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Computing the Land Subsidence of Shanghai by a Finite Element Method

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ABSTRACT An accurate simulation by a FEM code has been performed on the case of the subsiding area of Shanghai. In this big city, the subsidence phenomena, caused by groundwater withdrawals, reached an approximate total value of 2.5 meters since 1920. Since 1962, the recharge of water during winters contributed to decelerate the phenomena but a residual consolidation about 3 mm a year is still recorded.
The study area is limited to the central zone of Shanghai and to the upper 70 meters of loose sediments. Quaternary geology, engineering geology and hydrogeology aspects have been examined in details previously to the mathematical simulation, in order to prepare all the needed data. A Finite Element Method (FEM) code has been chosen, allowing a very accurate spatial discretization taking into account heterogeneities and facies variations of the layers. The simulation has comprised a 3D flow model giving as results the values and spatial distributions of the water pressures at each time step. Then, a coupled non-linear flow-compaction model has computed the subsidence in function of time, taking the pressure variations in the aquifers as stress data. After the calibration procedure, simulations have been computed with "neutral" conditions (recharge ≥ pumping) and with "intensive pumping" conditions (pumping = 1.3*recharge). Additional compactions of 1.4 to 7.9 cm are computed for the 1989-2000 period with these last conditions.

INTRODUCTION

The city of Shanghai is situated in the vast low-lying coastal plain (usually called the Yangtze delta) characterized by the lower reach of the Yangtze river (Baeteman & Schroeder, 1990). Due to ground-water withdrawal, mainly from an aquifer situated between 60 and 80 m of depth, land subsidence occurred drastically. It was noticed as from 1921 but it reached 2.5 m to 3 m with a maximum annual rate of 98 mm between 1956 and 1959. Although the total thickness of the loose sediments is about 300 m, the observation data have indicated that 65 % to 85 % of the total subsidence occurred in the upper 70 m portion.

In order to manage the ground-water production and hence control the land subsidence, a mathematical model based on finite element method has been designed. Therefore intensive investigations of the quaternary geology, the hydrogeology and the engineering geology were carried out simultaneously, collecting new data and numerous old data from Shanghai geologists and engineers.
This work has been completed together by the Shanghai Geological Center (P.R. China), the Belgian Geological Survey and the Laboratory of Engineering Geology, Hydrogeology and Geophysical Prospecting of the University of Liège (Belgium) (Monjoie et al. 1990). Very detailed data were thus available concerning the coastal lowland geology, the engineering geology and the hydrogeology, to provide the basic elements for the design of the mathematical model (Dassargues et al. 1990).
The geological setting of the Shanghai area (until a depth of about 70 m) has been subdivided into significant lithological units (Fig. 1) on basis of environmental analysis (Baeteman & Schroeder, 1990). The main hydro-engineering geology characteristics are summarized in the papers of Dassargues et al. (1990) and Baeteman & Schroeder (1990).
The total cumulative subsidence given by Fig.2 (Su, 1984 and Shi & Bao, 1979) shows a stabilization from 1963-1965 until 1985, due to the intensive recharge in the second aquifer (2A) in winter. Before the recharge has began the pore pressure maps showed two plate-shaped depression in the urban district, in accordance with the location of the main pumping. Fig.3 illustrates the lowest pore pressures reached in the subsoil of Shanghai before any recharge, in 1960. After 1965, a relative stabilization of the subsidence is obtained after the small elastic rebound, since this date a residual subsidence about 3 mm/year is still recorded and a lot of measurement data have been collected by the Shanghai Geological Center confirming this fact.

sedimentary environment		lithological unit	
fluvial	estuarine & coastal		
	intertidal and subtidal	1C unit	HOLOCENE
	salt marsh (supra tidal)	2C unit	
flood basin and backswamp		DCSC unit	UPPER PLEISTOCENE
natural levee and channel		1A upper unit	
	channel and sand bars	1A lower unit	
	intertidal and subtidal	3C unit	
	channel and sand bars	2A unit	

FIG. 1 Stratigraphic sequence of the Pleistocene and Holocene deposits (from Baeteman & Schroeder, 1990).

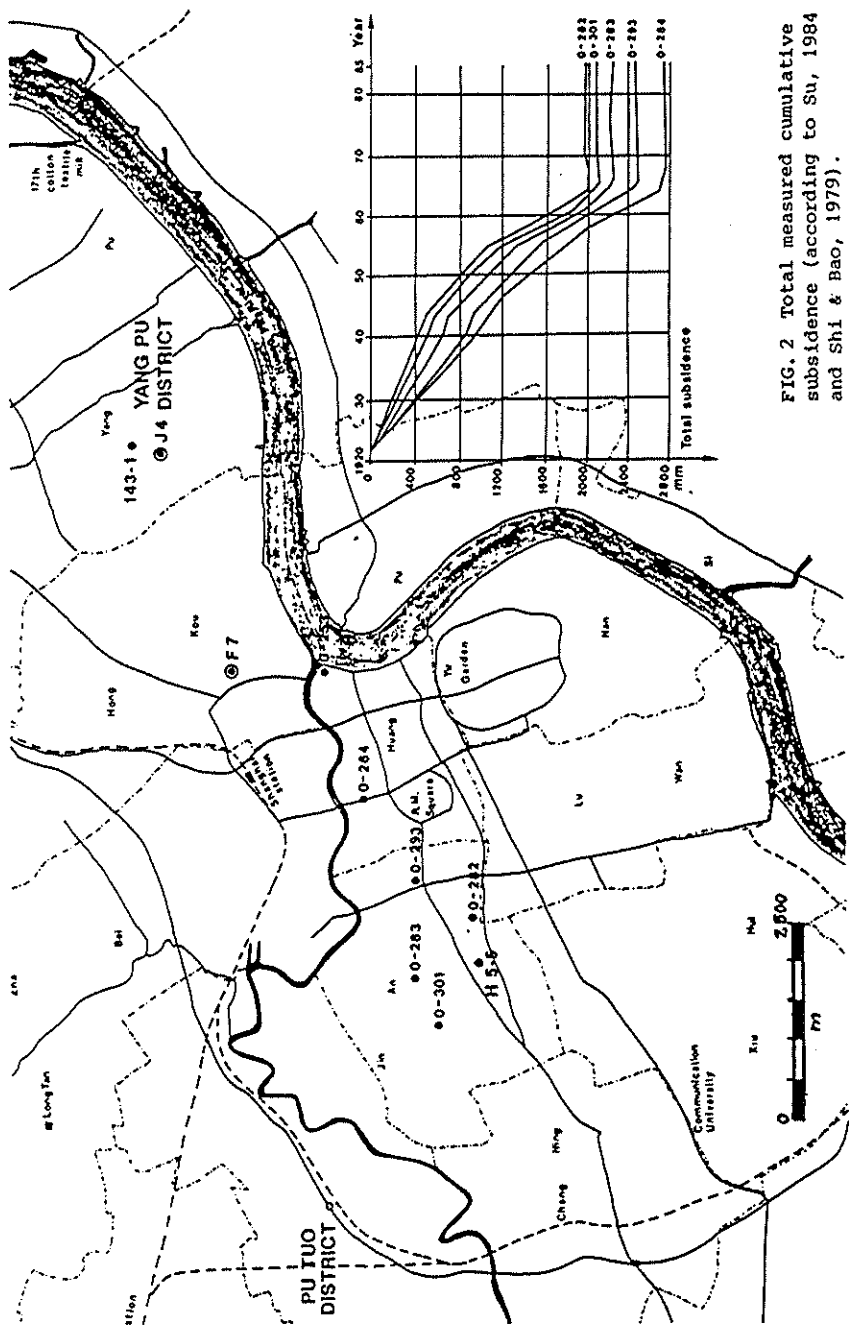

FIG. 2 Total measured cumulative subsidence (according to Su, 1984 and Shi & Bao, 1979).

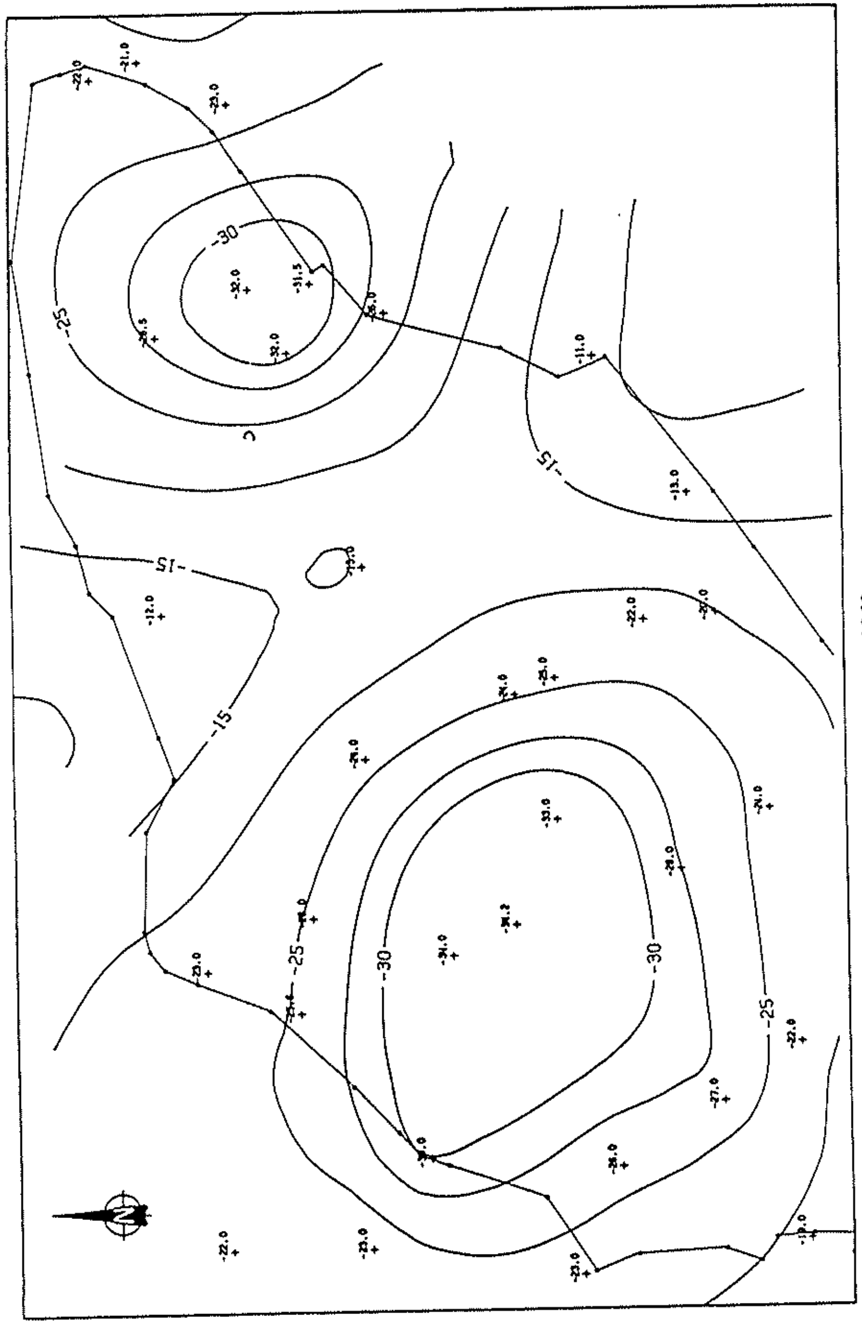

FIG. 3 Measured piezometric map in Shanghai, September 1960.

MAIN FEATURES OF THE NUMERICAL FORMULATION

The land subsidence model of Shanghai includes a transient 3D flow model and a coupled non-linear 1D flow-compaction model. The results of the flow model are the time dependent boundary conditions of the consolidation model. The models are implemented in the finite element code called LAGAMINE developed by the MSM department, University of Liège, since 1982. Large strains, both geometrical and material non-linear problems, mechanical problems, thermal conduction, seepage, ...can be modeled using this code (Charlier et al. 1988). It has been successfully applied in 1985-1987 to calculate the subsidence of the Ekofisk oil-field (Schroeder et al. 1988).

The Flow Model

The finite element method, based on the virtual power principle is applied. The internal virtual power (δW_i) and the external virtual power are expressed for a seepage problem (Charlier, 1987). The Gaussian numerical integration scheme is used on isoparametric 8-nodes brick-like elements.
Two constitutive laws are used : Darcy's law gives (Charlier et al. 1988) :

$$\underline{f} = - \frac{\underline{\underline{K}}}{\gamma_w} \, \underline{grad}\ p + \underline{grad}\ z \tag{1}$$

where K is the permeability tensor. γ_w the specific weight of water, p the pore pressure and z the vertical coordinate.
For the 3D flow model, the law is considered to be linear : K is isotropic and assumed to be constant. We suppose also that the the volumetric strain variation f^* is equal to the volume of water expelled during the compaction ('storage flow'): $f^* = \dot{V}/V = 3\dot{\epsilon}_m$ (2)
Using the Terzaghi principle expressed with mean stresses, $\sigma = \sigma' - p$ is written with tensions as positive stresses. If σ^* is supposed to be constant, we obtain $\dot{\sigma}' - \dot{p} = 0$ and $\dot{\sigma}^{*\prime} = \dot{p}$. Equation (2) can be written :

$$f^* = 3\,\dot{\sigma}'_m / \chi = 3\,\dot{p} / \chi = C_p \dot{p} \tag{3}$$

C_p is the storage coefficient expressed in terms of pressure (Dassargues et al. 1988), $C_p = 3/\chi$ with $\chi = E / (1-2\nu)$
Time integration must be realized in the transient flow problem; a Galerkin time scheme has been used for the present model.

Coupled flow-compaction finite element model
(Charlier et al. 1990).

The settlement of a "plate-like" aquifer system can be considered as essentially a unidimensional vertical problem. Hereafter, we use the Cauchy stress tensor σ and the Cauchy conjugated strain tensor $\underline{\epsilon}$. For the 'oedometric' behaviour of the clayey soils, the effective stress tensor σ' and strain tensor are reduced to :

$$\sigma'_{11} \neq 0\ ,\ \sigma'_{22} = \sigma'_{33} \neq 0 \text{ and } \sigma'_{12} = \sigma'_{23} = \sigma'_{31} = 0$$
$$\epsilon_{11} \neq 0\ ,\ \epsilon_{22} = \epsilon_{33} = 0 \text{ and } \epsilon_{12} = \epsilon_{23} = \epsilon_{31} = 0 \tag{4}$$

The virtual internal mechanical power is easily obtained by :

$$\delta W_{im} = \int_V \sigma_{ij}\, \delta\epsilon_{ij}\, dv = \int_V \sigma_{11}\, \delta\epsilon_{11}\, dv \tag{5}$$

The axial strain rate of an element is : $\dot{\epsilon}_{11} = \dot{L} / L_o$,
L represents the length variation rate, L_o is the initial length of the "element". Only flow in the vertical direction is allowed in oedometer tests, so that the expression of the internal

virtual flow power (δW_{if}) is reduced in consequence. We suppose that the elements are only subjected to gravity and the external virtual power (δW_E) is written consequently.
Based on the virtual power principle, equilibrium is obtained when $\delta W_{im} + \delta W_{if} = \delta W_E$ and nodal equivalent forces and flows are then easily got using the Gaussian numerical integration scheme (Charlier et al. 1990). A fully implicit time integration scheme has been adopted.
The following constitutive laws are concerned :
. The Terzaghi principle (Terzaghi, 1943) : $\sigma' = \sigma + p$ (6)
. The oedometer mechanical law (illustrated Fig. 4) can be transformed to an elastoplastic incremental form as follows :

$$\dot{\sigma}'_{11} = -\sigma'_{11} . A . \dot{\epsilon}_{11} \qquad \text{if } |\sigma'_{11}| < |\sigma'_c|$$
$$\left.\begin{array}{l} \dot{\sigma}'_{11} = -\sigma'_{11} . C . \dot{\epsilon}_{11} \\ \dot{\sigma}_c = \dot{\sigma}'_{11} \end{array}\right\} \qquad \text{if } |\sigma'_{11}| = |\sigma'_c| \qquad (7)$$
$$|\sigma'_{11}| > |\sigma'_c| \qquad \text{impossible}$$

where σ' is the extreme stress encountered by the soil through its whole history, σ'_{11} represents the yield threshold, it is an internal variable.

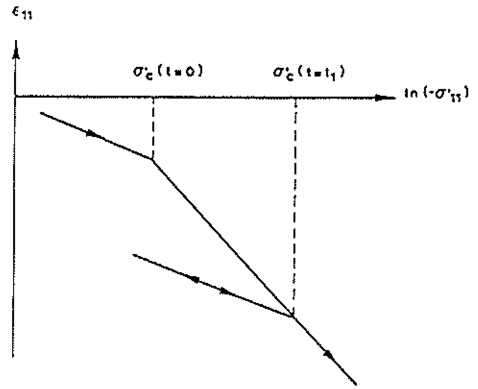

FIG. 4 Oedometer mechanical response.

.The storage is equal to the volumic strain rate because water and soil grains are assumed to be incompressible (equation 3).
.The Darcy's law with a non-linear relation concerning K is inducted. The chosen variation law was developed originally by Nishida & Nakagawa, 1969. Its transformed form is written (Dassargues, 1990) : $K = \exp(\alpha_e\, e + \beta_e)$ (K in m/s) (8)
where e is the void ratio, α_e and β_e are new material parameters related to plasticity index I_p and are obtained from laboratory tests providing I_p, e and K of the same samples.
We assume that volume variation is only due to void variation :
$$\dot{V}_{total} = \dot{V}_v = \dot{\epsilon}_{11}(V_v + V_s)$$
$$\dot{e} = \dot{V}_v/V_s = \dot{V}_{total}/V_s = \dot{\epsilon}_{11}.(V_v + V_s)/V_s = \dot{\epsilon}_{11}(1+e) \qquad (9)$$
In summary, the relations mentioned above form, non-linear constitutive coupled laws. Where A, C, α, β, γ, γ are the parameters which can be determined from laboratory test and σ, σ', σ' and e are internal variables.
The Newton-Raphson technique is used to find the solution.

APPLICATION TO SHANGHAI SUBSIDENCE

The 3D flow model mentioned above with a complete discretization of the soil layers is implemented in the Lagamine code to compute the spatial distribution of the pore pressure in function of time. The study area is divided into 10 layers of 205 8-nodes brick elements each. Fig. 5 and Fig. 6 show the discretization pattern.

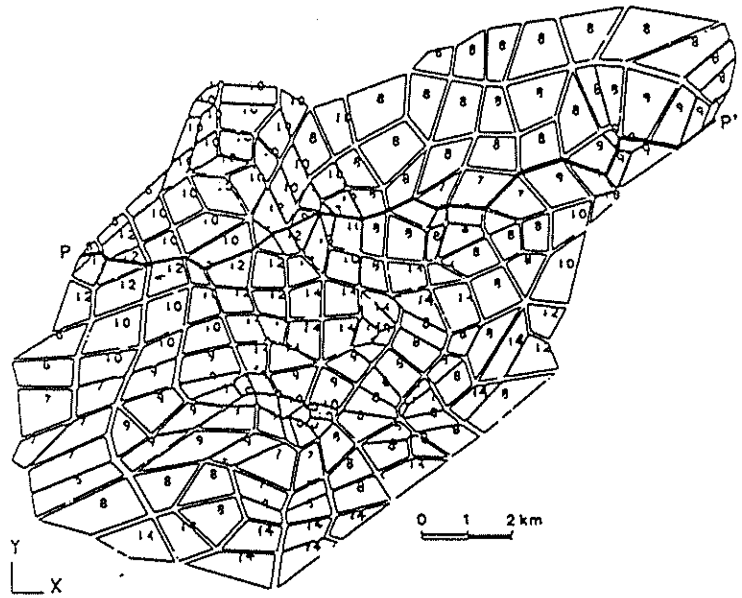

FIG. 5 Discretization of the fifth layer of the structure - Material classes distribution.

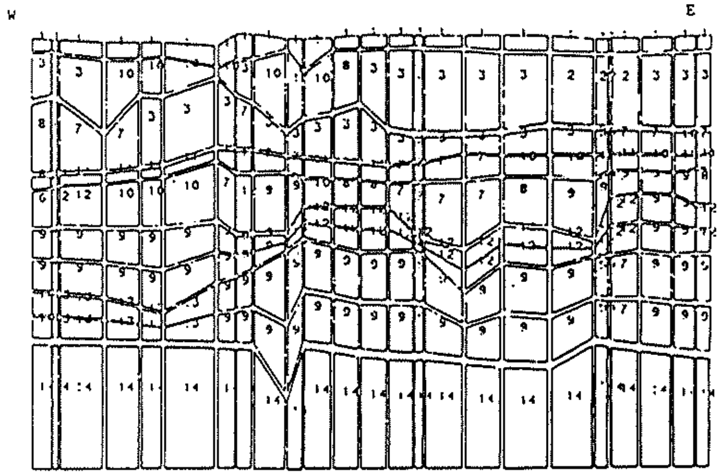

FIG. 6 One of the vertical cross-section in the mesh: section P-P'.

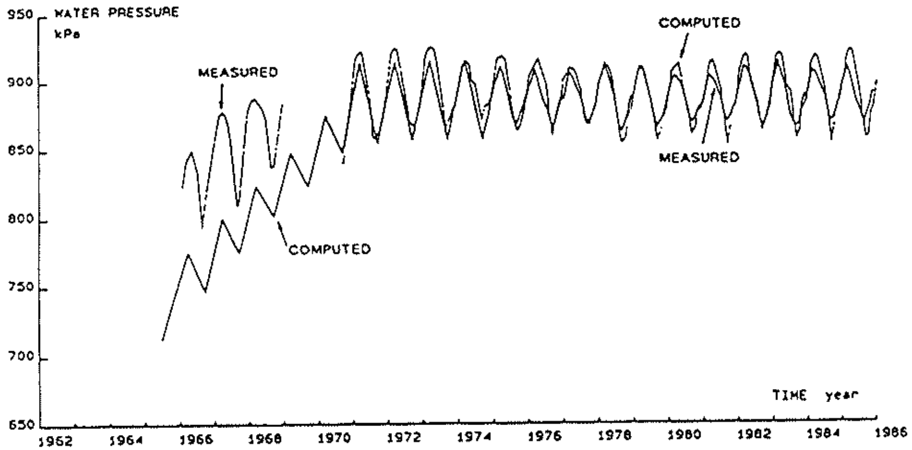

FIG. 7a Computed water pressure vs time at the column below the node 97.

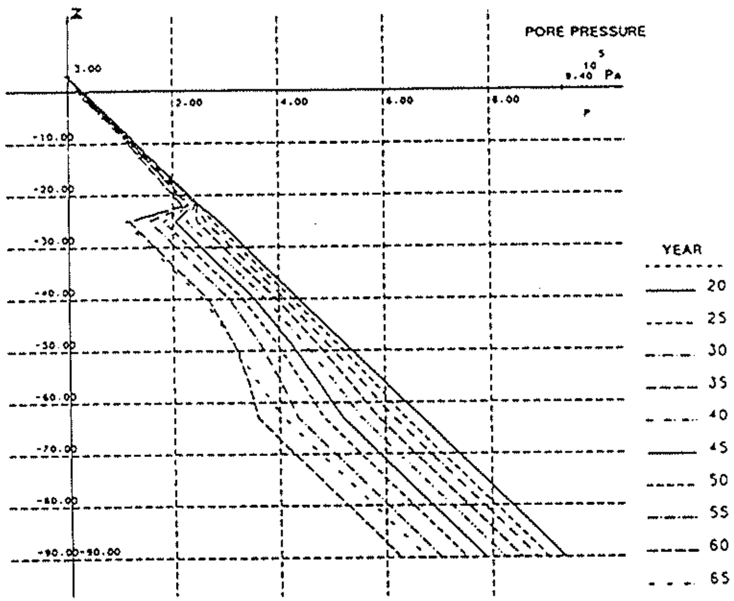

FIG. 7b Computed water pressure vs depth below the node 137 for different years.

The spatial distribution of the two parameters (K and S) is practically realized by using material classes for which the parameters are defined. Fourteen classes of materials are used in the model. As an example, Fig. 5 shows the material classes distribution in the 5th layer.

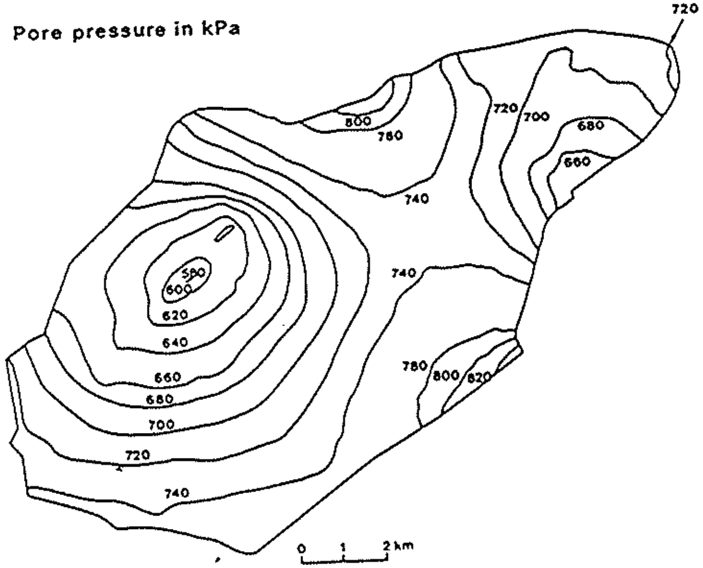

FIG. 8a Computed pore pressure map for the situation in 1960.

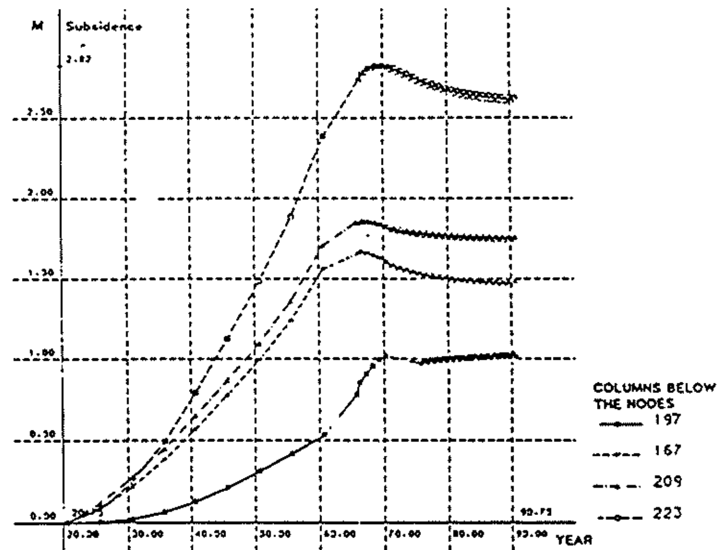

FIG. 8b Computed subsidence vs time in 4 of the 32 columns.

TABLE 1 Computed subsidence values for different running of the model.

1st -> 1990	Coupled and non-linear model (different running) 2nd	3rd	4th	1990	P=1.3xR -->2000	Additional subsidence
0.55	0.960	1.019	-	1.019	1.033	0.014
1.02	1.470	-	-	1.470	1.507	0.037
1.62	1.754	-	-	1.754	1.779	0.025
2.80	2.683	2.611	-	2.611	2.643	0.032
1.25	1.623	-	-	1.623	1.687	0.064
2.17	2.198	-	-	2.198	2.238	0.040
1.67	2.048	-	-	2.048	2.096	0.048
2.33	2.325	-	-	2.325	2.380	0.055
0.52	0.827	0.998	-	0.998	1.033	0.035
1.63	2.278	-	-	2.278	2.311	0.033
1.58	2.259	-	-	2.259	2.303	0.044
2.17	2.280	-	-	2.280	2.319	0.039
1.23	1.542	1.708	1.881	1.881	1.946	0.065
1.97	2.525	-	-	2.525	2.577	0.052
1.40	1.688	1.832	-	1.832	1.877	0.045
1.51	1.982	-	-	1.982	2.031	0.049
0.95	1.492	-	-	1.492	1.519	0.027
1.72	1.718	-	-	1.718	1.761	0.043
1.42	1.563	-	-	1.563	1.602	0.039
0.96	1.207	-	-	1.207	1.235	0.028
2.06	2.032	-	-	2.092	2.125	0.033
2.05	2.085	-	-	2.085	2.124	0.039
1.34	1.807	-	-	1.807	1.855	0.048
1.58	2.281	1.532	1.867	1.867	1.912	0.045
1.29	1.630	-	-	1.630	1.677	0.047
1.05	1.250	-	-	1.250	1.276	0.026
1.23	1.697	-	-	1.697	1.740	0.043
0.82	1.288	1.319	-	1.319	1.345	0.026
3.12	2.948	-	-	2.948	3.027	0.079
1.79	2.187	-	-	2.187	2.250	0.063
1.72	1.602	1.742	-	1.742	1.778	0.036
1.63	1.556	1.549	1.815	1.815	1.849	0.034

In our analysis, initial state of pore-pressure is chosen in geostatic equilibrium corresponding to the situation before 1920. The boundary are assumed impervious at the bottom and at the top of the model, and imposed varying pressure with time laterally.

K and S determined by various soil test data and several pumping test dâta, have been slightly modified during "trial and error" calibration. Fig. 7 shows examples of pore-pressure computed results. The choice of the time step is mainly based on the measurements frequency in the pumping/recharge data.
As mentioned above, the subsidence computations are completed with a 1D coupled flow-compaction model. In our study area, 32 columns are chosen to calculate the subsidence. Each column contains 60 elements. The simulation is carried out prescribing the water pressure (obtained by the flow-model), at the aquifer/aquitard boundaries of the modeled columns. These prescribed water pressure are variable in time. Moreover, the non-linear variations of the permeability coefficient K and the specific storage coefficient S are taken into account in the model. Both K and S are actualized at each time step. The variation of K is perfofmed by adapted Nishida relation and the S_s varies in the following way (Poland,1984 and Dassargues,1990):

$$S_s = \rho.g.\alpha \qquad (\alpha \text{ : compressibility of the porous media}) \qquad (10)$$

$$\text{and} \begin{cases} S_s = \gamma_w /A \,.\, \sigma'_{ii} & \text{(in the elastic part)} \\ S_s = \gamma_w /C \,.\, \sigma'_{ii} & \text{(in the plastic part)} \end{cases} \qquad (11)$$

The parameters such as A,C,the preconsolidation stress σ', α, β, the initial void ratio of 1920 e are calculated element by element using laboratory test results and some empirical relations. Unfortunately, we have only a small amount of target points available for the calibration, where total subsidence is recorded before 1965 (located on Fig. 2), moreover the recorded subsidence is relative to the compaction of the 300 meters of loose sediments. The part due to the upper 70 m is not known with accuracy. During the calibration, the computed subsidence are checked to be comprised in the measurement ranges.
For the columns situated for away from these measurement points, the lack of available data has forced to make a kind of "blind calibration" to stay in a range of assumed logical values. Of course, the reliability of detailed results is affected by this fact. Some of the results are drawn on Fig. 8 , detailed computed results are shown in table 1.

PREDICTION OF THE LAND SUBSIDENCE

A simulation of the future water pressure distribution is completed with pumping = 1.3 x Recharge, and computed future subsidences between 1988 - 2000 are found out. The computed additional compactions would be comprised between 1.4 and 7.9 cm (table 1).

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