-

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ent-based MMA (GBMMA-MMA) described by Bruyneel et al. (2002). The reasons for the bad convergence of the buckling optimisation problem are multiple and make it difficult to solve. Some of them are discussed hereafter.

The first reason is linked to an approximation in the computation of the sensitivities of the linear buckling loads. The eigenvalue problem solved in modal and linear buckling analyses is recalled in (2). The sensitivities of eigenvalues are obtained by solving (3), as reported, for example, by Adelman and Haftka (1991) and Lund (2006).

$$\mathbf{K}\boldsymbol{\Phi}_j - \lambda_j \mathbf{L}\boldsymbol{\Phi}_j = 0 \quad j = 1, 2, \ldots \tag{2}$$

$$\frac{\partial \lambda_j}{\partial x_i} = \boldsymbol{\Phi}_j^T \left(\frac{\partial \mathbf{K}}{\partial x_i} - \lambda_j \frac{\partial \mathbf{L}}{\partial x_i} \right) \boldsymbol{\Phi}_j \quad j = 1, 2, \ldots \tag{3}$$

K is the global stiffness matrix, $\boldsymbol{\Phi}_j$ the jth eigenvector and λ_j the corresponding eigenvalue. In modal (vibration) analysis (not considered in this paper), **L** is the mass matrix, which only depends on the design variables x, that is $\mathbf{L}(x)$. In buckling analysis (considered in this paper), **L** is the geometric stiffness matrix (also termed the initial stress stiffness matrix), which is dependent on both the design variables and the global displacements vector $\boldsymbol{q}$, that is $\mathbf{L}[x, \boldsymbol{q}(x)]$. However, the influence of the variation of the stress state in the sensitivity is often neglected to simplify the coding and to decrease the computational time (see Mateus et al. 1997). As a result, the sensitivity analysis of the buckling loads is conducted in the same way as in vibration problems. This is, of course, a source of (a minor) error in a gradient-based optimisation method, as there is no longer any strict correspondence between a function value and its gradient. Nevertheless, because a safety margin is used for industrial problems, a strict satisfaction of the restriction is not expected, and a percentage of infeasibility is often accepted (typically 1 or 2%, i.e. a constraint is satisfied as soon as RF>0.99 or RF>0.98). This approximation on the solution feasibility balances the error made in the derivation of the buckling loads sensitivities.

The authors are aware of the sensitivity and optimisation problems related to multiple eigenvalues, especially regarding the non-differentiability of the eigenvalues. See, for example, Kim and Wallerstein (1988), Seyranian et al. (1994), Wang et al. (2004), and Du and Olhoff (2005) for specific solutions. Although the treatment of such situations is possible with SAMCEF (Système d'Analyse des Milieux Continus par Eléments Finis. Samtech, Liège, Belgium http://www.samcef.com), the finite element code used in the following example; the details are omitted here for brevity and clarity. Additionally, the work of Thompson and Hunt (1984) on the effect of imperfection sensitivity in the case of coincidence of bifurcations loads must be mentioned.

A second source of convergence problems may come from an inadequate definition of the boundary conditions, leading to the apparition of parasitic modes of low value driving the optimisation problem. It is indeed well known that the boundary conditions strongly condition the results of a buckling analysis.

The selection of a reliable optimisation algorithm is important as well. Monotonous approximation schemes like Conlin (Fleury and Braibant 1986), which do not include an internal move-limit strategy, should be used with care. Approximation schemes like GBMMA-MMA (Bruyneel et al. 2002) use the information at two successive design points to build the approximation curvature. As modes crossing (this point is not discussed here) happens in buckling optimisation, such methods generate an envelope of the responses that can help in finding a solution. Additionally, the updating scheme of the moving asymptotes provides an adaptive move-limit strategy (internal to the optimiser) that can help in converging (but not necessarily in a monotonous way).

The aim of this note is primarily to bring to light another cause of erratic convergence, which is shown to come from an incomplete formulation of the optimisation problem. The example presented in the following section shows that, for some designs, the first buckling modes may not be representative of the overall structure. It turns out that, at a given iteration, the buckling modes may only influence a small part of the structure, which will be designed, while the remaining structural parts are not sensitive. The panel thickness in the sensitive part could increase to satisfy the stability criteria, while the thickness in the insensitive part will certainly reach its lower bound, because according to problem (1), the weight is to be minimised. At the next iteration, the low-thickness part is likely to become sensitive to buckling because of a small local stiffness, while the remaining part could become insensitive to the restrictions. If repeated, this scenario leads to oscillations and, depending on the optimisation method, either seriously slows down the optimisation process or leads to a lack of convergence.

When simple panels of limited size including few stiffeners are studied, local buckling modes are less likely to appear (in the set of the first modes). Additionally, in such cases, the thickness design variables may be defined over wide structural regions, and the evolution of their value is driven by global and possible local buckling modes. It turns out that the values of the design variables are not only driven by a target on a minimum weight but also by buckling considerations. When larger structures are studied, some design variables can become blind to the first (local) buckling modes used in the optimisation problem. In such a case, the convergence difficulty discussed above is prone to occur.

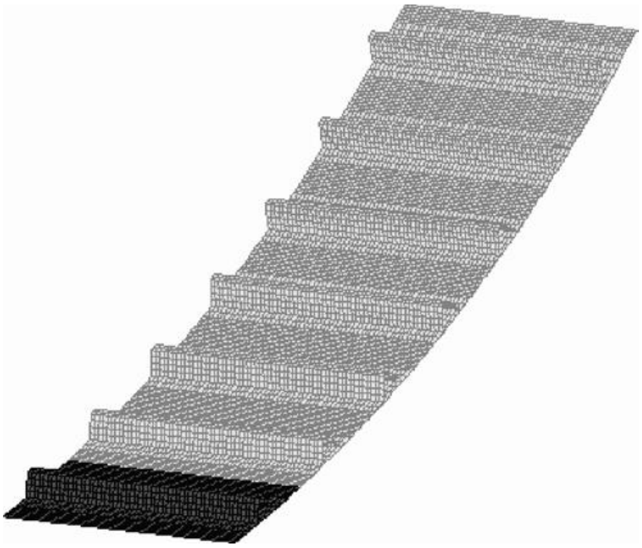


Fig. 1 Geometry of the stiffened panel and one super stiffener in *black*

3 Example

We consider a curved composite panel including seven hat stiffeners. The load case consists of compression along the long curved sides and in shear on the whole outline. The structure is simply supported on its edges. The stiffeners are fastened to the panel with bushing elements. In each super stiffener (in black in Fig. 1), three design variables are used for defining the thickness of the 0, 90 and ±45° plies in the panel and in the stiffener. Forty-two design variables are then defined. The structural and semi-analytical sensitivity analyses are carried out with SAMCEF. The Boss/Quattro optimisation tool box is used for defining and solving the optimisation problem (Radovcic and Remouchamps 2002). The results obtained with Conlin (Fleury and Braibant 1986) and GBMMA-MMA (Bruyneel et al. 2002; Bruyneel 2006) are compared. In Figs. 2, 3, 4, 5 and 6, the evolutions of the weight and the first buckling load λ_1 over the iterations are plotted.

In Fig. 2, the first 12 modes are considered in the optimisation problem (1), and Conlin is used. Oscillations appear, and a solution is never reached (λ_1 strongly violates the constraint). When the GBMMA-MMA approximation is selected (Fig. 3), the convergence is slow, but an optimum is obtained. A divergence in the process occurs between iterations 5 and 20. The general shape of the modes is provided in Fig. 3 for iteration 11. Those buckling modes only have an impact on given parts of the structure and are insensitive to some design variables. This explains the decrease of the weight at the next iteration given that some design variables are guided only by a weight minimisation and reach their minimum allowable value. Conlin undergoes a similar effect (see iterations 49 and 50 in Fig. 2) but is not able to reverse this bad tendency contrary to GBMMA-MMA, which seems more reliable certainly thanks to its non-monotonous behaviour, its ability to generate an envelope of the constraints responses and its internal adaptive move-limit strategy.

In Fig. 4, the first 100 modes are taken into account in the definition of the optimisation problem. The convergence of GBMMA-MMA is very fast because only six

Fig. 2 Iteration history for Conlin. Optimisation with the first 12 buckling modes

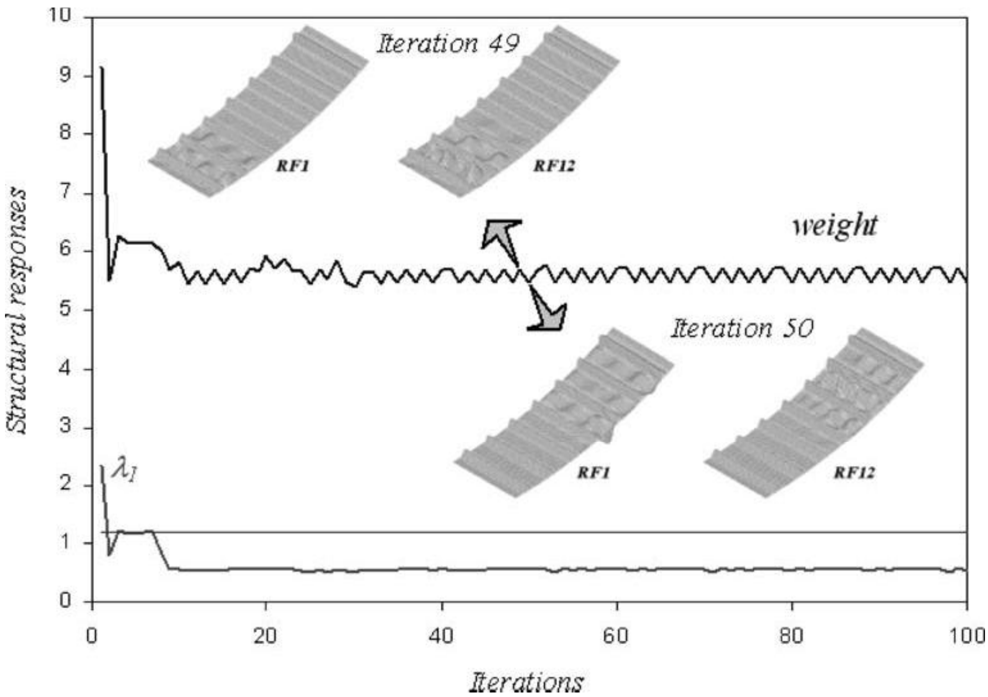
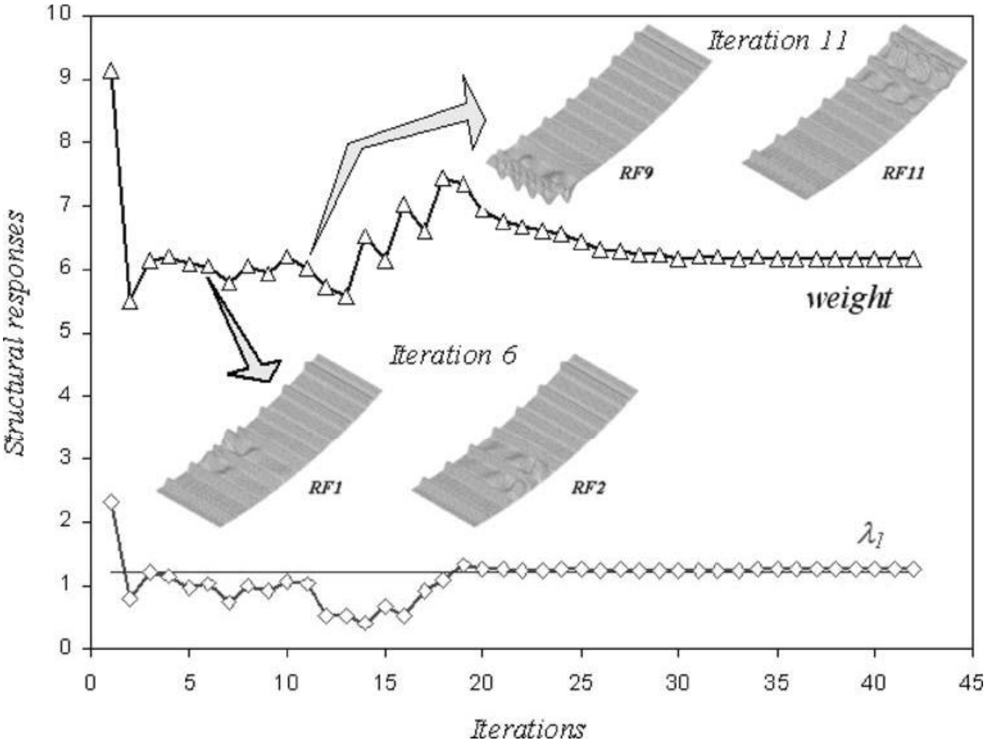
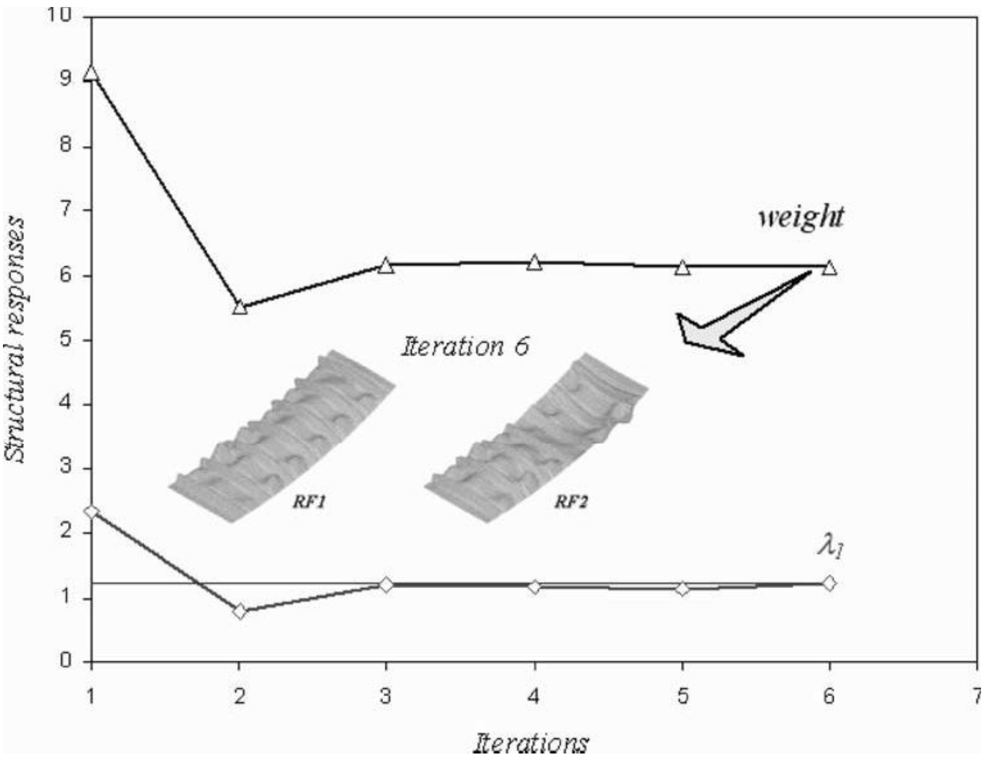


Fig. 3 Iteration history for GBMMA-MMA. Optimisation with the first 12 buckling modes



iterations are needed to satisfy the restrictions with a relative variation of weight less than 0.01%. Contrary to what is shown in Fig. 3 (buckling modes at iteration 6), modes exist that make the whole structure significantly sensitive to buckling. Note that the first two buckling modes at iteration 6 in Figs. 3 and 4 are not the same, as the number of restrictions is different as is the approximated optimisation problem built, in this case, with GBMMA-MMA. When Conlin is used, the process is not able to converge (within 100 iterations) even when a large number of modes is considered in the optimisation problem. However, large oscillations disappeared in the evolution

Fig. 4 Iteration history for GBMMA-MMA. Optimisation with the first 100 buckling modes



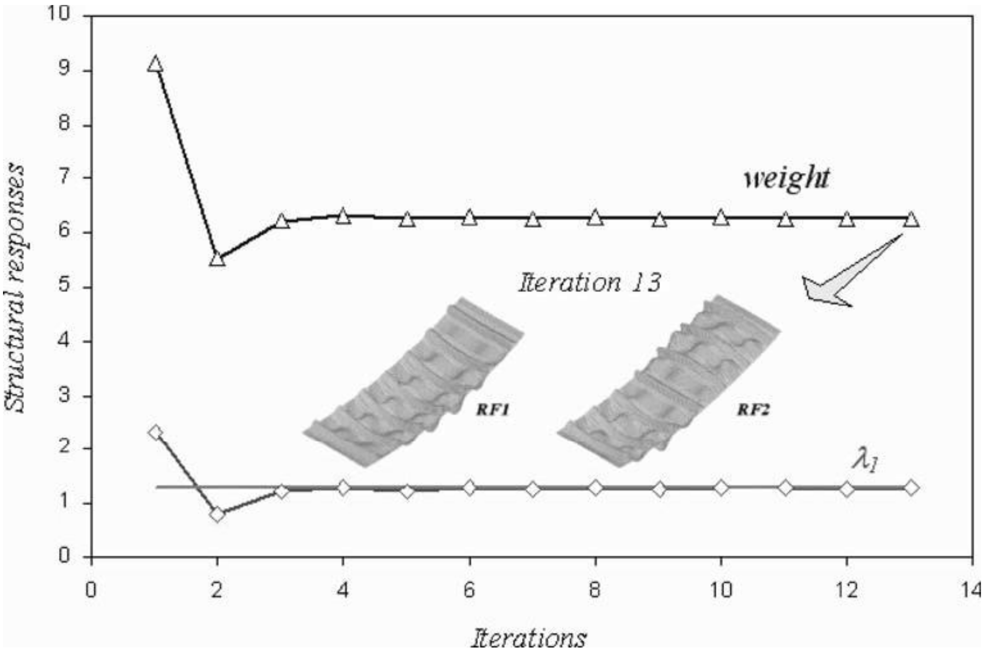


Fig. 5 Iteration history for GBMMA-MMA. Optimisation with the first 12 buckling modes and only six design variables

of RF_1, which moreover succeeds in reaching the value of 0.90. Some design variables change their value very slowly, and some others display significant oscillations. The use of external move limits with Conlin would not help in finding a solution. When the MMA approximation is used for solving this problem, a solution is found after 44 iterations, thanks to its adaptive internal move-limits strategy.

Figure 5 shows the convergence history for the curved composite panel of Fig. 1 including only six design variables (three for the plies in the full panel and three for the plies in the set of stiffeners). Twelve buckling modes define the set of design restrictions. When GBMMA-MMA is used, 13 iterations are sufficient to obtain a converged feasible solution. Additionally, the convergence is monotonous (no oscillations).

Finally, the problem of the maximisation of the buckling loads with 42 design variables is solved. The first 12 modes are taken as objective functions in a multi-objective formulation. The mass is restricted to the optimal value obtained earlier. It can be seen in Fig. 6 that the process converges monotonically. Large oscillations seem to be less prone to appear in such a formulation of the optimisation problem. It is interesting to note that, for such a problem, Conlin succeeds in finding a solution in 12 iterations without erratic convergence behaviour.

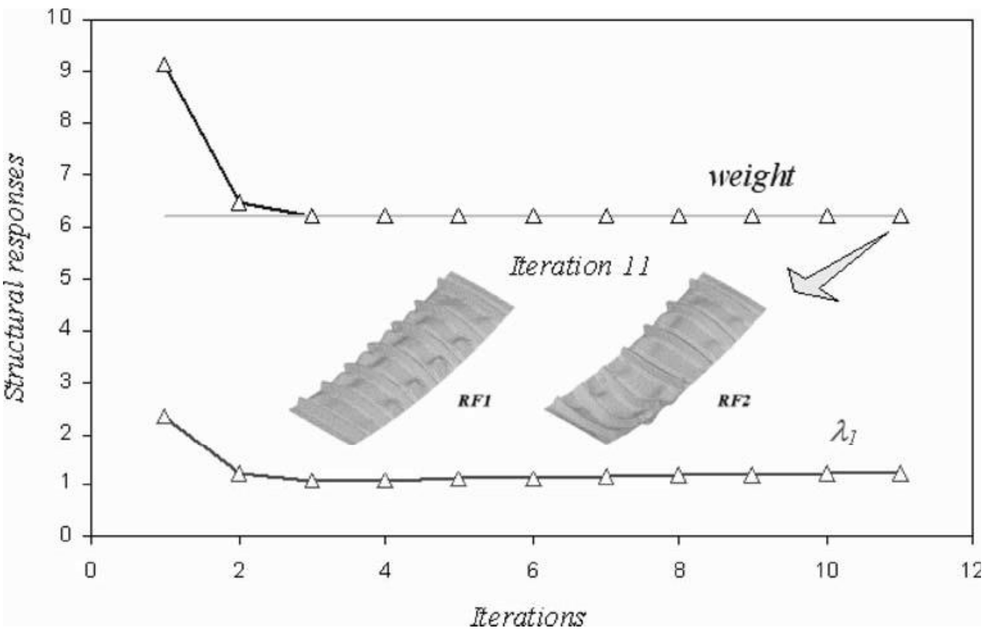


Fig. 6 Iteration history for GBMMA-MMA. Optimisation with the first 12 buckling modes as objective functions and 42 design variables