Mechanical buckling analysis of composite panels with/without cutouts

by
Oana Zenaida PASCAN, Weihong ZHANG, Jean Philippe PONTHOT
Engineering Simulation and Aerospace Computing, Northwestern Polytechnical University, P.O. Box 552 710072 Xi'an, Shaanxi, CHINA
Department of Aerospace and Mechanical Engineering, University of Liege, Chemin des Chevreuils, 1 B-4000 LIEGE 1, BELGIUM

ABSTRACT: *A simplified analytical solution suitable for simple stacking sequences was developed using the Euler buckling theory, the structure's equations of equilibrium and laminate panel mathematical formulation. Comparing these results with numerical results reveals the accuracy of the method and even more, allows us to validate the numerical analysis. Therefore, two important results are obtained : a simplified analytical solution for the buckling problem and validation of the numerical results. Another important and novel finding is related to the influence of the angle ply orientation and of the cutouts, on the buckling load. Under symmetrical boundary conditions and loading case, rectangular panels with elliptical cutouts, give better results for* $90°$ *oriented plies than for* $0°$ *oriented ones. With a compression load applied in the* X *direction, and with material properties* 10 *times better in* X *direction than in* Y *direction, the best results are obtained when the load is aligned with the* Y *direction associated to the material reference frame. Moreover, panels with cutouts seem to behave better than panels without cutouts under certain ply orientation angles.*
Keywords: Linear buckling, analytical solution, mechanical loading, angle ply orientation, cutouts

1 Introduction

In all the studied papers buckling analysis is performed using predefined stacking sequences ([1]-[5]). The influence of the angle of orientation of a single ply isn't studied independently. Using this aspect as a start point, in this paper certain important behavior trends that can be detected using a single layer composite panel, are discussed.

In the literature, there is a range of published studies on the buckling of composite plates ([7]-[13]). M. Aydin Komur, Faruk Sen, Akın Atac and Nurettin Arslan carried in [15] a buckling analysis of laminated composite plates with an elliptical/circular cutout using FEM. They conclude that the magnitudes of buckling loads are decreased by increasing thickness to length ratio, whereas it is increased by increasing width to length ratio. This means that big elliptical holes cause the weakest plates under the pressure. Secondly, the increasing of hole positioned angle causes the decrease of buckling loads. Lastly, the cross-ply $[(0°, 90°)_2]_s$ composite plates is stronger than other analyzed angle-ply $[(15°/75°)_2]_s$, $[(30°/60°)_2]_s$, $[(45°/-45°)_2]_s$ laminated plates.
Ghannadpour S.A.M., Najafi A. and Mohammadi B. discuss in [8] the buckling behavior of cross-ply laminated composite plates due to circular/elliptical cutouts. They concluded that the buckling load of the square plates containing a circular cutout reduces by the increment of cutout diameter. Moreover, small cutouts can be neglected from modeling and can reduce the meshing efforts.
In this paper a simplified analytical solution for solving buckling problems for particular laminate configuration and boundary conditions is proposed. By combining Euler's buckling theory with structural equilibrium equations and mathematical formulation of the laminates, an interesting novel solution is developed and compared with numerical results.

2 Simplified analytical model for mechanical buckling of composite panels without cutouts

Let us begin the analysis with the first test case. A simple rectangular plate without cutouts, under compression loading is considered at this first step of the analysis. A single layer graphite-epoxy plate with the material properties given in figure. 1 will be considered. The thickness of the plate is taken to be equal to 1 mm. A simple mesh configuration is chosen: 50 elements on each dimension, therefore 2500 mesh elements.
As discussed previously, the eigenvalue buckling analysis using ANSYS allows us to get the load factor, which multiplied with the applied load gives the critical buckling load factor. For simplicity reason the applied compression force equals $1N/\ mm^2$, therefore the load factor gives the critical compression force.
The first purpose is to get the evolution of the buckling load for different $\frac{a}{b}$ ratios.

Properties	Temperature ($°C$)				
	20	200	260	600	3316
E_1 (GPa)	141	141	141	141	141
E_2 (GPa)	13.1	10.3	0.138	0.0069	0.0059
G_{12} (GPa)	9.31	7.45	0.069	0.0034	0.0034
μ_{12}	0.28	0.28	0.28	0.28	0.28
$\alpha_1(10^{-6}/°C)$	0.018	0.054	0.054	0.054	0.054
$\alpha_2(10^{-6}/°C)$	21.8	37.8	37.8	37.8	37.8

Table 1: Lamina material mechanical properties ($G_{12} = G_{13} = G_{23}$)

2.1 Euler Buckling theory

The formula given by Euler for columns with no consideration for lateral forces is given by:

$$F = \frac{\pi^2 EI}{(KL)^2} \tag{1}$$

where

- F = maximum or critical force (vertical load on column);
- E = Young's modulus;
- I = area moment of inertia;
- L = unsupported length of column;
- K = column effective length factor, whose value depends on the conditions of end support of the column:
  -For both ends pinned, $K = 1$;
  -For both ends fixed , $K = 0.5$;
  -For one end fixed and the other pinned, $K = 0.7$;
  -For one end fixed and the other end free to move laterally, $k = 2$;

2.2 Flexural rigidity

For laminate composite materials the reduced stiffness matrix is expressed as a function of the engineering constants by the relations:

$$\mathbf{Q} = \begin{bmatrix} mE_l & mv_{yx}E_l & 0 \\ mv_{xy}E_t & mE_y & 0 \\ 0 & 0 & E_s \end{bmatrix} \tag{2}$$

where:

$$m = \frac{1}{1 - v_{xy}v_{yx}} \qquad v_{yx} = \frac{v_{xy}E_y}{E_x} \tag{3}$$

With respect to the kinematics of a Kirchhoff plate, at laminate level one can write:

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} \tag{4}$$

which gives the behavior of the laminate in the structural axes. Let us give a detailed form of the previous matrix: **N**- inplane loading, **M** moment loading, **A**, **B**, **D** stiffness matrix, ε^0 in plane strain and κ curvatures.

$$\begin{bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\ B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_3^0 \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix} \tag{5}$$

where all the plies have the same thickness h, and t gives the position of the ply:

$$A_{ij} = h \sum_{t=1-n/2}^{n/2} [Q_{ij}^t](t - (t-1)) \tag{6}$$

$$B_{ij} = \frac{h^2}{2} \sum_{t=1-n/2}^{n/2} [Q_{ij}^t](t^2 - (t-1)^2) \tag{7}$$

$$D_{ij} = \frac{h^3}{3} \sum_{t=1-n/2}^{n/2} [Q_{ij}^t](t^3 - (t-1)^3) \tag{8}$$

For a symmetric stacking sequence $B_{ij} = 0$. Therefore we get uncoupling between in and out of plane effects, flexion and extension. For a balanced stacking sequence, meaning the same proportion of $-45°$ and $+45°$ plies , we get in plane orthotropy, uncoupling between extension and shear.
For a homogenized material, D_{16} and D_{26} we get uncoupling between shear and torsion.

For an isotropic and homogeneous beam, the flexural rigidity is merely EI, however, for a beam made of composite material, the computation for the flexural rigidity is not so straightforward due to the various coupling terms.

The assumptions used in our approach consist of [7]:

- The amount of deflection of a point on the composite beam is only dependent on the position of this point on the composite laminate in the x direction.
- In equation. 5, onlyN_1 exists while N_2 and N_6 are taken to be zero.
- M_2 and M_6 in equation. 5 are negligible and therefore neglected.

With the above assumptions, from equation. 5, four expressions can be written:

$$N_1 = A_{11}\varepsilon_1^0 + A_{12}\varepsilon_2^0 + A_{13}\varepsilon_3^0 - B_{11}\frac{\partial^2 w}{\partial^2 x} \tag{9}$$

$$0 = A_{12}\varepsilon_1^0 + A_{22}\varepsilon_2^0 + A_{23}\varepsilon_3^0 - B_{12}\frac{\partial^2 w}{\partial^2 x} \tag{10}$$

$$0 = A_{13}\varepsilon_1^0 + A_{23}\varepsilon_2^0 + A_{33}\varepsilon_3^0 - B_{13}\frac{\partial^2 w}{\partial^2 x} \tag{11}$$

$$M_1 = B_{11}\varepsilon_1^0 + B_{12}\varepsilon_2^0 + B_{13}\varepsilon_3^0 - D_{11}\frac{\partial^2 w}{\partial^2 x} \tag{12}$$

$$\tag{13}$$

Two of the equations are used to find ε_2^0 and ε_3^0 in terms of ε_1^0 and $\frac{\partial^2 w}{\partial^2 x}$. The resultant expressions turn out to be:

$$\varepsilon_2^0 = \frac{(A_{13}A_{23} - A_{12}A_{33})}{(A_{22}A_{33} - A_{23}^2)}\varepsilon_1^0 + \frac{(A_{33}B_{12} - A_{23}B_{13})}{(A_{22}A_{33} - A_{23}^2)}\frac{\partial^2 w}{\partial^2 x} \tag{14}$$

$$\varepsilon_3^0 = \frac{(A_{12}A_{23} - A_{13}A_{22})}{(A_{22}A_{33} - A_{23}^2)}\varepsilon_1^0 + \frac{(A_{22}B_{13} - A_{23}B_{12})}{(A_{22}A_{33} - A_{23}^2)}\frac{\partial^2 w}{\partial^2 x} \tag{15}$$

The stability equation of the beam is given by:

$$-\frac{d^2 M}{dx^2} + N_1\frac{d^2 w}{dx^2} = 0 \tag{16}$$

The remaining equations plus the above two are the utilized together with expression 16 to form the resulting stability governing equation:

$$R\frac{d^4 w}{dx^4} + N_1\frac{d^2 w}{dx^2} = 0 \tag{17}$$

so the expression for the flexural stiffness derived using this method turns out to be:

$$F_{stiffness} = R = D_{11} - \beta B_{12} - \delta B_{13} - \frac{(B_{11} + \alpha B_{12} + \gamma B_{13})(B_{11} + \beta A_{12} + \delta A_{13})}{(A_{11} + \alpha A_{12} + \gamma A_{13})} \tag{18}$$

where, α,β, γ, δ are given by:

$$\alpha = \frac{(A_{13}A_{23} - A_{12}A_{33})}{(A_{22}A_{33} - A_{23}^2)} \tag{19}$$

$$\beta = \frac{(A_{33}B_{12} - A_{23}B_{13})}{(A_{22}A_{33} - A_{23}^2)} \tag{20}$$

$$\gamma = \frac{(A_{12}A_{23} - A_{22}A_{13})}{(A_{22}A_{33} - A_{23}^2)} \tag{21}$$

$$\delta = \frac{(A_{22}B_{13} - A_{26}B_{12})}{(A_{22}A_{33} - A_{23}^2)} \tag{22}$$

Going back to Euler buckling theory the only adjustment that has to be made is with respect to the flexural stiffness.
Therefore the formula becomes:

$$F = \frac{\pi^2 F_{stiffness}}{(KL)^2} \tag{23}$$

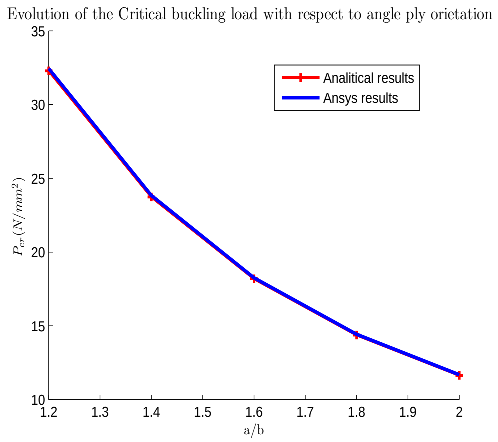

Figure 1: Evolution of the critical buckling load for different a/b ratio, for a single layer graphite-epoxy panel, 50 x 50 mesh, $1N/\ mm^2$ load, $h = 1\ mm$

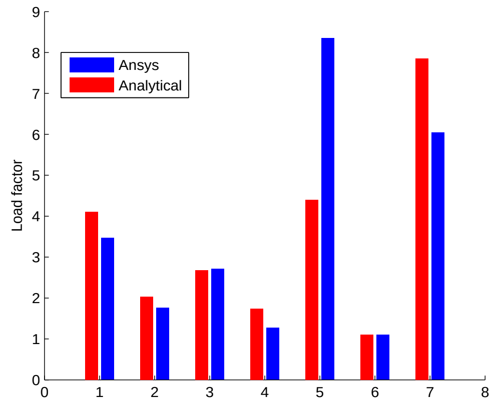

Figure 2: Analytical and eigenvalue comparison of the evolution of the critical buckling load for different stacking sequences, graphite-epoxy panel, 50 x 50 mesh, $1N/\ mm^2$ load, $h = 1\ mm$

2.3 Comparison to numerical solution (ANSYS)

Going back to our test case, we can see in figure. 1 the evolution of the buckling critical load for different $\frac{a}{b}$. As we could very well expect, given the formula used to calculate the critical compression force, as the length of the plate increases the critical force decreases. However, what stands out in this particular analysis is the very close results obtained for both types of the analysis. The analytical method although much more simple than the eigenvalue analysis performed using ANSYS gives perfectly matching results.

No.	α_1	α_2	α_3	α_4
1	45	-45	45	-45
2	60	-60	60	-60
3	45	-60	45	-60
4	90	-60	90	-60
5	0	90	0	90
6	80	90	80	90
7	15	45	45	15

Table 2: Stacking sequences

Still, it's a very simple case of one single layer panel. In order to validate the simplified analytical approach we will make a comparison for an 8 plies, symmetrical staking sequence. The following notation: $[\alpha_1, \alpha_2, \alpha_3, \alpha_4]_s$ is used, where α_1 is the orientation angle of the top layer. The total thickness of the laminate stays equal to 1 mm, so the ply thickness is equal to 0.125 mm. In table. 2 the different stacking sequences that have been tested are presented. In figure. 2 the values of the critical load found using the simplified analytical approach and the eigenvalue approach is noted.

The biggest difference between the two approaches is obtained for case no. 5, corresponding the stacking sequence that exhibits the greatest difference between two consecutive angles, in this particular case 90°. Moreover, as the difference between the orientation of consecutive sequences decreases so does the difference between the analytical and numerical obtained results.
This differences find their explanation in the assumptions accepted in order to obtain the simplified analytical solution. Most of the coupling affects are not considered, and the stiffness matrix is highly simplified. Some stacking sequence rules, like the maximum difference in orientation between to plies, or the positively orientated plies and negatively orientated plies, influence the values of the stiffness coefficients, and therefore the value of the analytical results.

Consequently, the best values, meaning the same results for both approaches, are obtained when the coupling effects are small enough to be neglected.

3 Single layer with elliptical cutout panel. Angle ply orientation optimization

Using the linear buckling frame the first goal is to find the optimal angle ply orientation of a single layer panel with elliptical cutout.
The initial ellipse has a major diameter of 30 mm and a minor diameter of 15 mm . The parametric analysis reveals a rather interesting result. The critical buckling load is maximal when the angle ply orientation equals 90°. This is contrary to what we would have expected. As the plate is loaded in compression and as the best properties are in the X direction, it is intuitively to set a 0° angle ply orientation for best results.
In order to understand this result let us check a few more detailed aspects of the analysis.
A static prestress analysis allows us to get the stress intensity for the two cases, for a 0°, respectively 90° angle ply orientation. In figure. 5 and figure. 6 the results can be compared. Not only that the stress intensity is lower for the 90° oriented ply, but the elliptical hole no longer acts as a strain and stress concentrator.

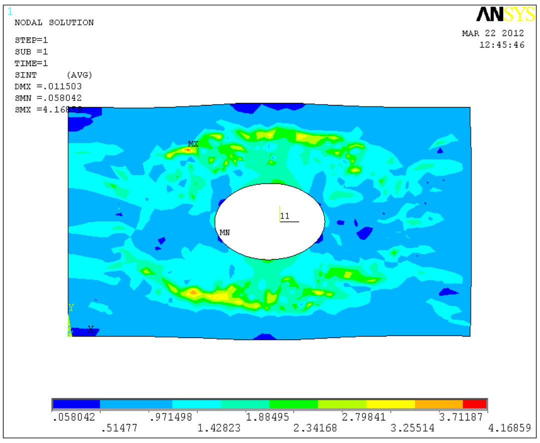

Figure 3: Stress intensity, static prestress analysis,single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30$, $r2 = 15$, ellipse angle 0°, angle ply0°

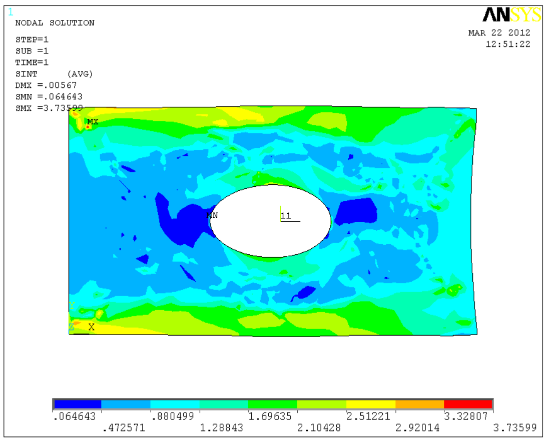

Figure 4: Stress intensity, static prestress analysis,single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30$, $r2 = 15$, ellipse angle 0°, angle ply90°

Note that $M1$ mesh means 50 elements for the length of the rectangular plate, 40 elements for the width and 80 elements for the elliptical cutout.

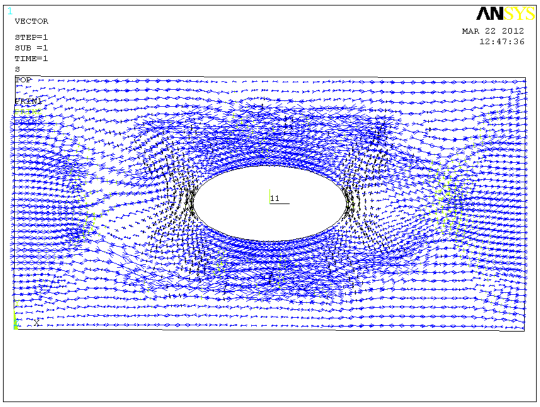

Figure 5: Vectorial representation of principal stresses, static prestress analysis,single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30$, $r2 = 15$, ellipse angle 0°, angle ply0°

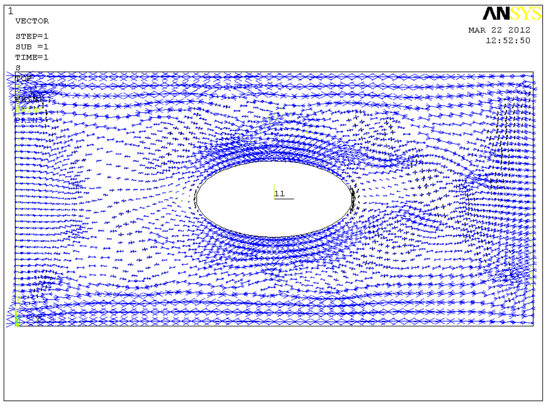

Figure 6: Vectorial representation of principal stresses, static prestress analysis,single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30$, $r2 = 15$, ellipse angle 0°, angle ply90°

The vector representation of the principal stresses emphasis the upper statement. The panel is more stressed when the 0° orientation of the fiber is used. Moreover, it can be seen that for the 90° oriented panel the stress has a continuous distribution, while for the 0° orientation the hole acts as a stress concentrator. The distribution of the stress is different for the two cases and generates different behaviors of the plate. We could say that for the 90° orientation the useful section of the plate is much higher than for the 0° one. Moreover the fibers act as stiffeners in this case. If for the 0° ply the fibers are subject to buckling, for the 90° the fibers become subject to bending, but it still can get the benefits of the matrix properties.

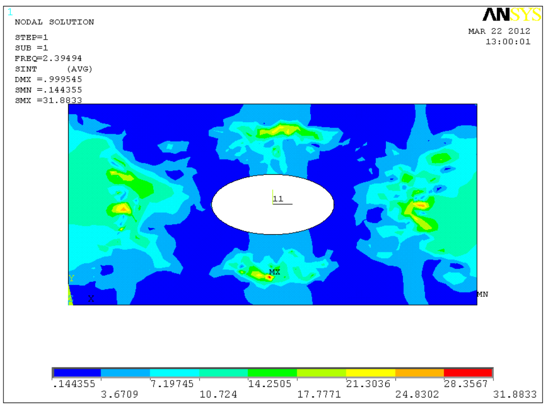

Figure 7: Stress intensity, eigenvalue analysis,mode1, single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30\ mm$, $r2 = 15\ mm$, ellipse angle 0°, angle ply 0°

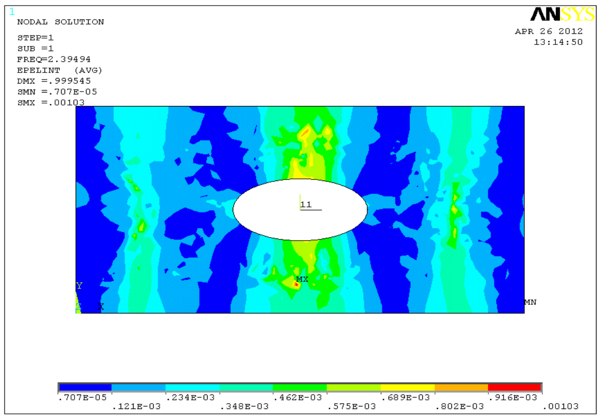

Figure 8: Elastic strain intensity, eigenvalue analysis,mode 1, single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30\ mm$, $r2 = 15\ mm$, ellipse angle 0°, angle ply 0°

In figures. 7,8,9, 10, we can see the stress intensity for the first buckling modes for 0°, respec-

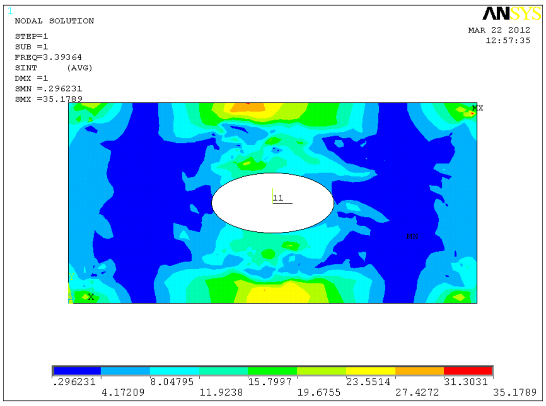

Figure 9: Stress intensity, eigenvalue analysis, single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30\ mm$, $r2 = 15\ mm$, ellipse angle $0°$, angle ply$90°$

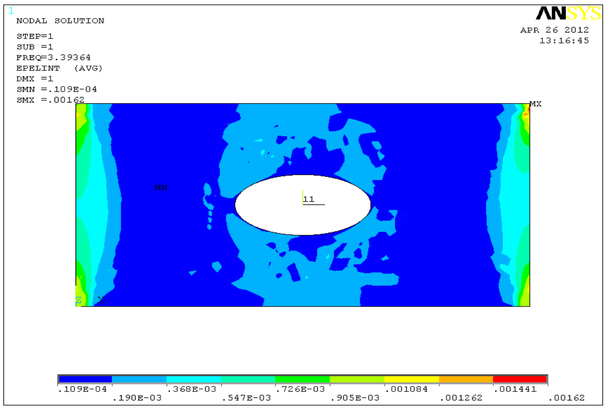

Figure 10: Elastic strain intensity, eigenvalue analysis,mode 1, single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $r1 = 30\ mm$, $r2 = 15\ mm$, ellipse angle $0°$, angle ply$90°$

tively $90°$ orientation of the fibers. The same boundary conditions and numerical parameters were used for the eigenvalue buckling analysis. As we can see that the buckling shape changes between the 2 cases. For the $0°$ angle ply orientation the buckling load is equal to $2.39N/\ mm^2$, while for the $90°$ the critical load is $3.39N/\ mm^2$. This means that an improvement of $1N/\ mm^2$ is achieved just by changing the orientation of the fibers.

Because this novel result is against any intuition, a nonlinear analysis was performed. The result can be seen in figure. 11.

The nonlinear analysis proves the validity of the results given by the linear analysis. It is also

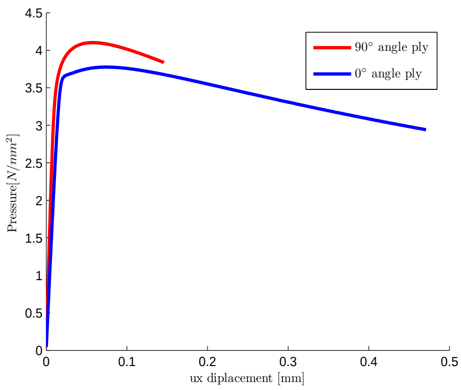

Figure 11: Nonlinear analysis,single layer graphite-epoxy panel, $40, 20, 10$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, $0°$ ellipse angle

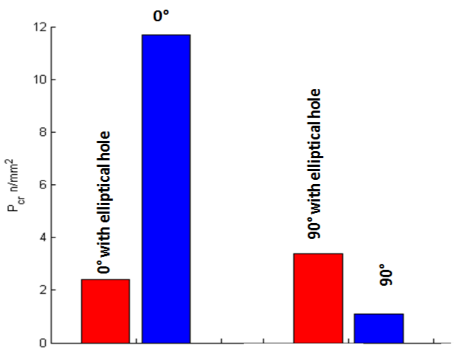

Figure 12: Critical buckling load for panels at $0°$ and $90°$ orientation of fiber, with and without a hole

very interesting to compare the critical buckling load for a panel with and without the elliptical hole. We can see in figure. 12 the values obtained for the two cases of interests, the panel with $0°$ orientation of the fibers, $90°$ orientation of the fibers respectively. While for $0°$ the introduction of the ellipse makes the critical load to drop from$11.68N/\ mm^2$ to $2.39N/\ mm^2$, for the $90°$ the buckling load increases from $1.08N/\ mm^2$ to $3.39N/\ mm^2$. If in the first case the change seems intuitive, the structure becoming less stiff if a cutout is introduced, quite the opposite happens for the second case.

A possible explication could be related to the radius of gyration of the plate. As the elliptical hole is introduced the 0° panel acts as if the effective length of the panel increases. On the other hand for the 90° the panel gives the same buckling load as a panel with a considerably reduced length. The effective length for a panel subject to buckling is given by his geometrical length divided by the radius of gyration:

$$L_{eff} = \frac{L}{\lambda} \tag{24}$$

$$\lambda = \sqrt{\frac{I_x}{A}} \tag{25}$$

where I_x is the plane moment of inertia and A is the area of the plate.

Although this formula seems quite simple, it appears that for a composite panel with an elliptical hole things are more complicated. We therefore think that the effective length is a function of angle ply orientation, dimensions and orientation of the ellipse, length and width of the plate.

$$L_{eff} = f(r_1, r_2, a, b, angle_{ply}, angle_{ellipse}) \tag{26}$$

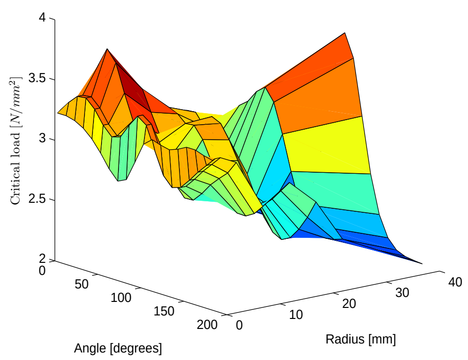

Figure 13: Parametrization study of ,single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, 0° ellipse angle, eigenvalue analysis

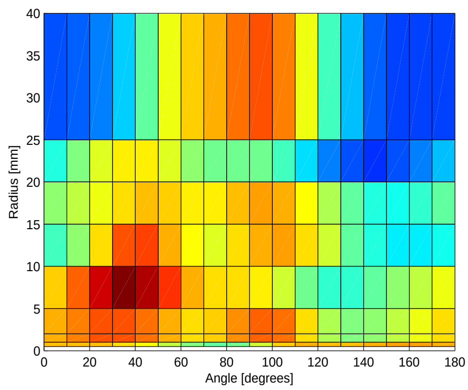

Figure 14: Parametrization study of ,single layer graphite-epoxy panel, $M1$ mesh, $1N/\ mm^2$ load, $h = 1\ mm$, 0° ellipse angle, eigenvalue analysis, radius- angle view

The next interest is to see how the critical buckling load evolves when the shape dimensions change. The minor diameters is a function of the major diameter,$r_1 = 0.5r_2$. A 2 variable parametrization was performed using BOSS-Quattro. The first variable is the fiber orientation and the second variable is the the major diameter of the ellipse, r_2. The results can be seen in figure. 13 and figure. 14. It is obvious that the best results are obtained for 5 mm diameter and 40° angle. Very close results are obtained for 40 mm diameter and 90° angle. It clear that as the ellipse's dimension decrease the value of the fiber orientation tends to 0° which is actually the optimal fiber orientation for a panel without cutout. This trend can be seen in figure. 15.
On the other hand as the ellipse's dimensions decrease, the hole becomes over meshed, meaning that the mesh becomes problematic, leading to an inaccurate approximation of the stress state (over-approximation), and therefore to an under approximation of the buckling load. When the hole becomes very small, smaller then the average value of a mesh element, a singularity has to be considered, but due to over-meshing the analysis exhibits an orientation angle of the ply different from 0 as it can be seen in figure. 15 for $r1 = 0.5mm$. In this particular case we could assume that the the elliptical hole is small enough to be neglected, or considered to be just as an imperfection. It could slightly influence the value of the buckling load, but no remarkable

influence would appear with respect to the angle of orientation of the fibers.

Consequently there are cases when the introduction of a hole improves the performances of a composite panel.

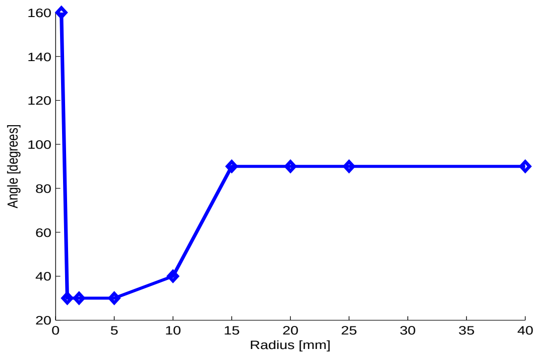

Figure 15: Evolution of the optimal angle with respect to the ellipses major diameter

4 Conclusions

Several important results were obtained in this work.
A simplified analytical solution suitable for simple stacking sequences was developed using the Euler buckling theory, structure's equations of equilibrium and laminate panel mathematical formulation. Comparing these results with results obtained performing a numerical analysis reveals the accuracy of the method and even more, allows us to validate the numerical analysis. Therefore, this part of the work exhibits two important results: a simplified analytical solution for the buckling problem and validation of the numerical results.

Another important finding is related to the influence of the angle ply orientation and of cutouts, on the buckling load. Under symmetrical boundary conditions and loading case, rectangular panels with elliptical cutouts, give better results for 90° oriented plies than for 0° oriented ones. This is a very important results because it means that the use of the composite laminate could be reconsidered. With a compression load applied in the X direction, and with material properties 10 times better in X direction than in Y direction, the best results are obtained when the load is aligned with the Y direction associated to the material reference frame. Moreover, panels with cutouts seem to behave better than panels without cutouts under certain ply orientation angles.

Acknowledgments

This is a sincere expression of my profound gratefulness to all those who directly or indirectly made it possible for me to successfully complete my project.
This paper is the result of an ERASMUS internship for master thesis realization at the laboratory of Engineering Simulation and Aerospace Computing of Northwestern Polytechnical University of China.
I would like to thank Prof. Jean Philippe Ponthot for making my internship possible and for having been a solid support throughout my period as a student at University of Liège.
I would like to thank Prof. Zhang Weihong for providing me with a professional and welcoming working environment and practical guidance throughout the entire research programme.

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