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EXPLOITING SEMIANALYTICAL SENSITIVITIES FROM LINEAR AND NONLINEAR FINITE ELEMENT ANALYSES FOR COMPOSITE PANEL OPTIMIZATION

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This paper presents a solution procedure developed in the SAMCEF finite element code for the advanced optimal design of stiffened composite panels of an aircraft fuselage. The BOSS Quattro, a task manager and optimization toolbox, is used for defining and running the optimization problem. The objective function to be minimized is the weight, and the restrictions depend on structural stability requirements, such as buckling and collapse. The design variables are the panel and stringer thicknesses of the conventional proportions (i.e. $0^\circ, 90^\circ$ and $\pm 45^\circ$) in a homogenized laminate. Since a collapse analysis introduces geometric nonlinearities into the design process, the function evaluation can take a long time. In order to obtain a rapid optimal solution, a gradient-based method is used, and the first order derivatives need to be computed, in this case with an original semianalytical approach. The sensitivity analysis of buckling and collapse is reviewed. Numerical tests on an industrial case study demonstrate the possibility and the reliability of the approach. Solving such problems is clearly difficult and remains a challenge. Through the applications, this paper provides the opportunity to discuss convergence issues and the use of such advanced optimization techniques in the overall aircraft design process.

Keywords: Composite panels; buckling, collapse; sensitivity analysis; optimization.

1. Introduction

Modern aeronautical structures are increasingly made of composite materials. In order to take advantage of their anisotropy, their high stiffness and strength-to-weight

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ratios, to benefit from further weight reductions and to propose safe designs, complex structural analysis is needed. Numerical optimization techniques may further support experienced users in finding mass efficient design solutions.

The goal of structural optimization is to automatically determine optimal designs satisfying structural requirements by modifying the values of selected design variables. Optimization has reached a certain maturity and is now well established in niche applications at an industrial level.[1–3] Several methods can be used to solve a structural optimization problem, including genetic algorithms,[4–6] response surface methods, coupling surrogate models and genetic algorithms (surrogate-based optimization with neural networks, for instance),[7–11] or gradient-based methods, as is the case in this paper. A comparison of such methods for solving buckling and collapse optimization in industrial test cases is conducted in Ref. 12.

The finite element approach is essential for simulating the behavior of mechanical systems and components of complex geometry and material properties. As long as compression and shear are present in a structure, it must be designed to withstand buckling.[13] Classically the buckling load factors are obtained by solving an eigenvalue problem around a linearized configuration. Despite a great deal of effort devoted to this topic,[14–16] handling buckling optimization for industrial applications is still an issue. Oscillations usually appear during the iterative process of minimizing the mass for buckling loads larger than a prescribed value, leading to a slow convergence process or, even worse, no convergence at all.[17] Mode switching,[18,19] multiple eigenvalues,[20] and local or global influence of certain modes make the problem more complicated. On top of that, the reliability of a linear buckling analysis is questionable for structures capable of withstanding large displacements observed in the postbuckling range, or assuming a limit point in the equilibrium path. To simulate such behaviors and approach reality, a nonlinear analysis is needed, which requires a specific continuation method,[21] for identifying the collapse (limit) load of the structure. Lighter and safer composite structures may be obtained by simulating buckling, postbuckling and collapse. Solving such problems remains challenging. Proposals for an efficient solution to this problem are relatively new, since buckling, postbuckling and collapse optimizations have only been of interest to researchers quite recently.[11,22–25]

In this paper, we describe the solution procedure made available around the BOSS Quattro,[26] an optimization toolbox for optimizing composite fuselage panels with complex structural analyses. Buckling and collapse are simulated with the SAMCEF finite element code.[27] The efficiency of the methodology is demonstrated on an industrial test case.

The paper is organized as follows. First, the gradient-based optimization method used in this paper is briefly presented. The optimization problem, which consists in minimizing the weight of a section of a composite fuselage with respect to restrictions on buckling and collapse, is then formulated in Sec. 3. Buckling and collapse analyses are reviewed in Sec. 4, and sensitivity analyses are reported in Sec. 5. Finally, the

June 12, 2010 4:32:20pm WSPC/165-IJSSD 00379 ISSN: 0219-4554 1st Reading

methodology is applied to the optimal design of the curved stiffened composite panel in Sec. 6.

2. The Optimization Algorithm

The gradient-based methods used in the paper are part of the sequential convex programming methods.[28,29] These are not purely mathematical programming methods,[30] which would require too many iterations to obtain the solution (and therefore structural analyses), but rather an approach where the solution of the initial nonlinear optimization problem is replaced by the solution of successive convex approximated problems, based on specific Taylor series expansions.

The initial optimization problem is defined as follows:

$$\begin{aligned} &\min_{\mathbf{x}} g_0(\mathbf{x}) \\ &g_j(\mathbf{x}) \geq \underline{g}_j, \qquad j = 1, \ldots, m, \\ &\underline{x}_i \leq x_i \leq \overline{x}_i, \quad i = 1, \ldots, n. \end{aligned} \tag{1}$$

It is illustrated in Fig. 1, where the isovalues of the objective function $g_0(\mathbf{x})$ and the constraints $g_j(\mathbf{x})$, $j = 1, \ldots, m$, limiting the feasible design space are drawn, $\mathbf{x} = \{x_i, i = 1, \ldots, n\}$ being the vector of the design variables. Besides the general restrictions on $g_j(\mathbf{x})$, side constraints on the design variables are also taken into account, to reflect manufacturing issues or physical limitations (e.g. positive thickness).

Typically, the problem (1) is nonlinear, nonconvex and implicit in the design variables. Indeed, in our problem, the functions $g_j(\mathbf{x})$ (buckling and collapse) cannot be expressed analytically and can be evaluated only with the finite element approach. Using a mathematical programming method to solve this problem would result in a prohibitively long computational time, since a large number of iterations (typically linked to the number of design variables) would be required to find a solution. At the current design point $\mathbf{x}^k$ (k is the iteration index for the optimization cycles), all the

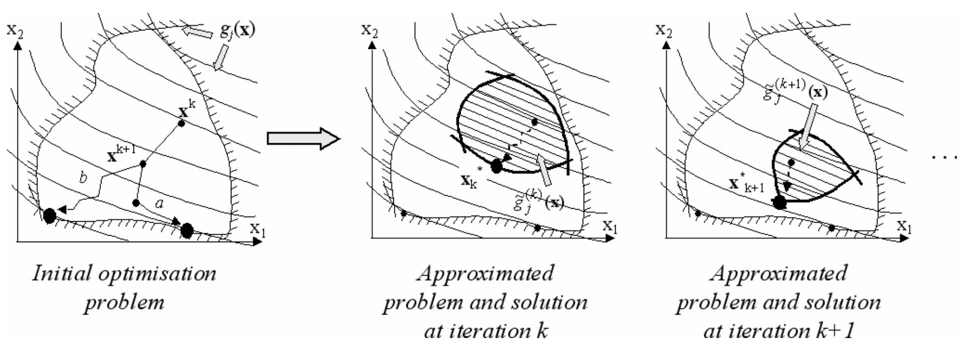

Fig. 1. Illustration of the sequential convex programming approach: the initial optimization problem (on the left) and its successive approximations with the intermediate optimal solutions x_k^*. k is the iteration index for the optimization cycles.

functions involved in the problem are rather approximated by convex functions denoted as $\tilde{g}_j(\mathbf{x})$. These approximations are based on zero and first order information, i.e. the functions' values and their first order derivatives. These values are obtained from structural and sensitivity analyses, respectively. Each approximated optimization problem (2) is now convex and explicit in terms of the design variables:

$$\begin{aligned}
&\min_{\mathbf{x}} \tilde{g}_0^k(\mathbf{x})\\
&\tilde{g}_j^k(\mathbf{x}) \geq \underline{g}_j, \quad j = 1, \ldots, m, \\
&\underline{x}_i^k \leq x_i \leq \overline{x}_i^k, \quad i = 1, \ldots, n.
\end{aligned} \tag{2}$$

Efficient mathematical programming methods[30] can now be used on the explicit subproblem, without any further finite element analysis, to find the related intermediate optimal solution $\mathbf{x}_k^*$. Successive approximations are built until convergence to a desired accuracy is achieved (Fig. 1).

The number of iterations needed to reach the optimal solution clearly depends on the quality of the approximations. A generalization of the method of moving asymptotes,[31] presented in Ref. 32 and called GBMMA (gradient-based MMA), is used here. This approximation was specially developed for composite structure optimization and has proven to be reliable in solving complex industrial applications.[3,33] In Ref. 3, this optimization algorithm is used for the preliminary design of a complete composite wing box in an optimization problem including around 1000 design variables and 300,000 constraints, such as buckling, damage tolerance, reparability and various geometric design rules. This method is available in the BOSS Quattro,[26] a task manager and optimization toolbox. Without going into the details of Refs. 32 and 33, this approximation scheme adapts itself to the problem features by checking the variation of the signs of the first order derivatives over successive iterations. As a result monotonous, nonmonotonous and nearly linear approximations can be developed at a given iteration k, for each function with respect to each design variable, based on the following tests [(3)–(5)]:

$$\frac{\partial g_j(\mathbf{x}^{(k)})}{\partial x_i} \times \frac{\partial g_j(\mathbf{x}^{(k-1)})}{\partial x_i} > 0 \quad \Rightarrow \text{monotonous approximation;} \tag{3}$$

$$\frac{\partial g_j(\mathbf{x}^{(k)})}{\partial x_i} \times \frac{\partial g_j(\mathbf{x}^{(k-1)})}{\partial x_i} < 0 \quad \Rightarrow \text{nonmonotonous approximation;} \tag{4}$$

$$\frac{\partial g_j(\mathbf{x}^{(k)})}{\partial x_i} - \frac{\partial g_j(\mathbf{x}^{(k-1)})}{\partial x_i} = 0 \quad \Rightarrow \text{locally linear approximation.} \tag{5}$$

This strategy was found to efficiently optimize composite structures with respect to both ply thickness and fibers orientation.[32,33] In this case monotonous structural responses are typically observed with respect to ply thickness, while nonmonotonous behaviors appear when orientations are considered. Generally speaking, this method is efficient for problems including high nonlinearities, which is the case for buckling and collapse optimization. For comparison, the Conlin[34] approximation is also tested, in which only monotonous approximations are built. Conlin is a first order Taylor series expansion, using linear approximation over x_i when the first order derivative is positive, and linear approximations otherwise with respect to the reciprocal (inverse) variables, $1/x_i$. As reported in Ref. 29, the sequential convex programming approach can be efficiently applied to large scale optimization problems. Moreover, the optimal solution is typically obtained in few iterations (i.e. few structural analyses), irrespective of the number of design variables. However, and contrary to genetic algorithms, a gradient-based method is more likely to be trapped in local optima, and will probably follow the path denoted as a in Fig. 1, instead of the path b toward the global optimum. This is the price to pay for a fast optimization strategy.

3. Test Case and Formulation of the Optimization Problem

In this paper, the optimization problem consists of minimizing the weight of a thin-walled composite-stiffened panel subjected to compression and shear, while satisfying some stability requirements — for example, buckling and collapse loads must be larger than a prescribed value. Local (stress) constraints are not taken into account. The section of an aircraft fuselage made of a curved composite-stiffened panel is studied (see Fig. 2).

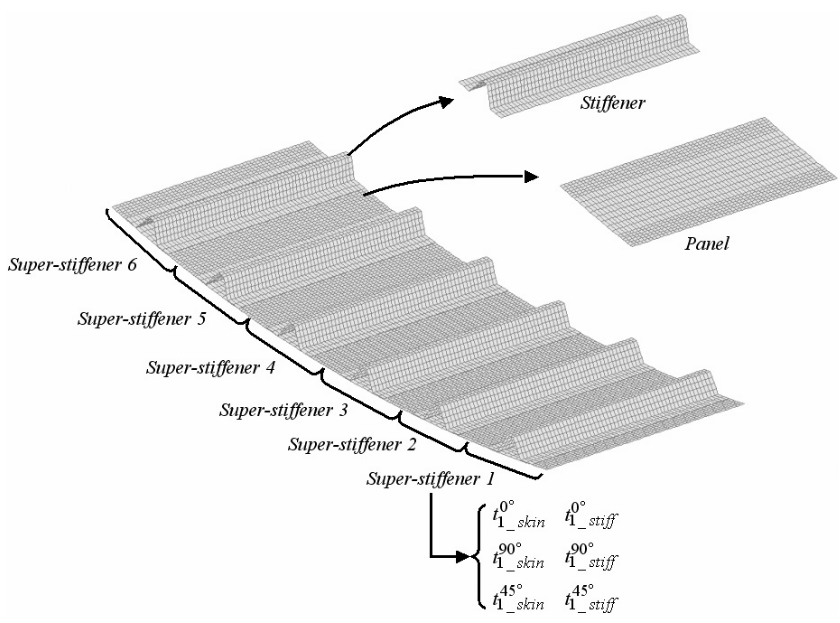

Fig. 2. The finite element model of the composite fuselage section madeup of six superstiffeners: location of the design variables.

Shell elements are used to model the skin and the longitudinal omega (hat) stiffeners, which are assembled with the skin using specific rivet finite elements. The frames are not modeled. The model includes 11,424 composite Mindlin shell elements and 92,639 degrees of freedom. Since numerical models are involved, the sensitivity of the solution to the mesh should be studied, but this point is not covered in this paper. The structure is simply supported on the edges with additional locked rotations, in order to simulate an embedded component. It is loaded in shear along the four edges, and subjected to longitudinal compressive forces along the curved boundaries and the stiffeners. The stiffened panel is divided into n single elements, called superstiffeners, consisting of one stiffener and the related piece of skin. In this paper, n is equal to 6. In order to limit the number of design variables, a homogenized material, called blackmetal, is used. The laminates of the skin and the stiffeners are madeup of plies oriented only at 0°, 90° and ±45°, and the resulting laminates are balanced (i.e. $A_{16} = A_{26} = 0$). The coefficients of the out-of-plane stiffness matrix are given by

$$D_{ij} = \frac{A_{ij}t^2}{12},$$

where t is the total thickness of the laminate. Each laminate is assumed to be symmetric and the coefficients B_{ij} are equal to zero. This way of modeling the material avoids the notion of stacking sequence and decreases the number of design variables, which now simply represent the thickness of the 0°, 90° and ±45° plies, i.e. t^{0°, t^{90° and t^{45°. The bending–twisting coupling is, however, lost in the model. This is clearly a limitation that should be removed in future work. For each superstiffener, three design variables are associated with the skin, and three with the stiffener. The optimization problem therefore includes $6 \times n$ design variables, i.e. 36 in our case. With these definitions, the problem (1) can now be written as

$$\begin{aligned}
&\min_{\mathbf{t}} w(\mathbf{t}) \\
&\lambda_j(\mathbf{t}) \geq \underline{\lambda}, \quad j = 1, \ldots, m, \\
&\lambda_{\text{collapse}}(\mathbf{t}) \geq \underline{\lambda}_{\text{collapse}}, \\
&\underline{t}_i \leq t_i \leq \bar{t}_i, \quad i = 1, \ldots, 6n, \\
&\mathbf{t} = \left\{t^{\theta}_{i_\text{skin}}, t^{\theta}_{i_\text{stiff}}, i = 1, \ldots, n; \theta = 0^\circ, 90^\circ, 45^\circ\right\},
\end{aligned} \tag{6}$$

where w is the structural weight to be minimized, λ_j is the jth buckling load, $\lambda_{\text{collapse}}$ is the collapse load, and $\mathbf{t}$ is the set of ply thicknesses, which must satisfy the side constraints. At the optimum, the buckling and collapse loads must be larger than the prescribed values $\underline{\lambda}$ and $\underline{\lambda}_{\text{collapse}}$, respectively. Here, the buckling modes are not tracked during the optimization and are therefore not included as constraints in the problem (6).

Finally, as explained in Ref. 17, a large number of buckling load factors are taken into account in the optimization problem (and not only the first few) in order to avoid or at least to limit oscillations in the convergence history. Indeed, at a given

June 12, 2010 4:32:24pm WSPC/165-IJSSD 00379 ISSN: 0219-4554 1st Reading

iteration, the first buckling modes may influence only a small part of the structure. Since weight is to be minimized, the thickness in the insensitive part will certainly reach its minimum allowable value. At the next iteration, the low-thickness part becomes sensitive to buckling and its thickness is then increased by the optimizer. If repeated, this scenario leads to oscillations, and possibly a lack of convergence. Including enough buckling modes allows one to keep the whole structure sensitive to buckling. In the application, the first 100 buckling loads are computed and included in the optimization problem, i.e. m is equal to 100 in (6).

4. Stability Analysis of Stiffened Composite Panels

As underlined in Ref. 13, stability is clearly an important issue in the design of composite aircraft structures as far as compressive and shear loads are concerned. In this section, buckling and collapse analyses are reviewed. The finite element method is used to model the problem, and the solution procedure is developed in the SAMCEF program,[27] an implicit finite element code.

4.1. *Linear buckling analysis*

In the finite element formulation, the buckling loads λ_j $(j = 1, \dots, m)$ are the first m eigenvalues of the problem (7). The Lanczos method is used to solve this problem. The buckling loads are ordered by magnitude as $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_m$. $\mathbf{K}$ is the global stiffness matrix. $\mathbf{S}$ is the initial stress stiffness matrix (also called the geometric stiffness matrix) obtained from an initial static stress analysis, and representing the initial stress stiffening effects due to the loads applied on the structure. $\boldsymbol{\Phi}_j$ in (7) is the eigenmode representing the displacement field under the load factor λ_j (Fig. 3).

$$(\mathbf{K} - \lambda_j \mathbf{S})\boldsymbol{\Phi}_j = 0, \quad j = 1, \dots, m. \tag{7}$$

The jth buckling load factor, λ_j, is the factor by which the applied load must be multiplied for the structure to become unstable with respect to the corresponding eigenmode, $\boldsymbol{\Phi}_j$. In this approximate analysis it is assumed that the stiffness matrix $\mathbf{K}$ is constant, and therefore the structural behavior is linear up to the bifurcation point, where the structure fails suddenly. It is possible, to some extent, to take into account in the analysis the second order effects due to the initial rotations. However, the analysis remains limited in its applications and may lead, as demonstrated in the application of Sec. 6, to nonconservative results.

4.2. *Collapse analysis*

Although a buckling analysis allows one to estimate the bifurcation points, it is based on a linearized approach and is therefore only an approximation. Moreover, a stiffened structure can usually sustain a higher load level after possible bifurcation and can work in the postbuckling range. In this case, large displacements appear and a

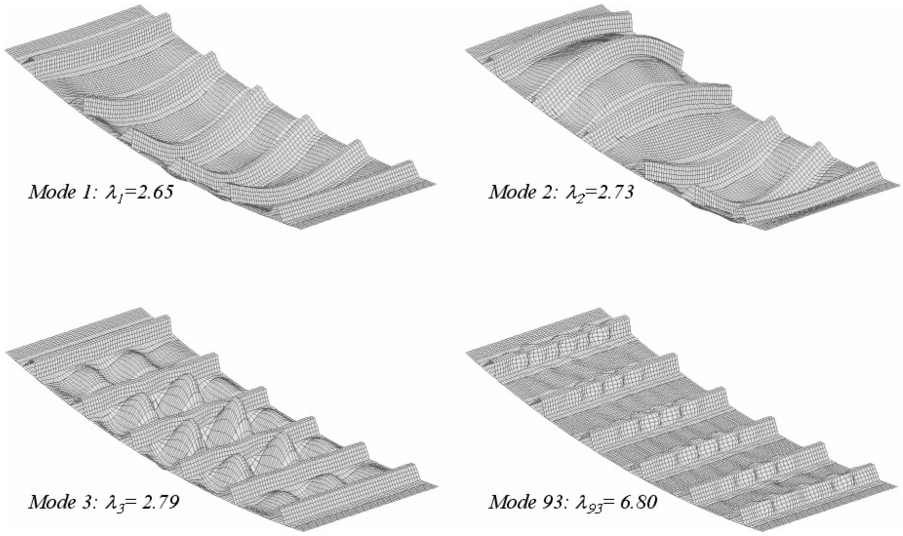

Fig. 3. Illustration of the buckling modes of a fuselage section: global and local buckling modes (for the initial values of the design variables, given in Table 1).

nonlinear analysis is required to follow the equilibrium path during the loading, up to collapse.

Classical Newton methods can present problems when passing a limit point. Indeed, the generalized load−displacement curve might have a decreasing load factor along the curve, and the method will not be able to find a solution. To solve this problem and to identify the collapse (limit) load, a continuation method, also called the arclength or Riks method,[21] must be used. In this method, the load factor λ is an additional unknown, and the arclength, denoted as s, is controlled over the iterative process instead of the load factor. A complementary equation is therefore added to the system to be solved [Eq. (8)]. This additional equation, (9), connects the generalized displacements $\mathbf{q}$, the load factor λ and the arclength s.

$$\mathbf{F}(\mathbf{q}, \lambda) = \mathbf{F}^{\text{ext}}(\lambda) - \mathbf{F}^{\text{int}}(\mathbf{q}) = 0, \tag{8}$$

$$\beta(\mathbf{q}, \lambda) = 0. \tag{9}$$

This additional constraint equation takes the general form (10). In the Riks method, $\mathbf{a} = \mathbf{n}$, and this additional equation represents a hyperplane perpendicular to the predictor.

$$\beta = \mathbf{a}^T \Delta\mathbf{q} + g\Delta\lambda - \Delta s. \tag{10}$$

During the iterative solution procedure, the unknowns are updated according to (11):

$$\mathbf{q}^{i+1} = \mathbf{q}^i + \Delta\mathbf{q}^i, \quad \text{and} \quad \lambda^{i+1} = \lambda^i + \Delta\lambda^i. \tag{11}$$

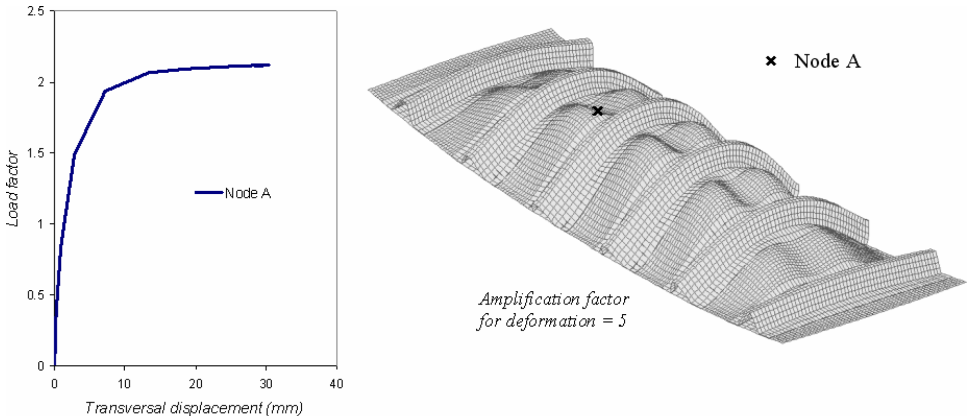

Fig. 4. Illustration of the collapse mode of a fuselage section: equilibrium path (for the initial values of the design variables, given in Table 1).

The increments in (11) are obtained by solving (12), where the right-hand side member is the residue vector (to be minimized) at the iteration i:

$$\begin{bmatrix} -\dfrac{\partial \mathbf{F}^{\text{int}}}{\partial \mathbf{q}} & \dfrac{\partial \mathbf{F}^{\text{ext}}}{\partial \lambda} \\ \dfrac{\partial \beta}{\partial \mathbf{q}} & \dfrac{\partial \beta}{\partial \lambda} \end{bmatrix}^i \begin{Bmatrix} \Delta\mathbf{q} \\ \Delta\lambda \end{Bmatrix}^i = -\begin{Bmatrix} \mathbf{F} \\ \beta \end{Bmatrix}^i \Rightarrow \begin{bmatrix} \mathbf{K}_T & -\tilde{\mathbf{f}} \\ \mathbf{a}^T & g \end{bmatrix}^i \begin{Bmatrix} \Delta\mathbf{q} \\ \Delta\lambda \end{Bmatrix}^i = \begin{Bmatrix} \mathbf{F} \\ -\beta \end{Bmatrix}^i. \quad (12)$$

In practice, the set of equations (12) is solved in two steps. The first line of (12) is first considered, omitting the index i:

$$\mathbf{K}_T\Delta\mathbf{q} - \tilde{\mathbf{f}}\Delta\lambda = \mathbf{F} \Rightarrow \Delta\mathbf{q} = \mathbf{K}_T^{-1}\mathbf{F} + \mathbf{K}_T^{-1}\tilde{\mathbf{f}}\Delta\lambda = \mathbf{q}_1 + \mathbf{q}_2\Delta\lambda.$$

After the factorization of $\mathbf{K}_T$, we solve the system twice, for $\mathbf{q}_1$ and for $\mathbf{q}_2$:

$$\mathbf{K}_T\mathbf{q}_1 = \mathbf{F}, \quad \mathbf{K}_T\mathbf{q}_2 = \tilde{\mathbf{f}}.$$

The value of $\Delta\lambda$ is then given by considering the second line of (12):

$$\Delta\lambda = -\frac{\beta + \mathbf{a}^T\mathbf{q}_1}{g + \mathbf{a}^T\mathbf{q}_2}.$$

5. Sensitivity Analyses

Since a gradient-based optimization method is used (see Sec. 2) to quickly solve large scale optimization problems, the first order derivatives of the functions must be computed. This is the role of the sensitivity analysis. These derivatives are used to build the approximations of the problem (6), to select the kind of approximation (monotonous, nonmonotonous, linear, if relevant) according to the tests (3)−(5), and to find the intermediate optimum of the approximated problem with a mathematical programming approach, as illustrated in Fig. 1.

5.1. *Linear buckling semianalytical sensitivity analysis*

The first order derivative of the buckling load factor is well known,[14,35] and is given by (13), where x_i is the considered design variable. This expression is based on the eigenmodes $\mathbf{\Phi}_j$, obtained when solving (7), and on the derivatives of the stiffness and geometric matrices, $\mathbf{K}$ and $\mathbf{S}$:

$$\frac{\partial \lambda_j}{\partial x_i} = \mathbf{\Phi}_j^T \left(\frac{\partial \mathbf{K}}{\partial x_i} - \lambda_j \frac{\partial \mathbf{S}}{\partial x_i} \right) \mathbf{\Phi}_j. \tag{13}$$

In an industrial finite element code, the sensitivity of $\mathbf{K}$ and $\mathbf{S}$ is carried out at the element level with a finite difference scheme in order to provide a general procedure applicable to the whole library of finite elements. The resulting approach is then called semianalytical sensitivity analysis, since it is based on the analytical expression (13) including derivatives obtained from finite differences.

$$\frac{\partial \mathbf{K}}{\partial x_i} \cong \frac{\Delta \mathbf{K}}{\Delta x_i} = \frac{\mathbf{K}(x_1, x_2, \ldots, x_i + \Delta x_i, \ldots, x_n) - \mathbf{K}(x_1, x_2, \ldots, x_i, \ldots, x_n)}{\Delta x_i}, \tag{14}$$

$$\frac{\partial \mathbf{S}}{\partial x_i} \cong \frac{\Delta \mathbf{S}}{\Delta x_i} = \frac{\mathbf{S}(x_1, x_2, \ldots, x_i + \Delta x_i, \ldots, x_n) - \mathbf{S}(x_1, x_2, \ldots, x_i, \ldots, x_n)}{\Delta x_i}. \tag{15}$$

The approach described above is rigorous for computing the sensitivity of the eigenfrequency in a modal analysis (not studied here), where the matrix $\mathbf{S}$ corresponds to the mass matrix $\mathbf{M}$, which depends only on the design variables $\mathbf{x}$, i.e. $\mathbf{M}(\mathbf{x})$. However, the presented approach (15) is an approximation for linear buckling analysis for nonisostatic structures, since in this case $\mathbf{S}$ depends not only on the design variables $\mathbf{x}$ but also on the stresses $\boldsymbol{\sigma}$, themselves functions of $\mathbf{x}$, i.e. $\mathbf{S}(\mathbf{x}, \boldsymbol{\sigma}(\mathbf{x}))$. In order to reduce the cost of evaluation, the influence of the variation of the stress state with the design variable is often neglected in industrial software, as proposed in Ref. 14. This simplification is of course a source of (minor) error, since there is no longer a strict correspondence between a function value and its gradient. In practice, however, a safety margin is used and a percentage of infeasibility is accepted for the constraints in the optimization problem (a few percent, e.g. 2.5%). This approximation balances the error made in the computation of the derivatives of the buckling loads.

The sensitivity analysis of multiple eigenvalues requires a specific treatment, as described in Ref. 20. Moreover, mode-tracking techniques[18] may sometimes be necessary when buckling modes are also included in the optimization problem, but this is not the case in the optimization problems addressed in this paper.

5.2. *Collapse semianalytical sensitivity analysis*

The goal of this sensitivity analysis is to compute the value $\partial\lambda/\partial\mathbf{x}$ at the collapse load, where $\mathbf{x}$ denotes the vector of design variables and λ is the load factor. The equilibrium equation (8) and its derivatives take the forms

$$\begin{gathered}\mathbf{F}(\mathbf{q},\lambda,\mathbf{x})=0,\\ \frac{\partial\mathbf{F}}{\partial\mathbf{q}}d\mathbf{q}+\frac{\partial\mathbf{F}}{\partial\lambda}d\lambda+\frac{\partial\mathbf{F}}{\partial\mathbf{x}}d\mathbf{x}=0.\end{gathered}\tag{16}$$

To be consistent with the system of equations (8)−(9) and to obtain an accurate measure of $\partial\lambda/\partial\mathbf{x}$ along a vector $\mathbf{t}$ orthogonal to the load−displacement curve,[24] the following equation is added to the set (16):

$$\beta(\mathbf{q},\lambda,\mathbf{x})=\mathbf{t}^T d\mathbf{q}+d\lambda=0.\tag{17}$$

Based on (16)−(17), the following system of equations is obtained, which has the same form as (12):

$$\begin{bmatrix}\dfrac{\partial\mathbf{F}}{\partial\mathbf{q}} & \dfrac{\partial\mathbf{F}}{\partial\lambda}\\ \mathbf{t}^T & 1\end{bmatrix}\begin{Bmatrix}d\mathbf{q}\\ d\lambda\end{Bmatrix}=-\begin{Bmatrix}\dfrac{\partial\mathbf{F}}{\partial\mathbf{x}}\\ 0\end{Bmatrix}d\mathbf{x}.$$

Since $\partial\mathbf{F}/\partial\mathbf{q}=-\mathbf{K}_T$ and $\partial\mathbf{F}/\partial\lambda=\tilde{\mathbf{f}}$ (see Subsec. 4.2), using (17) and after some algebra, it can be shown that:

$$\frac{\partial\lambda}{\partial\mathbf{x}}=-\frac{\mathbf{t}^T\mathbf{K}_T^{-1}\frac{\partial\mathbf{F}}{\partial\mathbf{x}}}{1+\mathbf{t}^T\mathbf{K}_T^{-1}\tilde{\mathbf{f}}},\tag{18}$$

where the inverse of the tangent stiffness matrix is known from the solution of (8)−(9). The derivatives of the forces with respect to the design variables in (18) are computed by finite differences, leading to a semianalytical approach to computing the sensitivity. For improved accuracy, a central finite difference scheme is used. The sensitivity $\partial\lambda/\partial\mathbf{x}$ is computed all over the loading up to the collapse, identified by a certain decrease in the load increment. This value of the derivatives is used to feed the optimizer.

6. Applications

The solution procedure described above is illustrated in the framework of a real industrial test case. The reader will appreciate that some data and results are omitted here for confidentiality reasons. The results are more qualitative than quantitative. In any case the proposed application illustrates the difficulty of the topic and the complexity of an industrial test case. For all applications, a section of the fuselage including six superstiffeners is considered. The optimization problem (6)

Table 1. Initial values of the design variables (in mm).

	$t^{0°}_{i\_\mathrm{skin}}$	$t^{45°}_{i\_\mathrm{skin}}$	$t^{90°}_{i\_\mathrm{skin}}$	$t^{0°}_{i\_\mathrm{stiff}}$	$t^{45°}_{i\_\mathrm{stiff}}$	$t^{90°}_{i\_\mathrm{stiff}}$
Superstiffener i ($i = 1, \ldots, 6$)	2	1.04	0.52	1	0.5	1

contains 36 design variables, and 100 buckling modes are used, when buckling is considered. The weight is minimized, with respect to either buckling only, or collapse only, or both kinds of restrictions. The initial values of the design variables are given in Table 1. According to our experience, selecting other initial values should not compromise the success of the optimization. Their minimum and maximum allowable values are 0.35 mm and 2 mm, respectively. The limiting values $\underline{\lambda}$ and $\underline{\lambda}_{\mathrm{collapse}}$ depend on the application but, as mentioned in Subsec. 5.1, an infeasibility of 2.5% is allowed at the optimum. This means that for $\underline{\lambda} = 0.8$ the design is supposed to be feasible when $\lambda_j \geq 0.78$ for all j. The model is given in Fig. 2; it includes 92,639 degrees of freedom. A single load case is considered; it includes shear along the edges and normal forces in the direction of the stiffeners. The optimal design is presumed to be obtained when, for a feasible design, the relative variation of the design variables or the objective function first becomes lower than 0.1%. The GBMMA[32,33] optimization method is used. A comparison with Conlin[34] is conducted in Subsec. 6.3. Comparisons for buckling optimization are given in Ref. 17.

6.1. *Buckling optimization*

In this first numerical test, only buckling is considered in the optimization problem (Subsecs. 4.1 and 5.1). Here, the buckling loads must be larger than 1.2. The structure is therefore designed to avoid any buckling at the nominal loading, with a safety margin of 20%. The optimal solution is obtained after 12 iterations. The convergence history is illustrated in Fig. 5. The weight decreases by 31%. The total thicknesses obtained are provided in Table 2. This kind of optimization is quite fast,

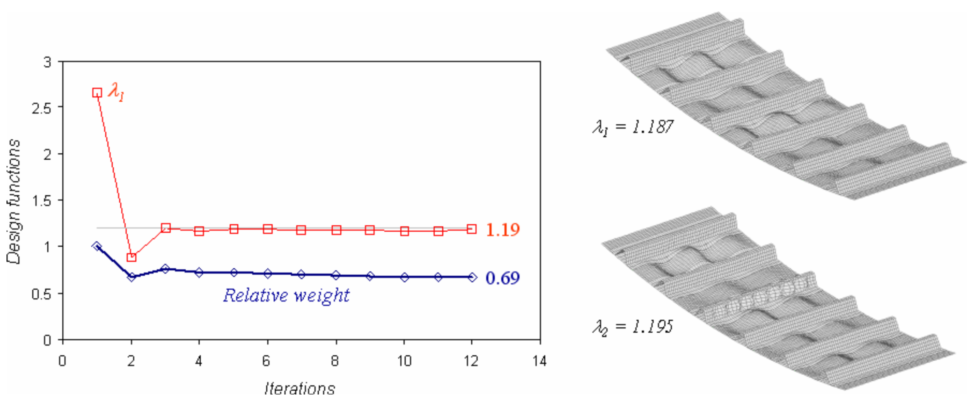

Fig. 5. Convergence history for the buckling optimization with GBMMA.

Table 2. Thicknesses at the solution of the buckling optimization problem (in mm).

	Skin panel	Stiffener
Superstiffener 1	3.50	1.42
Superstiffener 2	3.67	1.41
Superstiffener 3	3.60	1.41
Superstiffener 4	3.52	1.41
Superstiffener 5	3.77	1.40
Superstiffener 6	3.39	1.43

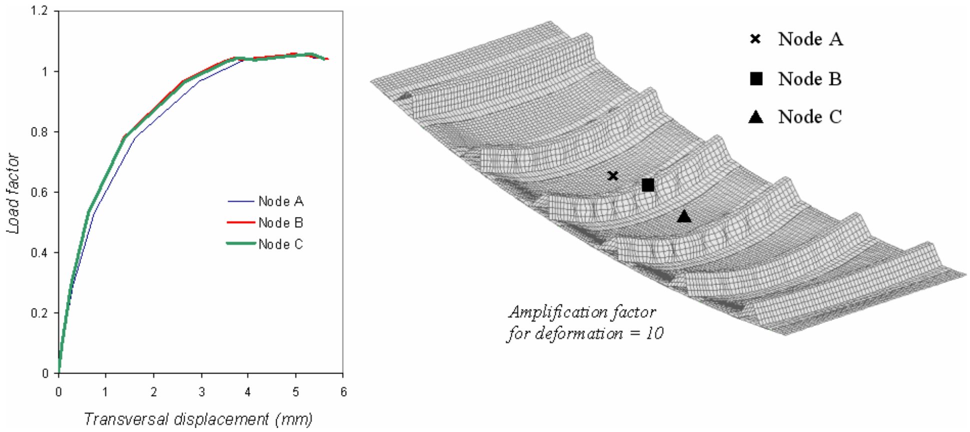

Fig. 6. Equilibrium load–displacement curves for three specific nodes of the composite panel.

since one iteration generally takes less than 5 min on today's computers. Note in Fig. 5 that a stiffener buckling appears for mode 2.

This solution is now checked with respect to collapse. A nonlinear analysis is conducted with the optimal values previously obtained for the design variables. The equilibrium load–displacement curve for three specific nodes is plotted in Fig. 6. The result from this analysis is that the collapse load is equal to 1.05, which is below the minimum buckling load factor λ_1 previously obtained at 1.19. The structure is designed to withstand buckling, but a more accurate nonlinear analysis predicts that it will reach a limit point and fail before local buckling occurs. Moreover, the assumed 20% safety margin is finally reduced to 5%, which is certainly too low and will result in an unsafe design.

Designing an aircraft stiffened panel against buckling alone is therefore insufficient, since it can provide a nonconservative solution, and should be used with care. As this is the case when initial imperfections are present in the structure, the critical point is no longer related to a bifurcation but rather to a limit point, as depicted in Fig. 6. Moreover, this result suggests that the design of structural components that are not usually designed to withstand collapse (e.g. the wing skins) should, however, include such restrictions since buckling alone gives a poor estimation of structural stability.

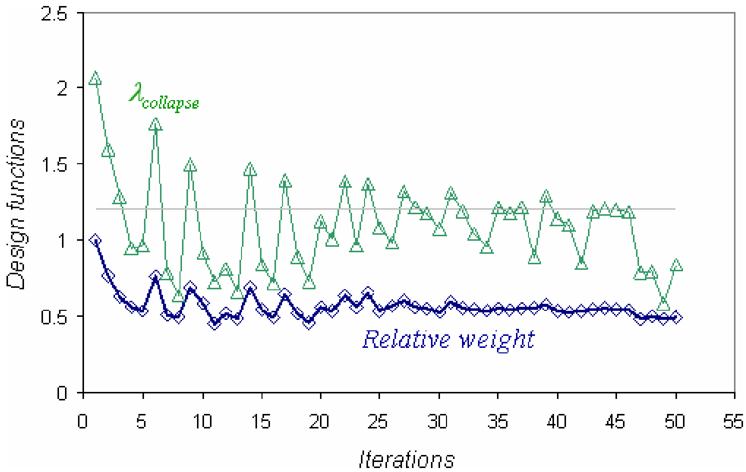

Fig. 7. Convergence problems for collapse optimization with GBMMA.

6.2. *Collapse optimization*

When collapse is considered only in the design problem (Subsecs. 4.2 and 5.2), large oscillations appear during the iterative optimization procedure, as illustrated in Fig. 7, where $\underline{\lambda}_{\text{collapse}} = 1.2$. As is often the case for buckling, when only λ_1 is constrained,[17] considering collapse only can lead to convergence problems. The best feasible design is obtained at iteration 46, with a gain of 46% in the weight. However, such a convergence history is clearly not expected with a gradient-based method. Generally 1−3 h CPU time is needed to run one iteration on a recent processor. The time spent in the optimizer is less than 1% of the total CPU time.

6.3. *Buckling and collapse optimization*

In this problem, $\underline{\lambda} = 0.8$ and $\underline{\lambda}_{\text{collapse}} = 1.2$. When buckling and collapse design functions enter the optimization problem, convergence is found after just nine iterations (Fig. 8). The relative weight at the solution is about 61% of its initial value. This solution is lighter than the one obtained previously with the buckling design functions alone (Subsec. 6.1). Moreover, it is feasible with respect to the collapse criterion. The total thicknesses obtained for each skin panel and stiffener are reported in Table 3. The optimum panel undergoes local buckling modes after a bifurcation point in the equilibrium path (i.e. buckling of the skin between the stiffeners) and then a global buckling mode (skin half-waves encompassing several stiffeners) resulting in collapse (Fig. 9).

It is interesting to note that the first optimization run provided the convergence history illustrated in Fig. 10. At iteration 9 we are close to convergence, since the design is feasible and the relative variation of the objective function is about 0.3% between two successive design steps. However, a divergence appears at iteration 10: the maximum allowable iteration number in the Newton scheme is reached, and a

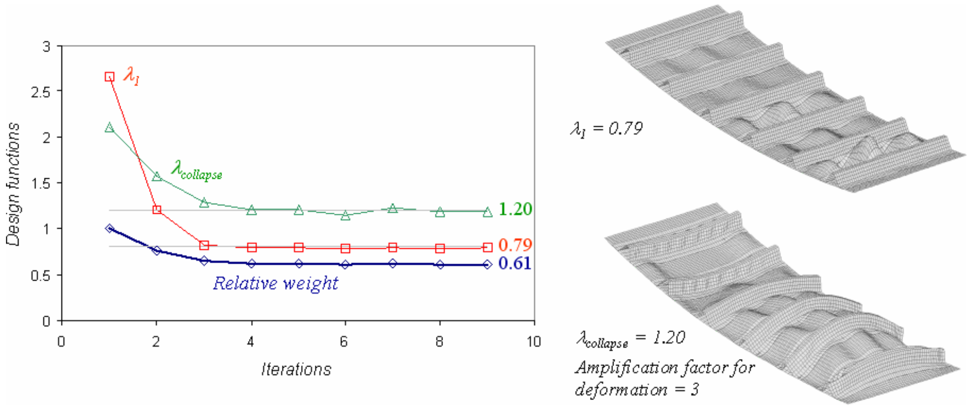

Fig. 8. Convergence history for the buckling and collapse optimization with GBMMA.

Table 3. Thicknesses at the solution of the buckling and collapse optimization problem (in mm).

	Skin panel	Stiffener
Superstiffener 1	2.75	1.40
Superstiffener 2	3.40	1.40
Superstiffener 3	2.85	1.93
Superstiffener 4	2.90	1.92
Superstiffener 5	3.41	1.40
Superstiffener 6	2.75	1.40

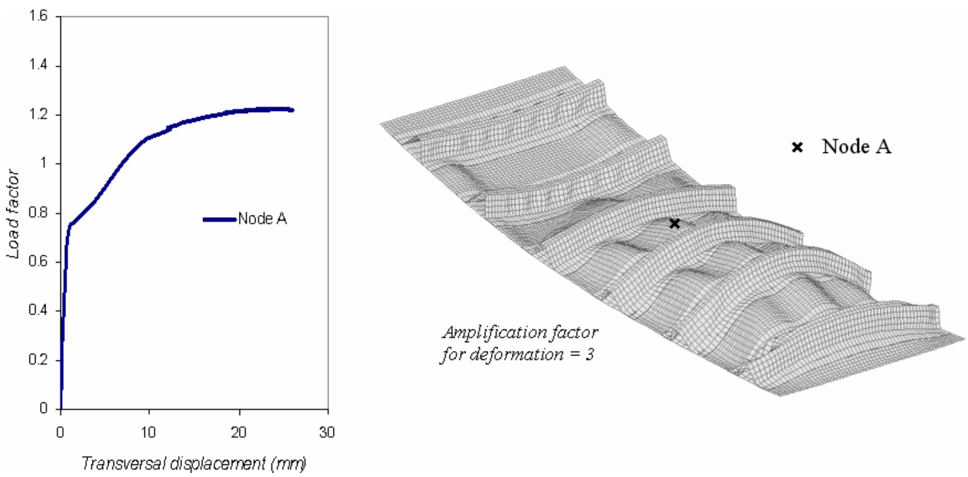

Fig. 9. Equilibrium load−displacement curve for the optimal composite panel.

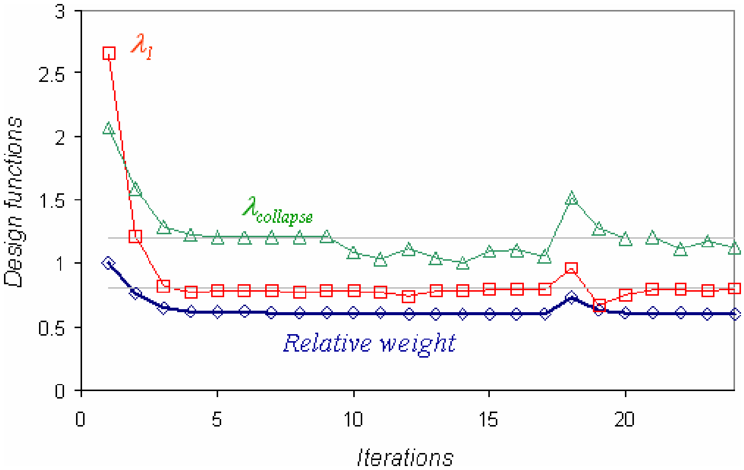

Fig. 10. Convergence problems for the buckling and collapse optimization.

destabilization appears in the solution procedure because of an automatic decrease of the time step. At this moment, we perhaps branch off into another equilibrium path, and reach a smaller value for the collapse load factor. This phenomenon occurs again later in the iterative process, which was intentionally stopped at iteration 24. Increasing the allowable number of Newton iterations allows us to converge more properly, and to find the solution as illustrated in Fig. 8. Including an imperfection in the structure based on the first buckling mode (possibly of the initial design) for the nonlinear analysis could perhaps avoid this phenomenon. This effect clearly needs to be understood and a solution strategy should be further investigated. In order to make the process more reliable, another strategy to manage convergence failure of the structural nonlinear analysis during the iterative optimization process should be developed.

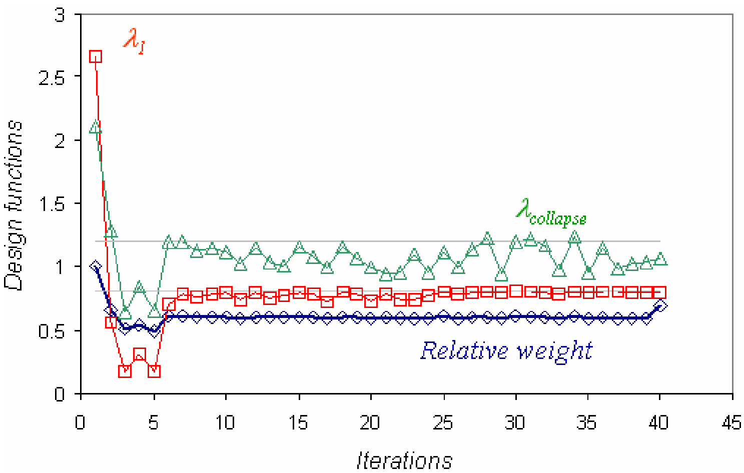

Fig. 11. Convergence problems for the buckling and collapse optimization with Conlin.

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The solution of nonlinear problems at the optimization and structural analysis levels remains a demanding task, being dependent on the convergence of iterative processes. These processes are sensitive to the solution scheme parameters, and furthermore nonlinear analysis can prove costly in computational time.

When Conlin[34] is used to solve the problem with the same conditions as in Fig. 8, it is not possible to find a solution within 40 iterations (Fig. 11). In this case, the collapse load often presents a value lower that unity. The best feasible solution is obtained at iteration 31, with a decrease of 40% in the weight.

This illustrates the fact that the selection of a suitable gradient-based optimization method — more precisely, of an approximation scheme — is another key issue in determining a reliable scheme for collapse and buckling optimization.

7. Conclusions

A solution procedure for buckling and collapse optimization has been presented. The developments were carried out in the SAMCEF finite element code, and the BOSS Quattro optimization toolbox was used to set up and solve the optimization problem. The optimal design of a section of a composite fuselage was conducted, which demonstrated that the methodology is available for solving industrial applications.

It is clear that buckling and collapse optimization is a difficult task. Through the applications it is shown that it is not sufficient to take into account a linear buckling analysis only, since a linear behavior cannot be assumed in the initial equilibrium path. On the other hand, such a linear analysis could provide a first buckling load larger than the collapse load predicted with a nonlinear analysis (see Subsec. 6.1). It is then concluded that, when possible, a more accurate nonlinear analysis should be part of the buckling optimization. Taking collapse into account in the optimization process can certainly improve the quality of the simulation and allows one to decrease the structural weight. Together with a linear buckling analysis, it allows the design of efficient stiffened panels for aircraft applications, which present first local buckling modes so that the structure can still carry loads in the postbuckling range. Moreover, the oscillations in the convergence history for collapse design can be decreased by considering additional buckling restrictions in the optimization problem.

However, since nonlinear analyses are required at both the optimization and the structural analysis levels, the procedure can lead to convergence difficulties, either in the analysis tool or in the optimizer. This kind of advanced optimization must then be undertaken with care. Moreover, the CPU time for the nonlinear analysis can become prohibitive.

Further work will investigate the influence of imperfections on the convergence process, the effect of local (stress) constraints as design criteria, the bending–twisting coupling issue, and the possible optimization of the stiffeners' dimensions.

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