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Trajectory optimization for 3D robots with elastic links

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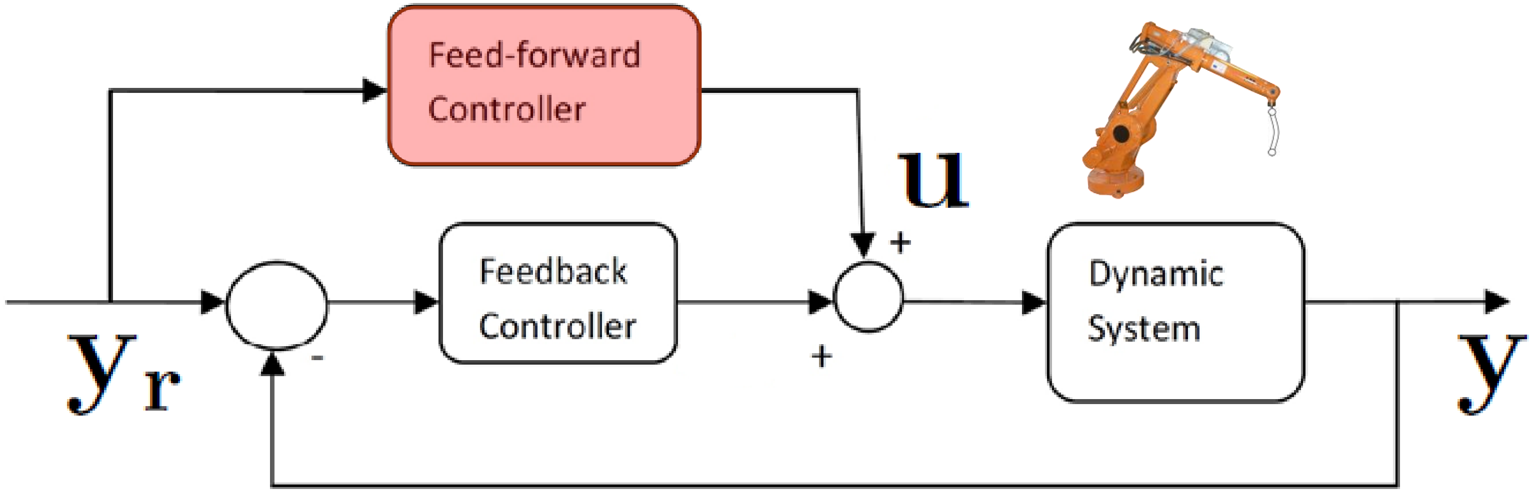
Context

- Industries have a growing interest for lightweight robots, but those can have flexibility and vibration issues…

⇒ To take care of these problems, we can act on the control system of such robots.



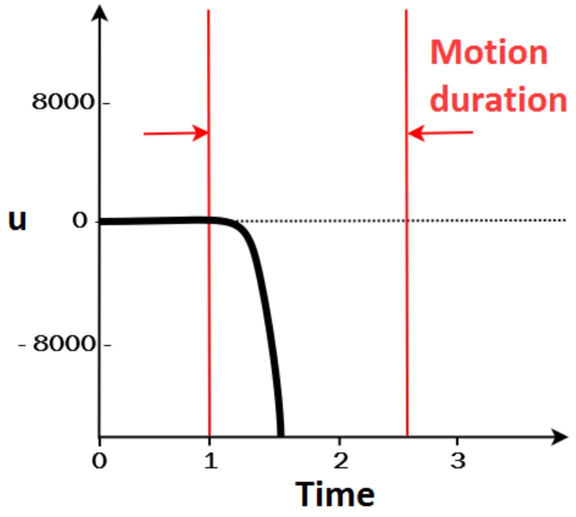
Reminder - 1

- Computation of the feedforward solution = Solve the inverse dynamic problem.

$\Rightarrow$ Find controls $\mathbf{u}$, when trajectory $\mathbf{y}$ is given

for the dynamical system:

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q},\dot{\mathbf{q}},t) + \boldsymbol{\Phi}_{\mathbf{q}}(\mathbf{q})\boldsymbol{\lambda} &= \mathbf{A}\mathbf{u} \\ \boldsymbol{\Phi}(\mathbf{q}) &= \mathbf{0} \\ \mathbf{y} &= \mathbf{D}\mathbf{q} \end{aligned}$$



- Time integration of the equation of motion can lead to unbounded solution, if the system is non-minimum phase (unstable internal dynamic).

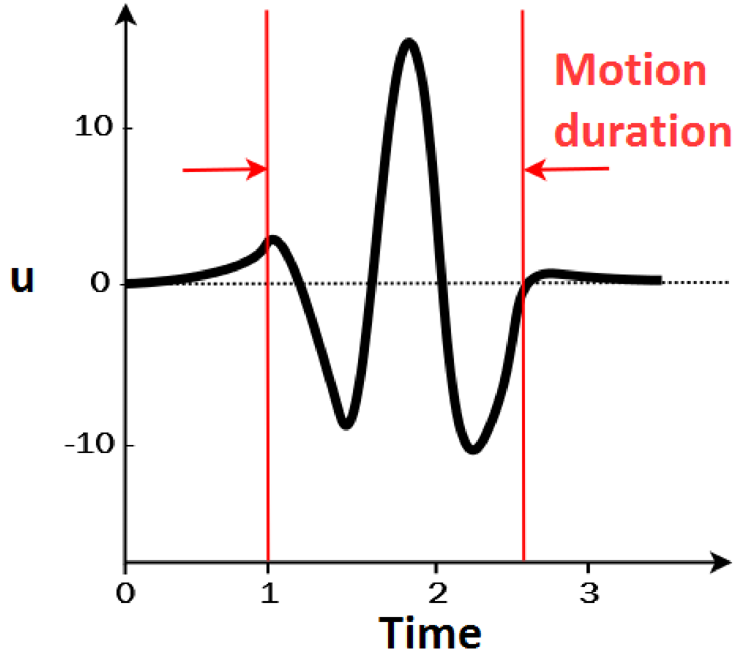
Reminder - 1

- Other ways to solve the inverse dynamic of non-minimum phase systems:
  - Stable inversion method

  [Seifried 2013], [Devasia et al 1996].
  - Optimal control approach

  [Bastos 2013] for 2D systems.

⇒ Bounded and non-causal solution.



Reminder - 2

- Flexible multibody systems can be modeled thanks to a finite element formulation.
- Usually the nodal variables q are stated in the inertial frame and have to be parameterized in order to solve the equation of motion (singularity issues).
- The use of Lie groups states the nodal variables q in the material frame:
  1. No more singularity issues.
  2. Nearly constant stiffness matrix and internal forces.
- In this work, we work with the Special Euclidian group SE(3), in which:

$$q = \begin{pmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0} & 1 \end{pmatrix} \in SE(3)$$

Objective and originality

- Solve the inverse dynamic of 3D flexible systems, formulated with the Lie group theory, using an optimal control approach (direct transcription method).

Method

- Constrained optimization problem where:

1. Objective function J : the time integral of strain energy E.

$$\min J = \min \int_{t_i}^{t_f} E \; dt$$

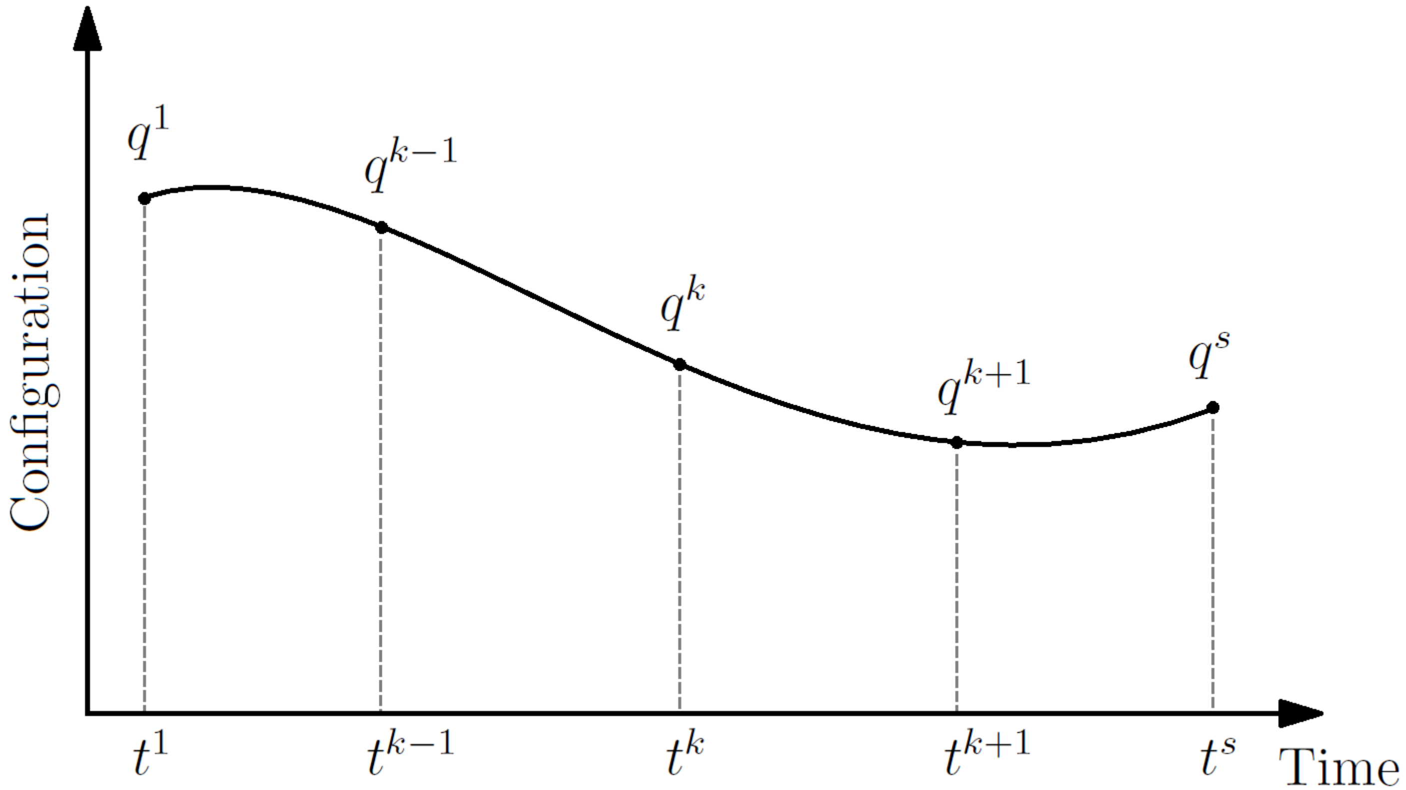
2. Optimization constraints:

$$\begin{aligned} \dot{q} &= q\tilde{\mathbf{v}} \\ \mathbf{M}\dot{\mathbf{v}} + \mathbf{g}(q, \mathbf{v}) + \mathbf{B}^T\lambda &= \mathbf{A}\mathbf{u} \\ \mathbf{\Phi}(q) &= 0 \\ \mathbf{y}_{\text{eff}}(q) - \mathbf{y}_{\text{presc}}(t) &= 0 \end{aligned}$$

where $q \in SE(3)$
$\mathbf{v} \in \mathbb{R}^n$

Method

- Direct transcription method for optimization:



1. Discretize into « s » time steps.
2. Optimise the configuration q at the given time steps.

Method

- For the direct transcription, we use the discrete form with:

1. Objective function J : the time integral of strain energy.

$$\min J = \min \sum_{k=1}^{s} E^k(q^k)\ \Delta t$$

2. Optimization constraints:
   a. Motion constraints: c_m
   b. Time integration equations (e.g. generalized- α): c_α

   Also formulated on a Lie group, which means that we have for the positions:

$$q^{k+1} = q^k \exp_{SE3}(\Delta Q^k) \quad \text{where } q \in SE(3)$$

Method

- The design variables would be the absolute variables at each time step k

$$(q^1, v^1, \dot{v}^1, \lambda^1, u^1, ..., q^k, v^k, \dot{v}^k, \lambda^k, u^k) \text{ where } q \in SE(3)$$

which means that the problem would need a Lie group optimization solver…

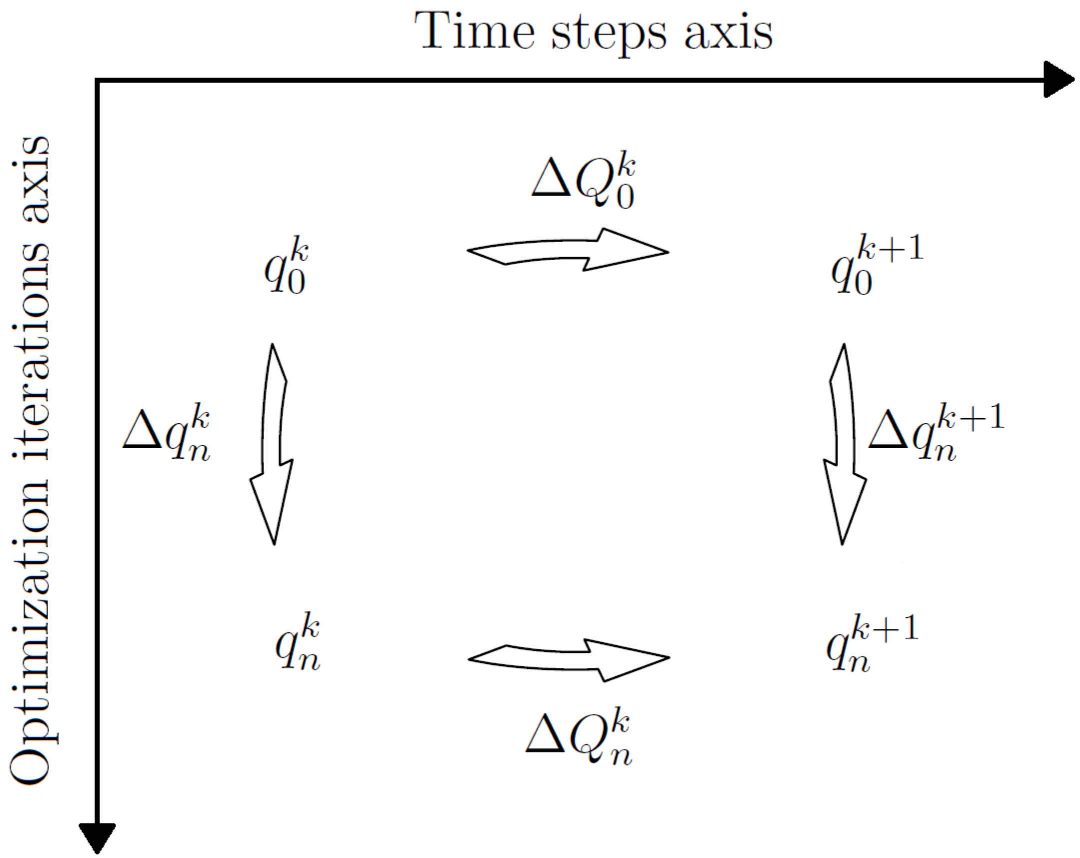
- Instead, we choose as design variables $x = (x^0, x^1, ..., x^k)$, some incremental variables with

$$x^k = (\Delta q^k, \Delta v^k, \Delta \dot{v}^k, \Delta a^k, \Delta \lambda^k, \Delta u^k)$$

where each component $\in \mathbb{R}^n$.

- Resulting optimization can be solved with a NLP algorithm.

Method



Δq_n^k : Optimization variables

q_0^k : Absolute coordinates variables

ΔQ_0^k : Change in absolute variables from time step k to k+1

Method

- Updates of the system's states thanks to the optimization variables:

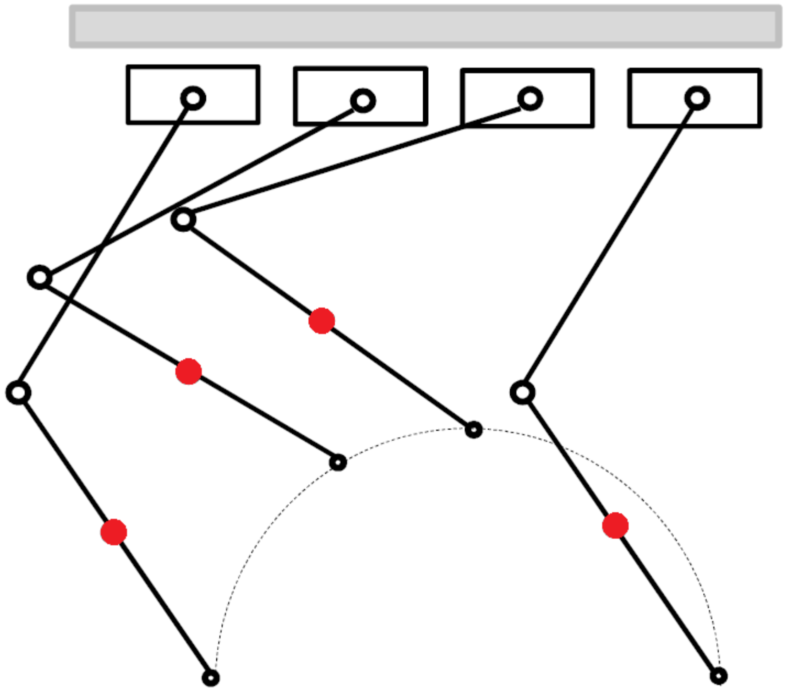
$$
\begin{aligned}
q_n^k &= q_0^k \exp_{SE3}(\Delta q_n^k) \\
v_n^k &= v_0^k + \Delta v_n^k \\
\dot{v}_n^k &= \dot{v}_0^k + \Delta \dot{v}_n^k \\
\lambda_n^k &= \lambda_0^k + \Delta \lambda_n^k \\
u_n^k &= u_0^k + \Delta u_n^k
\end{aligned}
$$

- Evaluate new objective and constraints (and gradients).
- Compute new optimization variables…

Examples

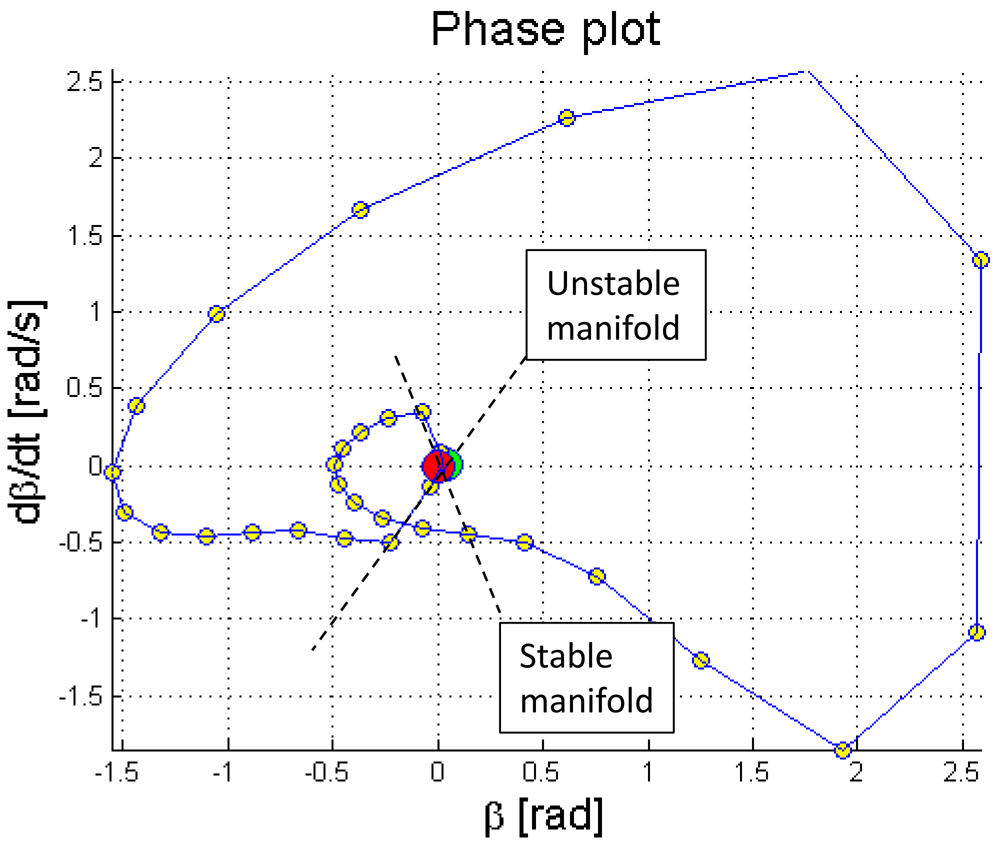
- [Bastos et al 2013] showed that the Optimal control approach works (2D flexible systems): rigid bodies with passive joint.
- We now want to show that it works in 3D, with the Lie group formalism.

⇒ We first want to validate the method with Lie formalism in a 2D system then extend to 3D problems.



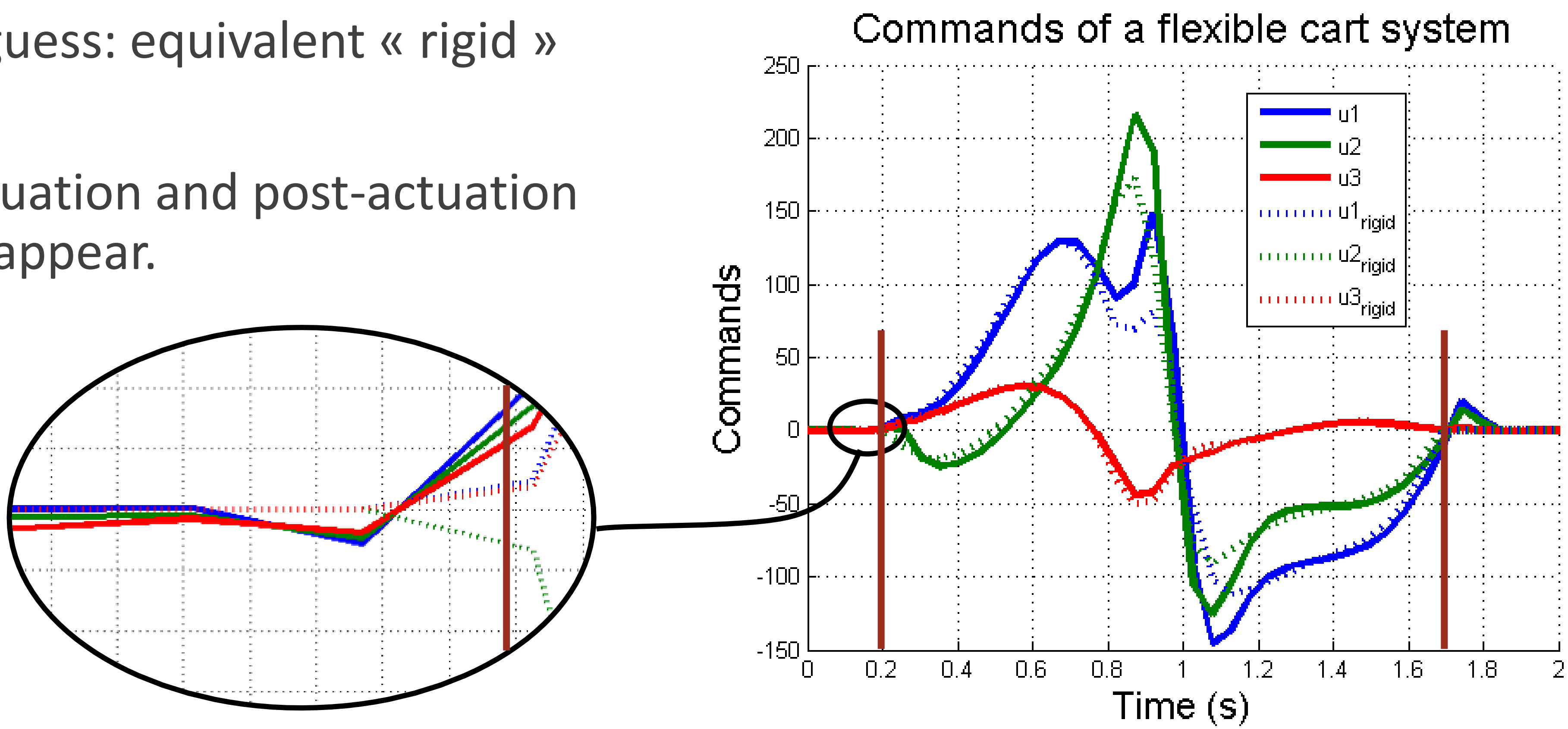
Example – Rigid bodies with passive joint

- Similarly to the stable inversion method, we see that the internal dynamics starts on a unstable manifold and ends on a stable manifold.



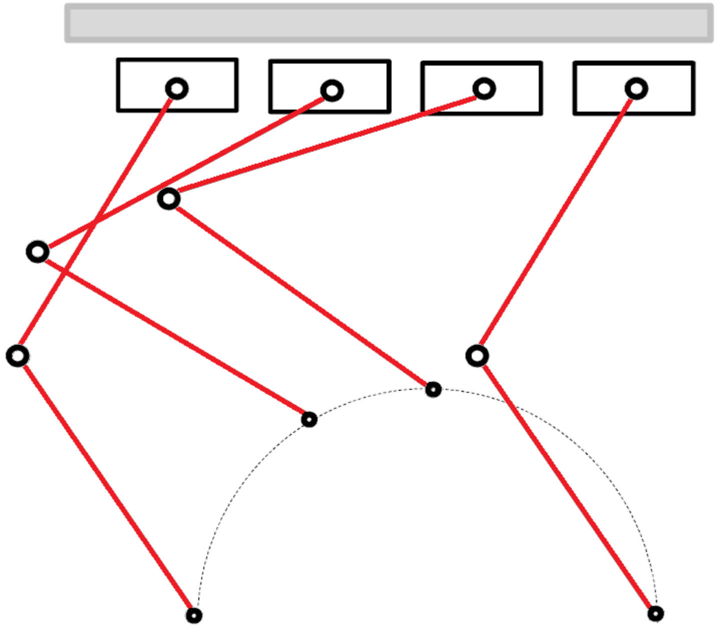
Example – Rigid bodies with passive joint

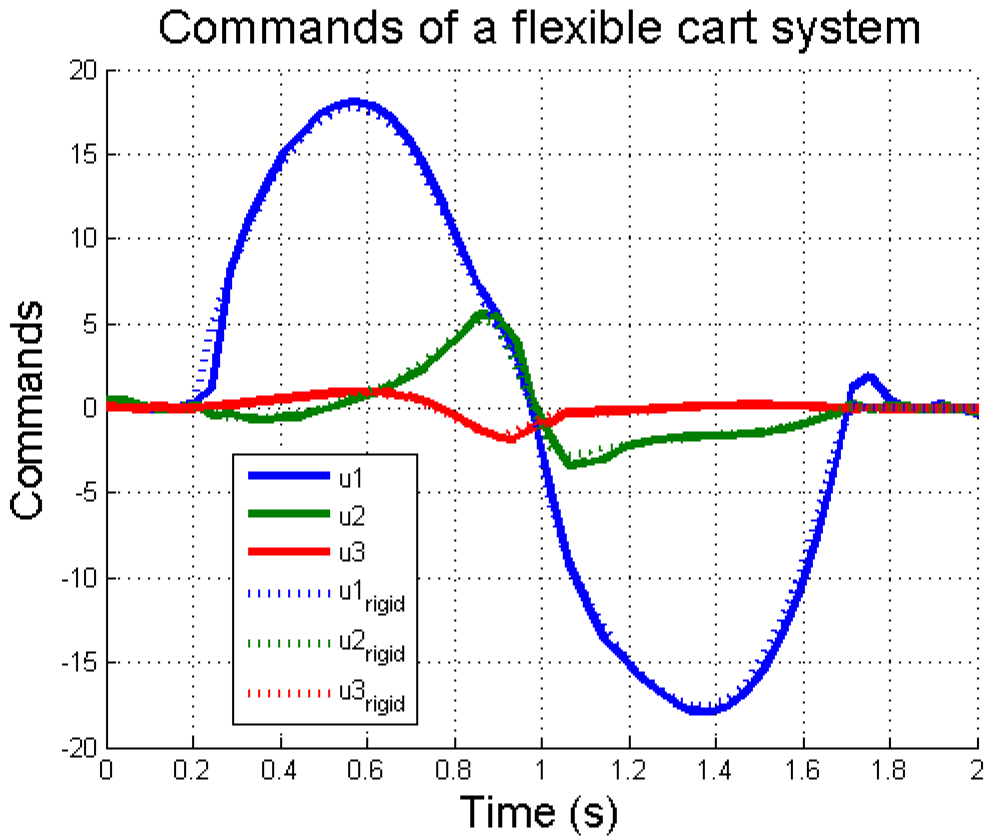
- Initial guess: equivalent « rigid » system.
- Pre-actuation and post-actuation phases appear.



Example – 2 flexible beam elements.

- Initial guess: equivalent « rigid » system.





Conclusions and Perspectives

- In this work:
  1. The inverse dynamic problem of flexible non-minimum phase system is solved with an optimal control approach.
  2. A Lie group formalism is used to model the system's dynamic.
  3. Successful analysis for 2D systems.
- On going work:
  1. Inverse dynamic of flexible 3D systems.
- Perspectives:
  1. Experimental testing of the method.

Thank you for your attention

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