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A viscoelastic-viscoplastic-damage constitutive model based on a large strain hyperelastic formulation for amorphous glassy polymers

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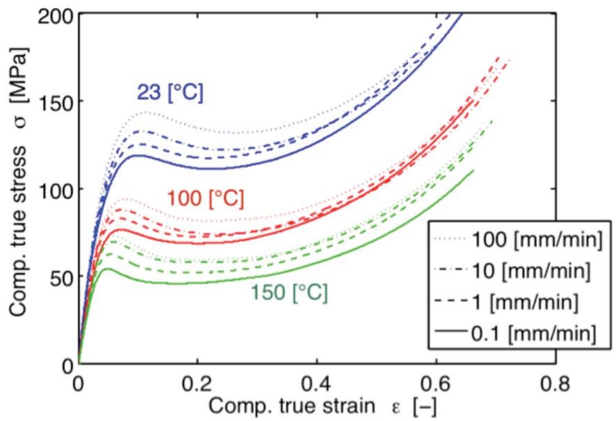
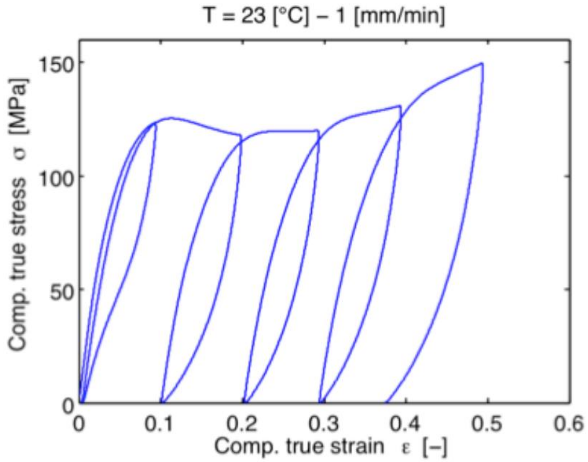
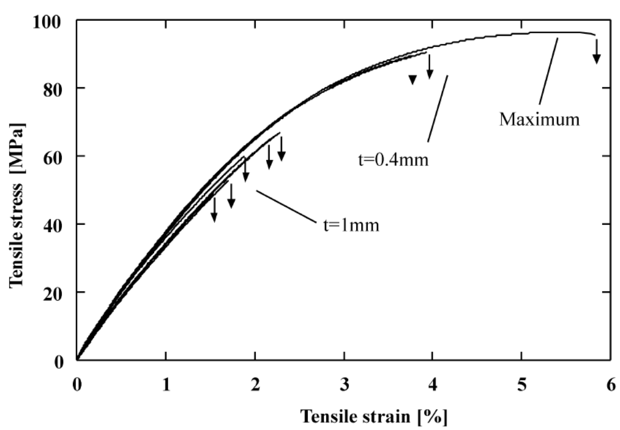
Introduction

- Complex behavior of glassy polymers
  - Rate-, pressure-, temperature-dependent
  - Multi-stage:
    - Viscoelastic at small strains
    - Viscoplastic at large strains
    - Irreversible saturation softening
    - Different failure points in tension and compression

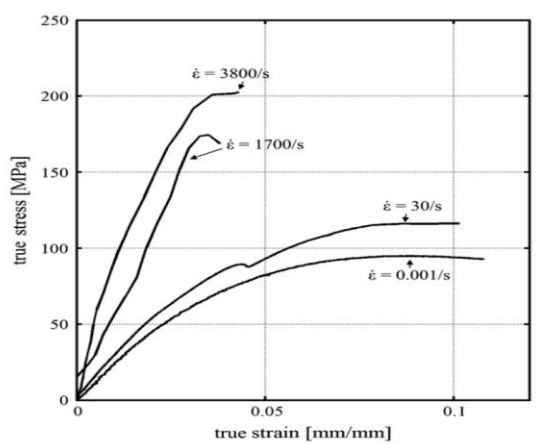




Compression behavior of RTM6 under monotonic and cyclic loadings (Morelle et al. 2012)



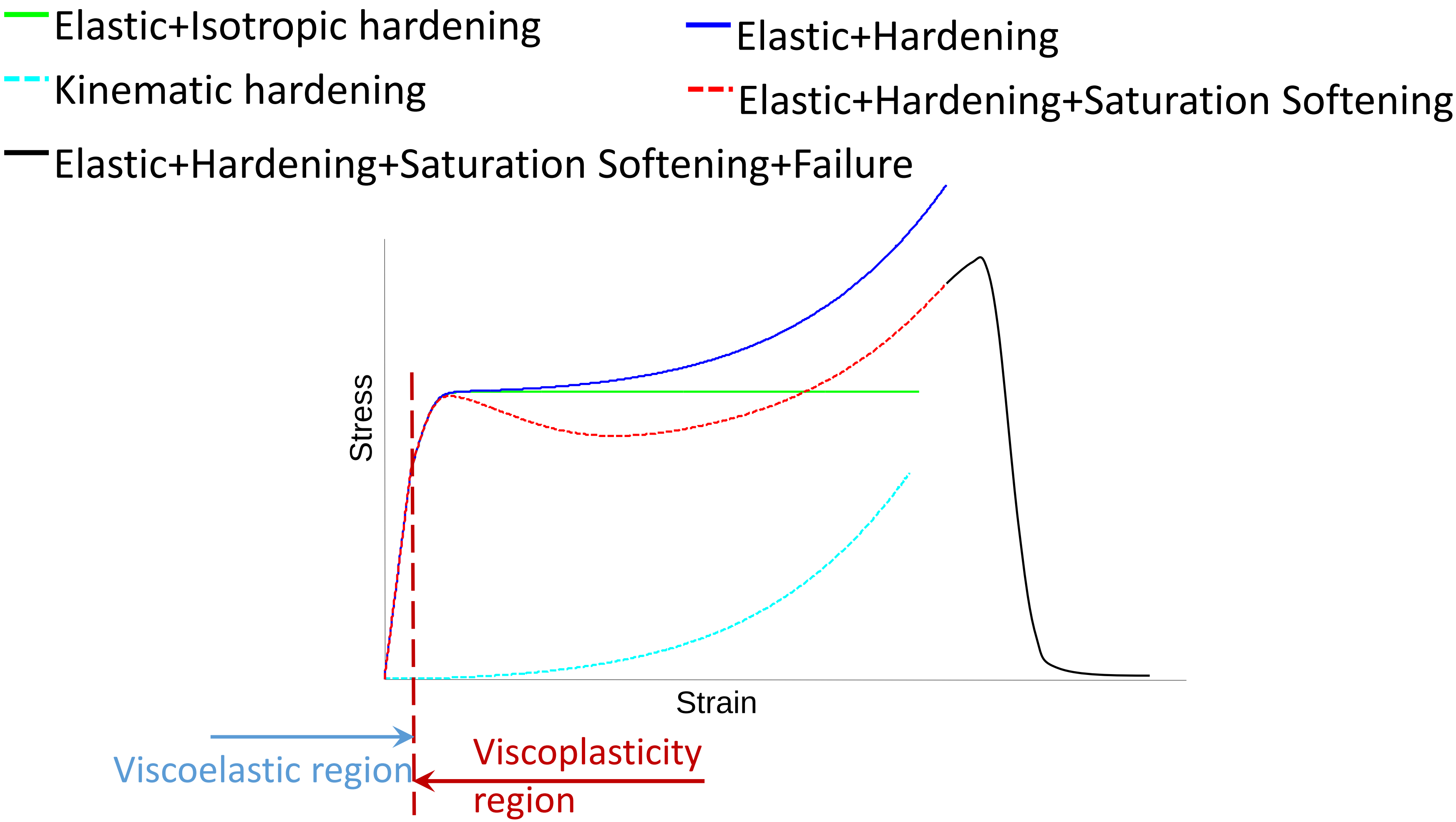
Tensile behavior of epoxy (Fiedler et al. 2001)



Tensile behavior of epoxy (Gerlachet al. 2008)

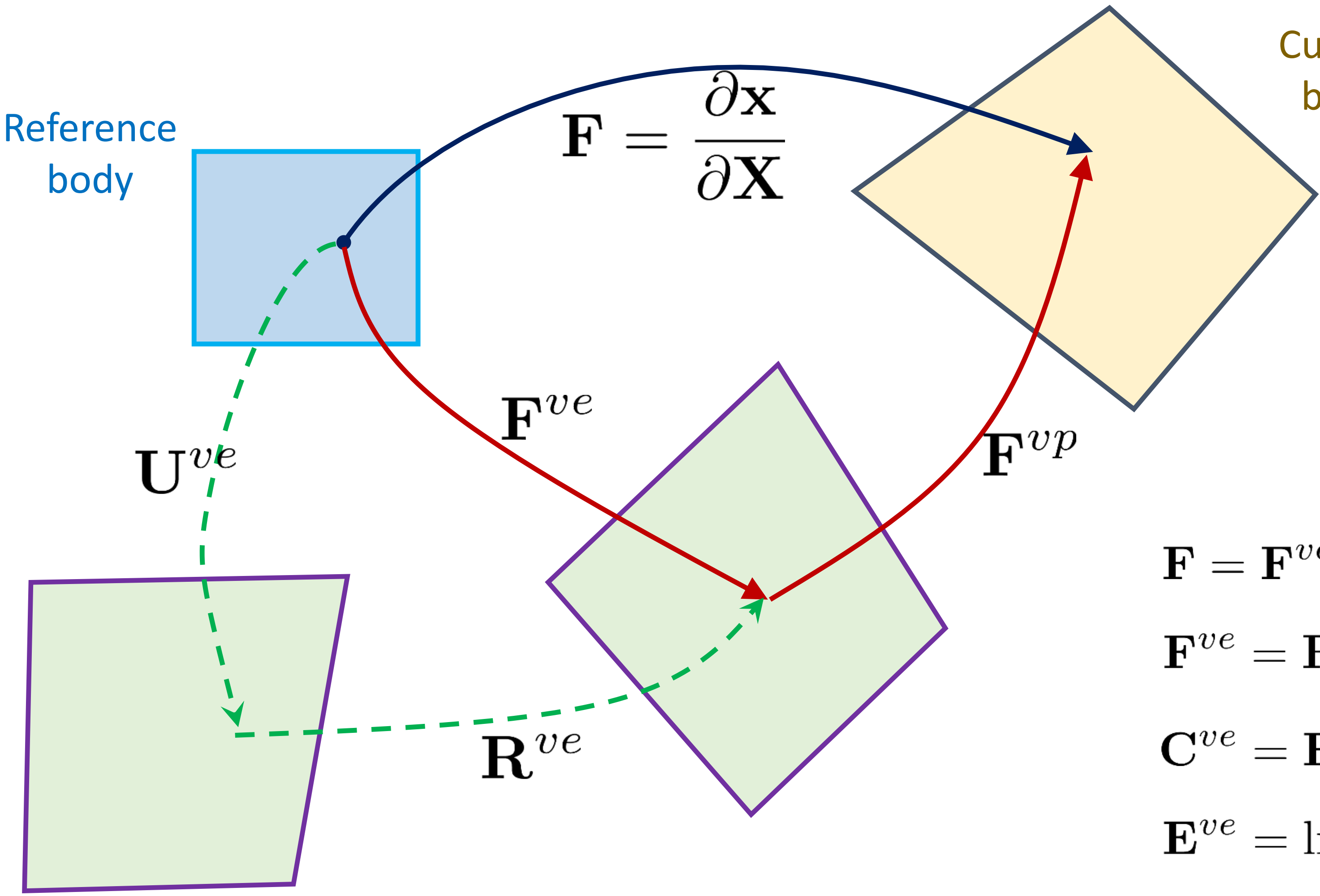
Introduction

- Modeling strategy:



Modeling strategy

- Finite deformation decomposition:



$$\mathbf{F} = \mathbf{F}^{ve} \cdot \mathbf{F}^{vp}$$

$$\mathbf{F}^{ve} = \mathbf{R}^{ve} \cdot \mathbf{U}^{ve}$$

$$\mathbf{C}^{ve} = \mathbf{F}^{veT} \cdot \mathbf{F}^{ve}$$

$$\mathbf{E}^{ve} = \ln \sqrt{\mathbf{C}^{ve}}$$

Modeling strategy

- Isotropic softening model:
  - Material degradation is modeled by the reduction of the load carrying surface with a softening parameter D

$$D = 1 - \frac{S_{\text{reduced}}}{S}$$

  - Isotropic 3D problems: $\hat{\mathbf{P}} = \dfrac{\mathbf{P}}{1 - D}$

- Constitutive modeling includes:
  - Undamaged constitutive law : $\hat{\mathbf{P}}(t) = \hat{\mathbf{P}}\left(\mathbf{F}(t), \mathbf{Q}(\tau) : 0 \leq \tau \leq t\right)$
  - Evolution of the softening variable: $D(t) = D\left(\mathbf{F}(t), \mathbf{Q}(\tau) : 0 \leq \tau \leq t\right)$

$\mathbf{Q}$ is a vector, contains internal variables

Modeling strategy

- Effective stress measures based on an hyperelastic approach:
  - Existence of a viscoelastic potential: $\Psi = \Psi\left(\mathbf{E}^{ve}\right)$
  - Stress measures:

$$\delta\Psi = \hat{\mathbf{P}} : \delta\mathbf{F} = \hat{\boldsymbol{\kappa}} \cdot \left(\delta\mathbf{F} \cdot \mathbf{F}^{-1}\right) = \hat{\boldsymbol{\tau}} : \delta\mathbf{E}^{ve}$$

    - First Piola Kirchhoff stress tensor: $\hat{\mathbf{P}}$
    - Kirchhoff stress tensor: $\hat{\boldsymbol{\kappa}}$
    - Co-rotational Kirchhoff stress tensor: $\hat{\boldsymbol{\tau}}$

Same form as small deformation theory

  - Relations between the stress measures:

$$\hat{\mathbf{P}} = 2\mathbf{F}^{ve} \cdot \left(\hat{\boldsymbol{\tau}} : \frac{\partial\mathbf{E}^{ve}}{\partial\mathbf{C}^{ve}}\right) \cdot \mathbf{F}^{vp-T}$$

$$\hat{\boldsymbol{\kappa}} = \hat{\mathbf{P}} \cdot \mathbf{F}^{T}$$

$$\hat{\boldsymbol{\kappa}} = \mathbf{R}^{ve} \cdot \hat{\boldsymbol{\tau}} \cdot \mathbf{R}^{veT}$$

→ Undamaged constitutive law is written in terms of the corotational stress

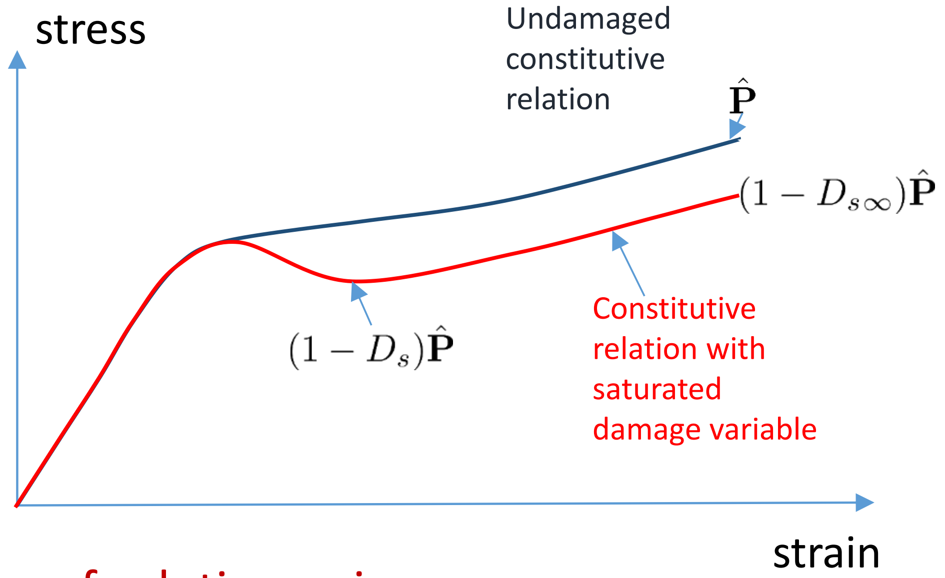
Modeling strategy

- Saturation softening stage:

D_s :saturation softening variable

$0 \leq D_s \rightarrow D_{s\infty} < 1$

when strains increase



- To avoid the problem of loss of solution uniqueness
  → implicit non-local formulation (Peerlings et al. 1996)

$$\bar{\gamma}_s - l_s^2 \nabla_0 \cdot \nabla_0 \bar{\gamma}_s = \gamma_s$$

γ_s , $\bar{\gamma}_s$ local& non-local variables associated to the saturation softening stage

l_s saturation non-local characteristic length

- Saturation damage evolution:

$$\dot{D}_s = F_s\left(D_s, \mathbf{F}, \bar{\gamma}_s\right)\dot{\bar{\gamma}}_s$$

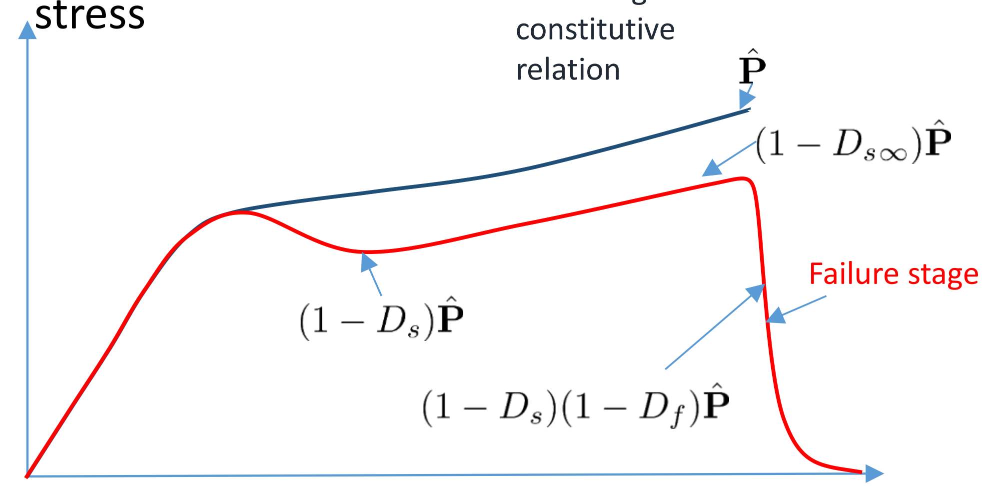
Modeling strategy

- Failure stage:

D_f : failure softening variable

$0 \leq D_f \to 1$

when strains increase



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- Failure damage evolution:

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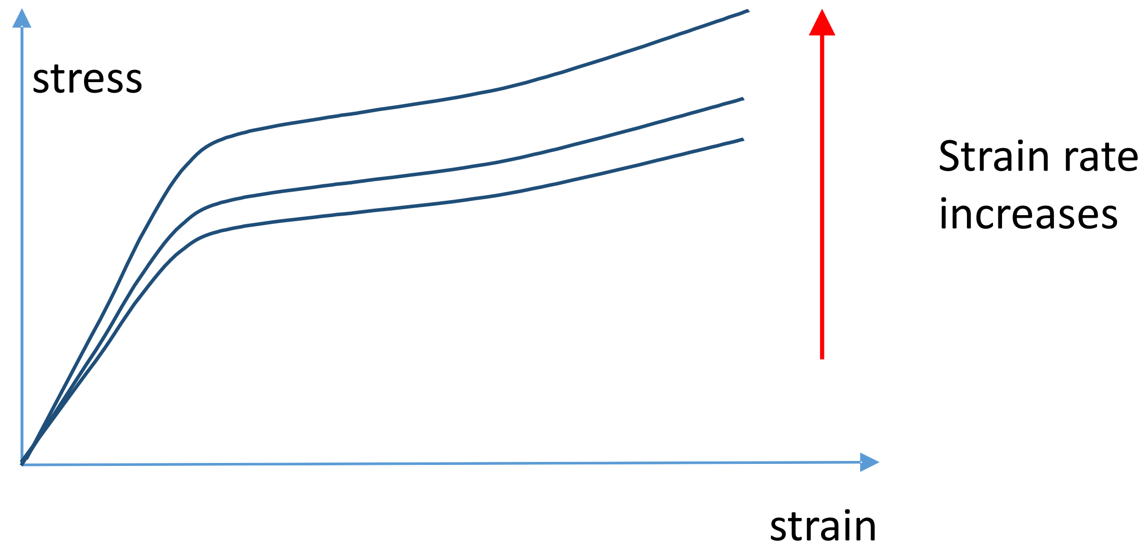
Modeling strategy

- Rate dependent behavior:
  - coupled viscoelastic/viscoplastic model for the undamaged constitutive law



Outline

- Viscoelastic modeling
- Viscoplastic modeling
- Selection of local variables for implicit non-local formulation
- Numerical examples
- Conclusions & perspectives

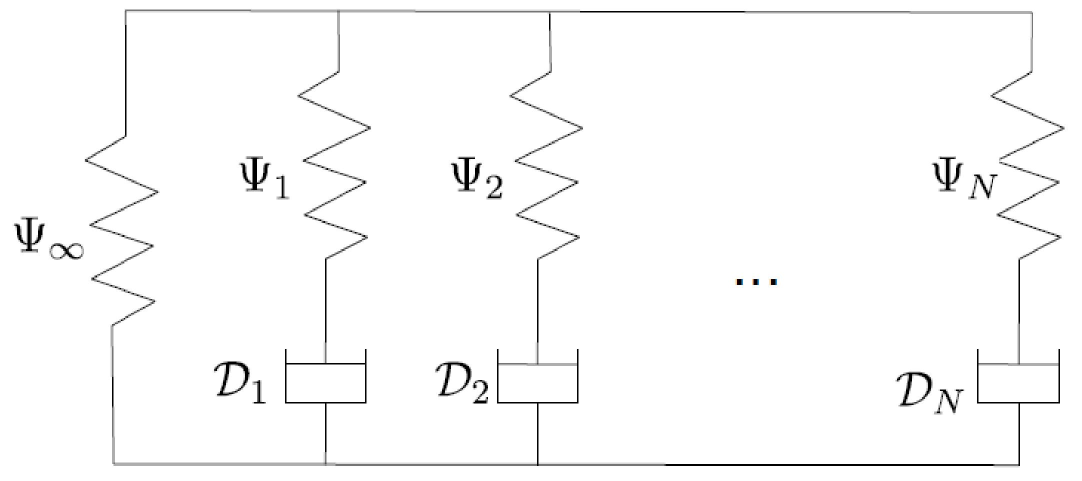
Viscoelastic modeling

- Generalized Maxwell model:
  - N+1 springs governed by N+1 bi-logarithmic potentials:

$$\Psi_i = \frac{K_i}{2} \ln^2\left(J^{ve}\right) + G_i \mathrm{dev}\, \mathbf{E}^{ve} : \mathrm{dev}\, \mathbf{E}^{ve}\ , i = \infty, 1 \ldots N$$

  - N dashpots governed by N dissipation functions:

$$\mathcal{D}_i,\ \ i = 1 \ldots N$$



Viscoelastic modeling

- Corotational Kirchhoff stress:

$$\Psi = \Psi_\infty + \sum_{k=1}^{N} (\Psi_k - \mathcal{D}_k) \qquad \hat{\boldsymbol{\tau}} = \frac{\partial \Psi}{\partial \mathbf{E}^{ve}} = \hat{\boldsymbol{\tau}}_\infty + \sum_{i=1}^{N} \hat{\boldsymbol{\tau}}_i$$

$$\hat{\boldsymbol{\tau}}_\infty = \frac{\partial \Psi_\infty}{\mathbf{E}^{ve}} = K_\infty \mathrm{tr}\, \mathbf{E}^{ve} \mathbf{I} + 2G_\infty \mathrm{dev}\, \mathbf{E}^{ve}$$

$$\hat{\boldsymbol{\tau}}_i = \frac{\partial \Psi_i}{\mathbf{E}^{ve}} - \frac{\partial \mathcal{D}_i}{\mathbf{E}^{ve}} = K_i \mathrm{tr}\, \mathbf{E}^{ve} \mathbf{I} + 2G_i \mathrm{dev}\, \mathbf{E}^{ve} - \mathbf{q}_i$$

- Evolution of internal variables $\mathbf{q}_i, \ i = 1 \ldots N$

$$\mathrm{dev}\, \dot{\mathbf{q}}_i = \frac{\mathrm{dev}\, \hat{\boldsymbol{\tau}}}{g_i} = \frac{2G_i}{g_i} \mathrm{dev}\, \mathbf{E}^{ve} - \frac{1}{g_i} \mathrm{dev}\, \mathbf{q}_i$$

$$\mathrm{tr}\, \dot{\mathbf{q}}_i = \frac{\mathrm{tr}\, \hat{\boldsymbol{\tau}}}{k_i} = \frac{3K_i}{k_i} \mathrm{tr}\, \mathbf{E}^{ve} - \frac{1}{k_i} \mathrm{tr}\, \mathbf{q}_i$$

Time constants: $k_i, g_i, \ i = 1 \ldots N$

Viscoelastic modeling

- Viscoelastic constitutive relation:
  - Deviatoric part:

$$\operatorname{dev}\hat{\boldsymbol{\tau}} = \int_{-\infty}^{t} 2G(t-s) : \frac{\mathrm{d}}{\mathrm{d}s}\operatorname{dev}\mathbf{E}^{ve}(s)\,ds$$

$$G(t) = G_\infty + \sum_{i=1}^{N} G_i \exp\left(-\frac{t}{g_i}\right)$$

  - Volumetric part:

$$\hat{p}^{cor} = \frac{1}{3}\operatorname{tr}\hat{\boldsymbol{\tau}} = \int_{-\infty}^{t} K(t-s) : \frac{\mathrm{d}}{\mathrm{d}s}\operatorname{tr}\mathbf{E}^{ve}(s)\,ds$$

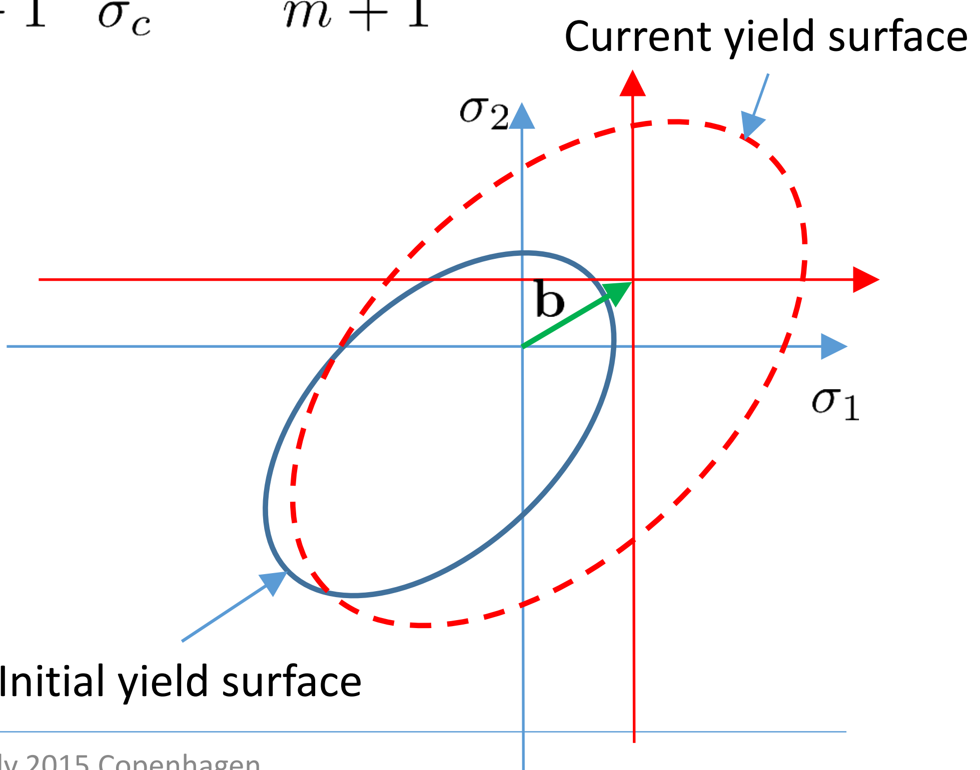
$$K(t) = K_\infty + \sum_{i=1}^{N} K_i \exp\left(-\frac{t}{k_i}\right)$$

Viscoplastic modeling

- Yield surface:
  - A generalized version of the Drucker –Prager yield surface with an exponent-enhanced octahedral term
  - In terms of the corotational Kirchhoff stress

$$F\left(\hat{\boldsymbol{\tau}}, \mathbf{b}, \sigma_c, \sigma_t\right) = \left(\frac{\phi_e}{\sigma_c}\right)^{\alpha} - \frac{m^{\alpha}-1}{m+1}\frac{3\phi_p}{\sigma_c} - \frac{m^{\alpha}+m}{m+1}$$

$$\begin{cases} F < 0 & \text{Elastic region} \\ F \geq 0 & \text{Plastic region} \end{cases}$$



$$\phi_e = \sqrt{\frac{3}{2}\text{dev}\left(\hat{\boldsymbol{\tau}} - \mathbf{b}\right) : \text{dev}\left(\hat{\boldsymbol{\tau}} - \mathbf{b}\right)}$$

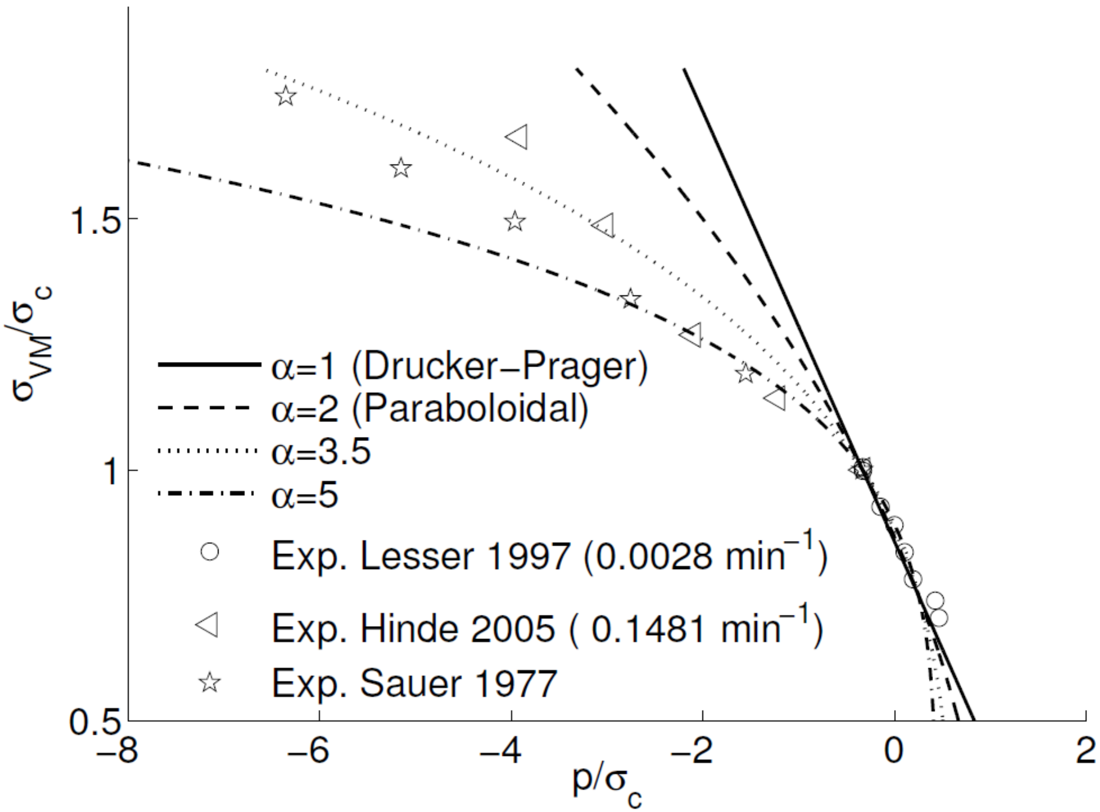
$$\phi_p = \frac{1}{3}\text{tr}\left(\hat{\boldsymbol{\tau}} - \mathbf{b}\right) \quad m = \frac{\sigma_t}{\sigma_c}$$

σ_t : Tensile yield stress

σ_c : Compression yield stress

Viscoplastic modeling

- Yield surface:
  - Influence of the yield exponent α

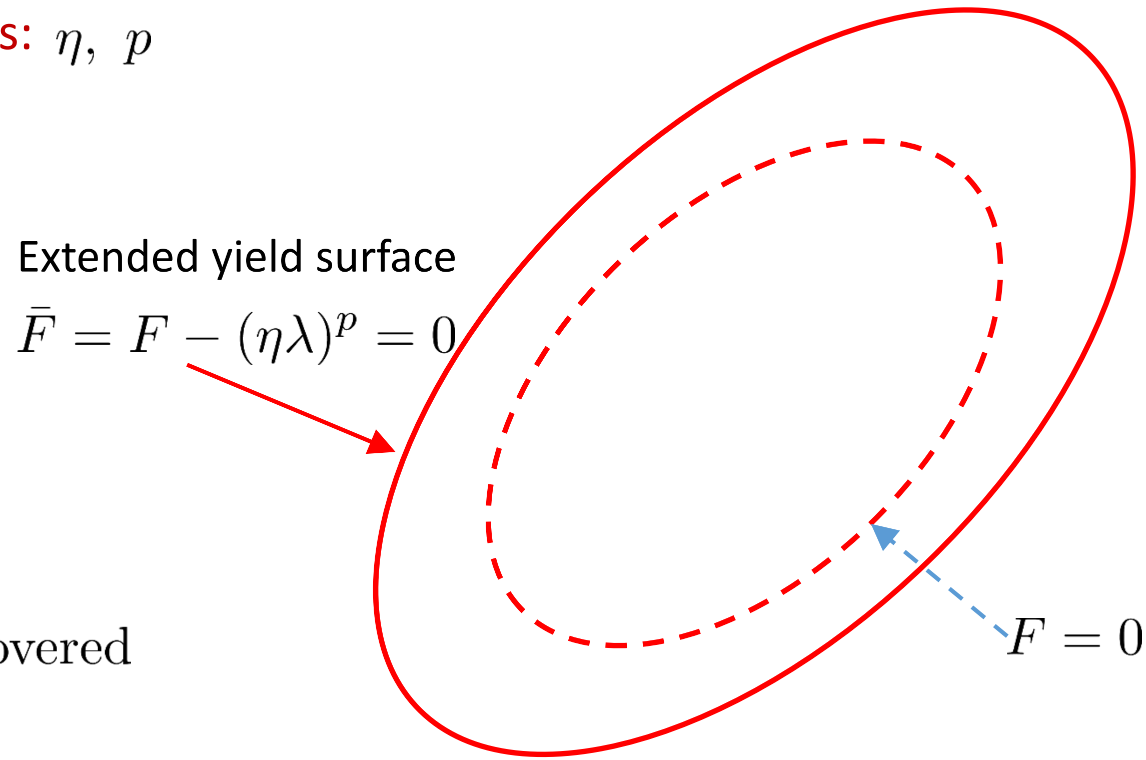


Viscoplastic modeling

- Non-associated Perzyna-type viscoplastic flow rule:

$$\mathbf{D}^{vp} = \lambda \frac{\partial P}{\partial \hat{\boldsymbol{\tau}}} \qquad \lambda = \frac{1}{\eta} < F >^{\frac{1}{p}} \qquad < F >= \begin{cases} F \text{ if } F \geq 0 \\ 0 \text{ if } F < 0 \end{cases}$$

  - Yield function: F
  - Plastic flow potential: P
  - Viscosity parameters: $\eta, \ p$



$$\text{If } \eta \to 0$$
$$\text{time-independent case is recovered}$$

Viscoplastic modeling

- Quadratic flow potential:

$$P = \phi_e^2 + \beta \phi_p^2$$

$$\phi_e = \sqrt{\frac{3}{2} \mathrm{dev}\left(\hat{\boldsymbol{\tau}} - \mathbf{b}\right) : \mathrm{dev}\left(\hat{\boldsymbol{\tau}} - \mathbf{b}\right)}$$

$$\phi_p = \frac{1}{3} \mathrm{tr}\left(\hat{\boldsymbol{\tau}} - \mathbf{b}\right)$$

  - Constant plastic Poisson ratio during plastic flow:

$$\nu_p = \frac{9 - 2\beta}{18 + 2\beta}$$

$$\beta = 0 \rightarrow \text{ incompressible plastic flow is recovered}$$

  - Equivalent plastic strain:

$$\dot{\gamma} = k\sqrt{\mathbf{D}^{vp} : \mathbf{D}^{vp}} = k\lambda \sqrt{\frac{\partial P}{\partial \hat{\boldsymbol{\tau}}} : \frac{\partial P}{\partial \hat{\boldsymbol{\tau}}}}$$

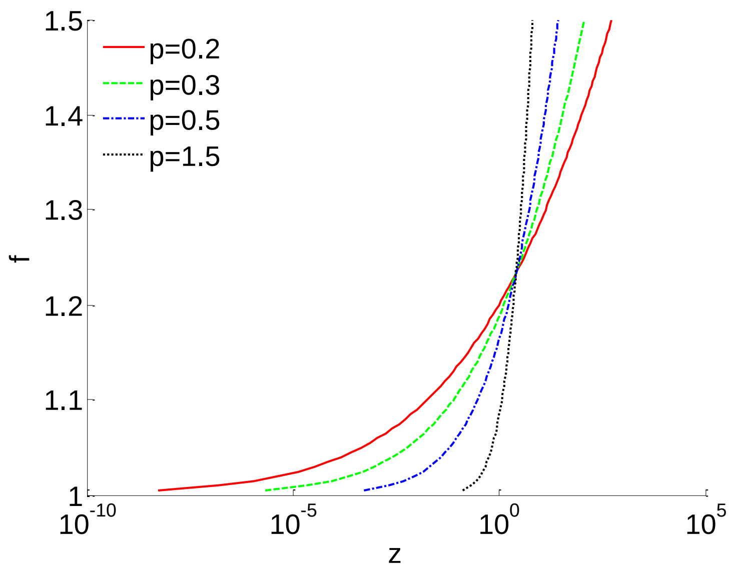
$$k = \frac{1}{\sqrt{1 + 2\nu_p^2}} \qquad \beta = 0 \rightarrow k = \sqrt{\frac{2}{3}}$$

Viscoplastic modeling

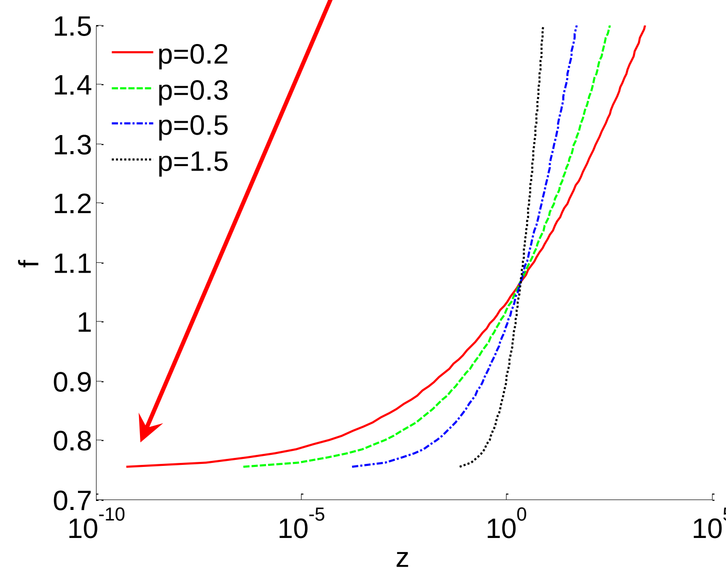
- Influence of the viscosity exponent in the uniaxial tests:

$$\bar{F} = f^{\alpha} \pm \frac{m^{\alpha} - 1}{m + 1} f - \frac{m^{\alpha} + m}{m + 1} - \left(z \frac{1}{f\sqrt{6 + \frac{4}{27}\beta^2}} \right)^p = 0$$

$$f = \frac{|\sigma|}{\sigma_c} \quad z = \frac{\eta \dot{\gamma}}{\sigma_c} \qquad \mathbf{b = 0} \qquad \alpha = 3.5, \; m = 0.75, \; \beta = 0.3$$



Uniaxial compression (+)



Uniaxial traction (-)

Application to epoxy resins

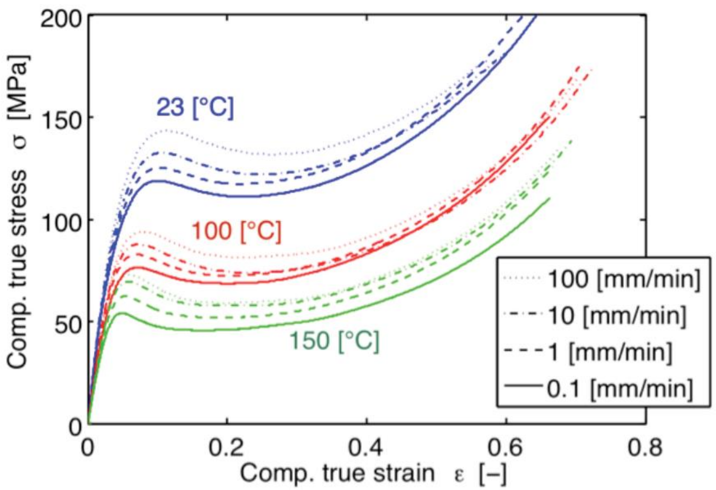
- Uniaxial compression tests

$$\left(\frac{\sigma_f}{\sigma_c}\right)^{\alpha} + \frac{m^{\alpha}-1}{m+1}\frac{\sigma_f}{\sigma_c} - \frac{m^{\alpha}+m}{m+1} - \left(\frac{\eta\dot{\gamma}}{\sigma_f\sqrt{6+\frac{4}{27}\beta^2}}\right)^{p} = 0$$

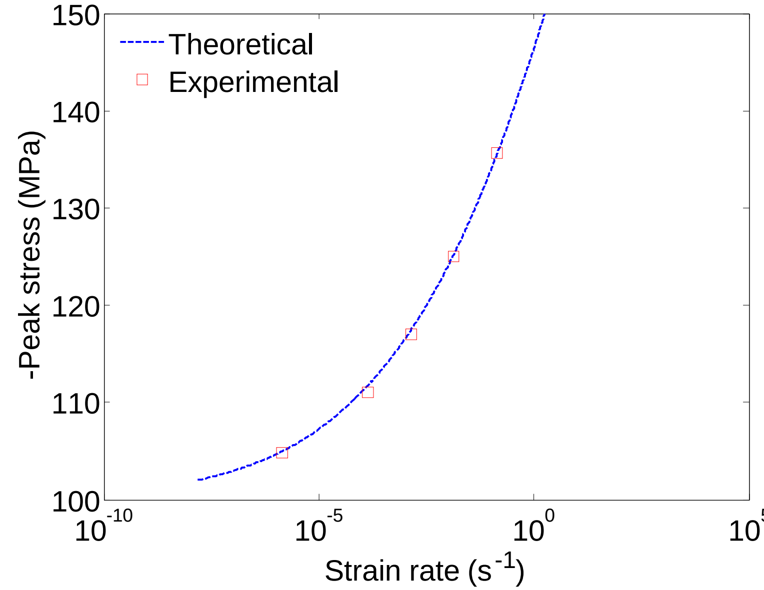
$\alpha = 3.5 \quad p = 0.21$

$m = 0.75 \quad \eta = 3.10^4 \text{ MPa.s}$

$\beta = 0.3 \quad \sigma_c = 100 \text{ MPa}$



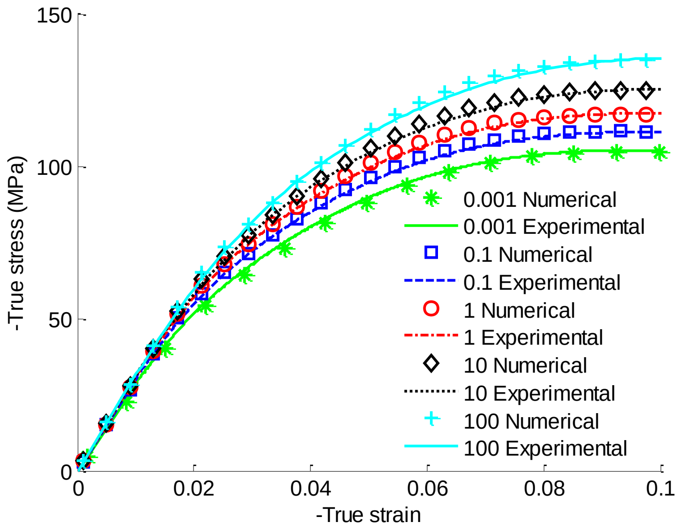
Compression behavior of RTM6 under monotonic loadings (Morelle et al. 2012)

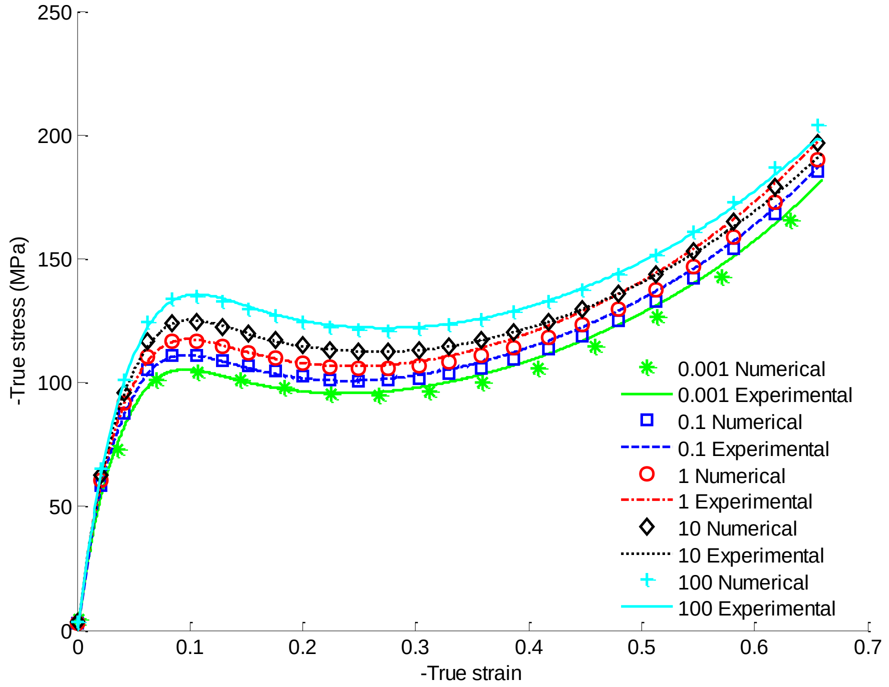


Compared to experiments at 23°C

Application to epoxy resins

- Local variable for saturation softening stage: $\dot{\gamma}_s = \dot{\gamma}$
- Uniaxial compression tests without considering failure damage
  - Saturation damage law: $\dot{D}_s = H_s \bar{\gamma}_s^{n_s} \left(D_{s\infty} - D_s \right) \dot{\bar{\gamma}}_s$





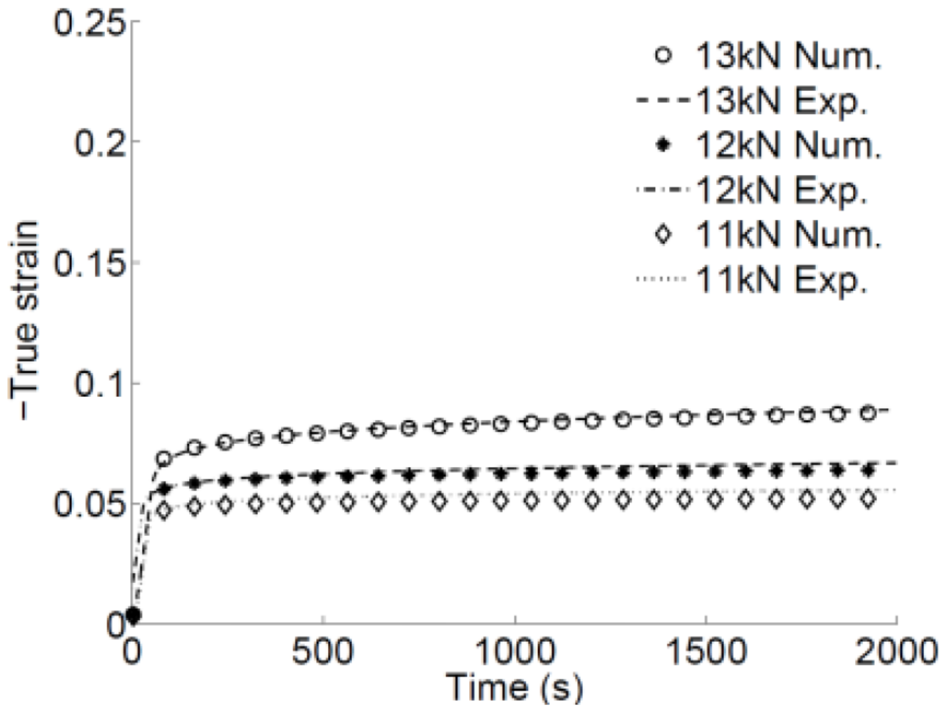
Pre-peak

Full-range

Experimental data from Morelle et al. 2012

Application to epoxy resins

- Creep tests without considering failure damage :
  - Saturation damage law: $$\dot{D}_s = H_s \bar{\gamma}_s^{n_s} \left(D_{s\infty} - D_s \right) \dot{\bar{\gamma}}_s$$



Experimental data from Morelle et al. 2012

Application to epoxy resins

- Local variable for failure stage: based on a pressure-sensitive failure criterion:
  - Failure criterion: $F_f = \Phi_f\left(\hat{\boldsymbol{\tau}}, \hat{X}_c, \hat{X}_t\right) - r \leq 0$
    - Power failure surface:

$$\Phi_f = \frac{m_c + 1}{m_c^{\alpha_f} + 1}\left(\frac{\hat{\tau}_e}{\hat{X}_c}\right)^{\alpha_f} + \frac{1 - m_c^{\alpha_f}}{m_c^{\alpha_f} + m_c}\frac{\mathrm{tr}\,\hat{\boldsymbol{\tau}}}{\hat{X}_c} - 1$$

$$\hat{\tau}_e = \sqrt{\frac{3}{2}\mathrm{dev}\,\hat{\boldsymbol{\tau}} : \mathrm{dev}\,\hat{\boldsymbol{\tau}}}$$

  - Failure stresses:

$$\hat{X}_{c,t} = \hat{X}_{c,t}(\dot{\gamma})$$

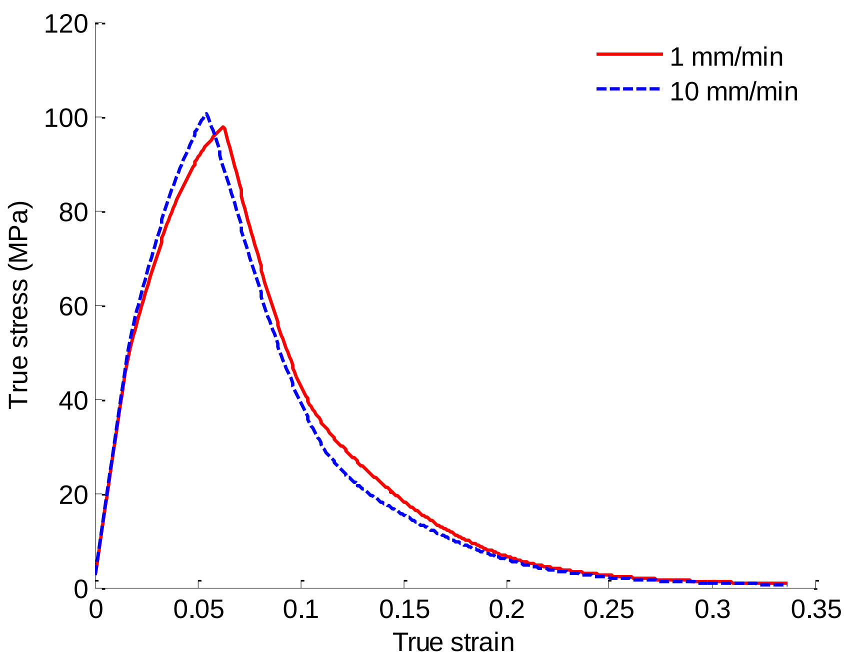
e.g. power law:

$$\hat{X}_c = \hat{X}_c^0 + A_c\dot{\gamma}^{\beta_c}$$
$$\hat{X}_t = \hat{X}_t^0 + A_t\dot{\gamma}^{\beta_t}$$

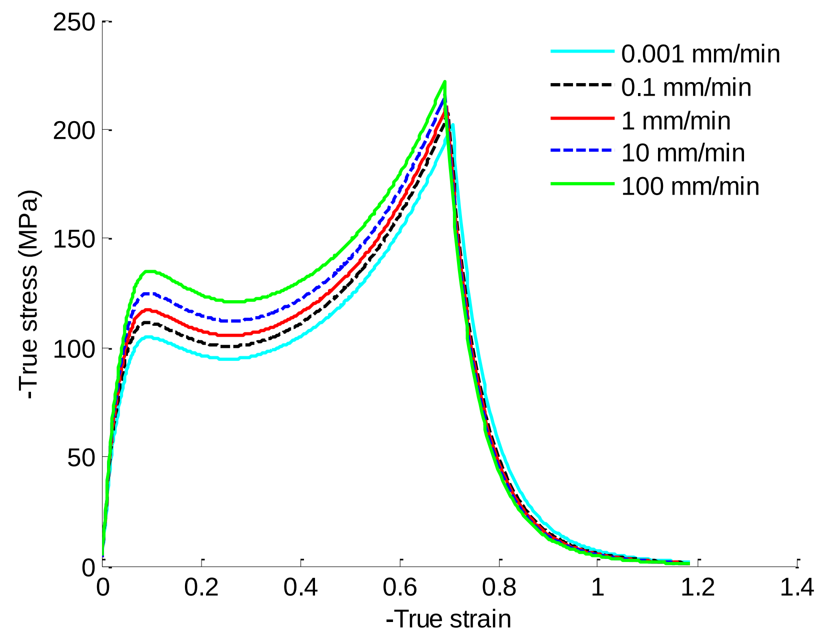
  - Failure onset evolution: $\dot{r} \geq 0,\ F_f \leq 0,\ \dot{r}F_f = 0$
  - Local variable for failure stage: $\dot{\gamma}_f = \begin{cases}\dot{\gamma} \text{ if } \dot{r} > 0\\ 0 \text{ if } \dot{r} = 0\end{cases}$

Application to epoxy resins

- Stress/strain behavior with failure:
  - Saturation damage law: $\dot{D}_s = H_s \bar{\gamma}_s^{n_s} \left(D_{s\infty} - D_s \right) \dot{\bar{\gamma}}_s$
  - Failure damage law: $\dot{D}_f = H_f \bar{\gamma}_f^{n_f} \left(1 - D_f \right) \dot{\bar{\gamma}}_f$



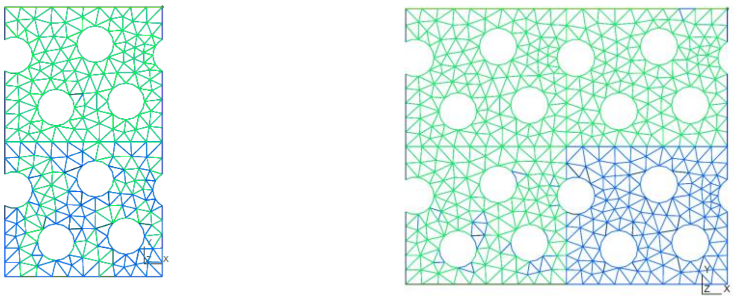
Traction



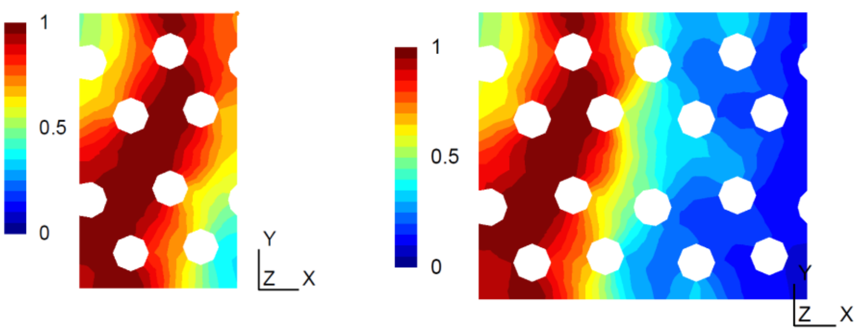
Compression

Application to epoxy resins

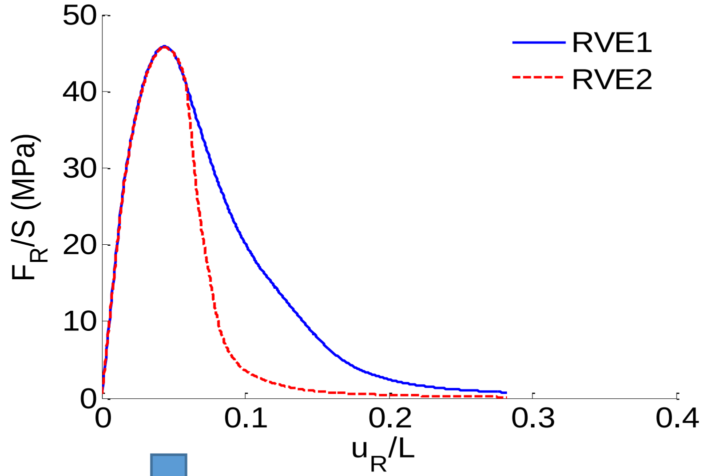
- Transition from the continuum damage to a cohesive law (Nguyen V.P et al. 2010)
  - Voided structure made of epoxy resin
  - Uniaxial traction (mode I) using the orthotropic BC



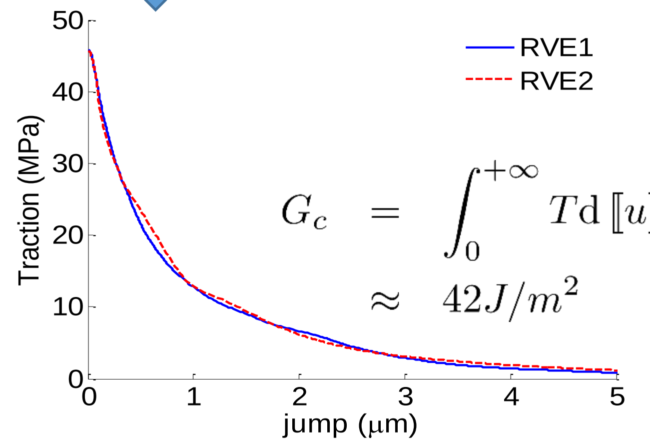
RVE1 RVE2



Damage pattern



Stress-strain



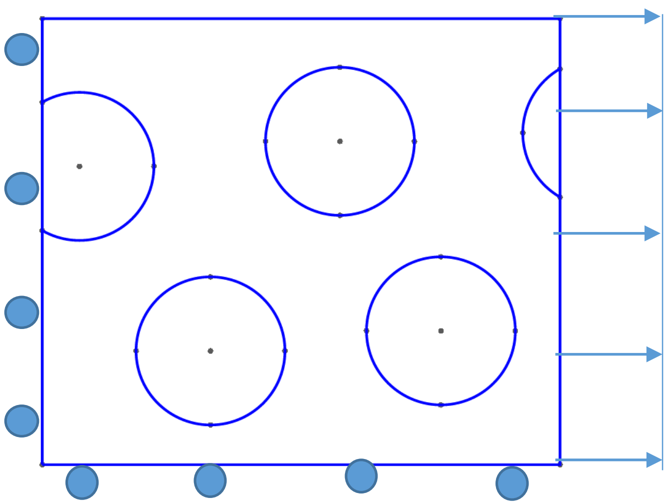
Traction/separation law

Application to epoxy resins

- Uniaxial traction test of the carbon fiber/epoxy composite

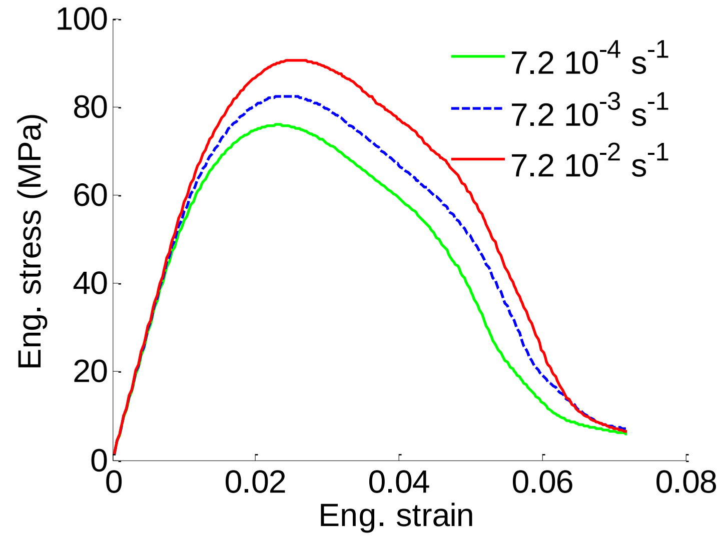
$$D = 1 - (1 - D_s)(1 - D_f)$$

$$\begin{aligned} f &= 30\% \\ R &= 0.05 \text{ mm} \\ L_x &= 3.48\,10^{-4} \text{ mm} \\ L_y &= 3.01\,10^{-4} \text{ mm} \\ L_z &= 1.\,10^{-5} \text{ mm} \end{aligned}$$



Prescribed displacement

7.2 e-3 s^{-1}



Conclusions & future works

- The proposed model can capture:
  - Rate-dependent full-range stress/strain behavior of epoxy
  - Rate-dependent stress/strain behavior of composites
  - Traction/separation law can be extracted on RVEs
- Future works
  - Study the failure behavior of fiber/epoxy composites using the computational homogenization method

Thank you for your attention!