A viscoelastic-viscoplastic-damage constitutive model based on a large strain hyperelastic formulation for amorphous glassy polymers


ICCM20 19-24 July 2015, Copenhagen, Denmark

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Introduction

- Complex behavior of glassy polymers
  - Rate-, pressure-, temperature-dependent
  - Multi-stage:
    - Viscoelastic at small strains
    - Viscoplastic at large strains
    - Irreversible saturation softening
    - Different failure points in tension and compression

Tensile behavior of epoxy (Fiedler et al. 2001)

Compressive behavior of RTM6 under monotonic and cyclic loadings (Morelle et al. 2012)
Introduction

• Modeling strategy:

- Elastic+Isotropic hardening
- Kinematic hardening
- Elastic+Hardening+Saturation Softening+Failure

![Stress vs Strain Graph]

Viscoelastic region
Viscoplasticity region

7/9/2015

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Modeling strategy

- Finite deformation decomposition:

\[ F = \frac{\partial x}{\partial X} \]

\[ F^{ve} = R^{ve} \cdot U^{ve} \]

\[ C^{ve} = F^{veT} \cdot F^{ve} \]

\[ E^{ve} = \ln \sqrt{C^{ve}} \]
Modeling strategy

• Isotropic softening model:
  • Material degradation is modeled by the reduction of the load carrying surface with a softening parameter $D$

\[ D = 1 - \frac{S_{\text{reduced}}}{S} \]

• Isotropic 3D problems: \[ \hat{P} = \frac{P}{1 - D} \]

• Constitutive modeling includes:
  • Undamaged constitutive law: \[ \hat{P}(t) = \hat{P}(F(t), Q(\tau) : 0 \leq \tau \leq t) \]
  • Evolution of the softening variable: \[ D(t) = D(F(t), Q(\tau) : 0 \leq \tau \leq t) \]

$Q$ is a vector, contains internal variables
Modeling strategy

- Effective stress measures based on an hyperelastic approach:
  - Existence of a viscoelastic potential: $\Psi = \Psi(\mathbf{E}^{ve})$
  - Stress measures:
    $$\delta \Psi = \mathbf{P} : \delta \mathbf{F} = \mathbf{\kappa} \cdot (\delta \mathbf{F} \cdot \mathbf{F}^{-1}) = \mathbf{\hat{\tau}} : \delta \mathbf{E}^{ve}$$
    - First Piola Kirchhoff stress tensor: $\mathbf{\hat{P}}$
    - Kirchhoff stress tensor: $\mathbf{\hat{\kappa}}$
    - Co-rotational Kirchhoff stress tensor: $\mathbf{\hat{\tau}}$

- Relations between the stress measures:
  $$\mathbf{\hat{P}} = 2\mathbf{F}^{ve} \cdot \left( \mathbf{\hat{\tau}} : \frac{\partial \mathbf{E}^{ve}}{\partial \mathbf{C}^{ve}} \right) \cdot \mathbf{F}^{vp-T}$$
  $$\mathbf{\hat{\kappa}} = \mathbf{\hat{P}} \cdot \mathbf{F}^{T}$$
  $$\mathbf{\hat{\kappa}} = \mathbf{R}^{ve} \cdot \mathbf{\hat{\tau}} \cdot \mathbf{R}^{ve T}$$

$\rightarrow$ Undamaged constitutive law is written in terms of the corotational stress

Same form as small deformation theory
Modeling strategy

- Saturation softening stage:

\[ D_s : \text{saturation softening variable} \]

\[ 0 \leq D_s \rightarrow D_s^\infty < 1 \]

when strains increase

- To avoid the problem of loss of solution uniqueness

\[ \rightarrow \text{implicit non-local formulation (Peerlings et al. 1996)} \]

\[ \tilde{\gamma}_s - l_s^2 \nabla_0 \cdot \nabla_0 \tilde{\gamma}_s = \gamma_s \]

- Saturation damage evolution:

\[ \dot{D}_s = F_s (D_s, F, \tilde{\gamma}_s) \dot{\tilde{\gamma}}_s \]

\[ l_s \] saturation non-local characteristic length

\[ \gamma_s, \tilde{\gamma}_s \] local & non-local variables associated to the saturation softening stage

\( \hat{\mathbf{P}} \)

\( (1 - D_s^\infty) \hat{\mathbf{P}} \)

\( (1 - D_s) \hat{\mathbf{P}} \)
Modeling strategy

• Failure stage:

\[ D_f : \text{failure softening variable} \]

\[ 0 \leq D_f \to 1 \]

when strains increase

• To avoid the problem of loss of solution uniqueness
  \[ \to \text{implicit non-local formulation (Peerlings et al. 1996)} \]

\[ \bar{\gamma}_f - l_f^2 \nabla_0 \cdot \nabla_0 \bar{\gamma}_f = \gamma_f \]

\[ \gamma_f, \bar{\gamma}_f \] local & non-local variables associated to the saturation softening stage

• Failure damage evolution:

\[ \dot{D}_f = F_f (D_f, F, \bar{\gamma}_f) \dot{\bar{\gamma}}_f \]

\[ l_f \] saturation non-local characteristic length
Modeling strategy

• Rate dependent behavior:
  • coupled viscoelastic/viscoplastic model for the undamaged constitutive law
Outline

• Viscoelastic modeling

• Viscoplastic modeling

• Selection of local variables for implicit non-local formulation

• Numerical examples

• Conclusions & perspectives
Viscoelastic modeling

- Generalized Maxwell model:
  - N+1 springs governed by N+1 bi-logarithmic potentials:
    \[ \Psi_i = \frac{K_i}{2} \ln^2 (J^{ve}) + G_i \text{dev } \mathbf{E}^{ve} : \text{dev } \mathbf{E}^{ve}, \ i = \infty, 1 \ldots N \]
  - N dashpots governed by N dissipation functions:
    \[ \mathcal{D}_i, \ i = 1 \ldots N \]
Viscoelastic modeling

- Corotational Kirchhoff stress:

\[ \Psi = \Psi_\infty + \sum_{k=1}^{N} (\Psi_k - D_k) \]

[\hat{\tau} = \frac{\partial \Psi}{\partial E^{ve}} = \hat{\tau}_\infty + \sum_{i=1}^{N} \hat{\tau}_i]

[\hat{\tau}_\infty = \frac{\partial \Psi_\infty}{E^{ve}} = K_\infty \text{tr} E^{ve} I + 2G_\infty \text{dev} E^{ve}]

[\hat{\tau}_i = \frac{\partial \Psi_i}{E^{ve}} - \frac{\partial D_i}{E^{ve}} = K_i \text{tr} E^{ve} I + 2G_i \text{dev} E^{ve} - q_i]

- Evolution of internal variables \( q_i, \ i = 1...N \)

[\text{dev} \dot{q}_i = \frac{\text{dev} \hat{\tau}}{g_i} = \frac{2G_i}{g_i} \text{dev} E^{ve} - \frac{1}{g_i} \text{dev} q_i]

[\text{tr} \dot{q}_i = \frac{\text{tr} \hat{\tau}}{k_i} = \frac{3K_i}{k_i} \text{tr} E^{ve} - \frac{1}{k_i} \text{tr} q_i]

Time constants: \( k_i, g_i, \ i = 1...N \)
Viscoelastic modeling

- Viscoelastic constitutive relation:
  - Deviatoric part:
    \[
    \text{dev } \hat{\tau} = \int_{-\infty}^{t} 2G(t - s) : \frac{d}{ds} \text{dev } \mathbf{E}^{ve}(s) \, ds
    \]
    \[
    G(t) = G_{\infty} + \sum_{i=1}^{N} G_i \exp \left(-\frac{t}{g_i}\right)
    \]
  - Volumetric part:
    \[
    \hat{\rho}^{\text{cor}} = \frac{1}{3} \text{tr } \hat{\tau} = \int_{-\infty}^{t} K(t - s) : \frac{d}{ds} \text{tr } \mathbf{E}^{ve}(s) \, ds
    \]
    \[
    K(t) = K_{\infty} + \sum_{i=1}^{N} K_i \exp \left(-\frac{t}{k_i}\right)
    \]
Viscoplastic modeling

- Yield surface:
  - A generalized version of the Drucker–Prager yield surface with an exponent-enhanced octahedral term
  - In terms of the corotational Kirchhoff stress

\[
F(\帽子{\tau}, b, \sigma_c, \sigma_t) = \left( \frac{\phi_e}{\sigma_c} \right)^\alpha - \frac{m^\alpha - 1}{m + 1} \frac{3\phi_p}{\sigma_c} - \frac{m^\alpha + m}{m + 1}
\]

\[
\begin{cases} 
F < 0 \quad \text{Elastic region} \\
F \geq 0 \quad \text{Plastic region}
\end{cases}
\]

\[
\phi_e = \sqrt{\frac{3}{2} \text{dev}(\hat{\tau} - b) : \text{dev}(\hat{\tau} - b)}
\]

\[
\phi_p = \frac{1}{3} \text{tr}(\hat{\tau} - b) \quad m = \frac{\sigma_t}{\sigma_c}
\]

\(\sigma_t\) : Tensile yield stress

\(\sigma_c\) : Compression yield stress

Current yield surface

Initial yield surface
Viscoplastic modeling

• Yield surface:
  • Influence of the yield exponent $\alpha$
Viscoplastic modeling

- Non-associated Perzyna-type viscoplastic flow rule:

\[ \mathbf{D}^{vp} = \lambda \frac{\partial P}{\partial \hat{\tau}} \]

\[ \lambda = \frac{1}{\eta} \langle F \rangle \]

\[ \langle F \rangle = \begin{cases} 
F & \text{if } F \geq 0 \\
0 & \text{if } F < 0 
\end{cases} \]

- Yield function: \( F \)
- Plastic flow potential: \( P \)
- Viscosity parameters: \( \eta, \quad p \)

**Extended yield surface**

\[ \overline{F} = F - (\eta \lambda)^p = 0 \]

If \( \eta \to 0 \),

time-independent case is recovered

\[ F = 0 \]
Viscoplastic modeling

- Quadratic flow potential:

\[ P = \phi_e^2 + \beta \phi_p^2 \]

\[ \phi_e = \sqrt{\frac{3}{2} \text{dev} (\mathbf{\hat{\tau}} - \mathbf{b}) : \text{dev} (\mathbf{\hat{\tau}} - \mathbf{b})} \]

\[ \phi_p = \frac{1}{3} \text{tr} (\mathbf{\hat{\tau}} - \mathbf{b}) \]

- Constant plastic Poisson ratio during plastic flow:

\[ \nu_p = \frac{9 - 2\beta}{18 + 2\beta} \]

\[ \beta = 0 \rightarrow \text{incompressible plastic flow is recovered} \]

- Equivalent plastic strain:

\[ \dot{\gamma} = k \sqrt{\mathbf{D}^{vp} : \mathbf{D}^{vp}} = k\lambda \sqrt{\frac{\partial P}{\partial \mathbf{\hat{\tau}}} : \frac{\partial P}{\partial \mathbf{\hat{\tau}}}} \]

\[ k = \frac{1}{\sqrt{1 + 2\nu_p^2}} \]

\[ \beta = 0 \rightarrow k = \sqrt{\frac{2}{3}} \]
Viscoplastic modeling

- Influence of the viscosity exponent in the uniaxial tests:

\[
\bar{F} = f^\alpha \pm \frac{m^\alpha - 1}{m + 1} f - \frac{m^\alpha + m}{m + 1} - \left( z \frac{1}{f \sqrt{6 + \frac{4}{27} \beta^2}} \right)^p = 0
\]

\[f = \frac{|\sigma|}{\sigma_c}, \quad z = \frac{\dot{\gamma}}{\dot{\gamma}_c}, \quad b = 0 \quad \alpha = 3.5, \quad m = 0.75, \quad \beta = 0.3\]
Application to epoxy resins

• Uniaxial compression tests

\[
\left( \frac{\sigma_f}{\sigma_c} \right)^{\alpha} + \frac{m^\alpha - 1}{m+1} \frac{\sigma_f}{\sigma_c} - \frac{m^\alpha + m}{m+1} - \left( \frac{\eta \dot{\gamma}}{\sigma_f \sqrt{6 + \frac{4}{27} \beta^2}} \right)^p = 0
\]

\[
\alpha = 3.5 \quad p = 0.21 \\
m = 0.75 \quad \eta = 3.10^4 \text{ MPa.s} \\
\beta = 0.3 \quad \sigma_c = 100 \text{ MPa}
\]

Compression behavior of RTM6 under monotonic loadings (Morelle et al. 2012) Compared to experiments at 23°C
Application to epoxy resins

- Local variable for saturation softening stage: $\gamma_s = \gamma$
- Uniaxial compression tests without considering failure damage
  - Saturation damage law: $\dot{D}_s = H_s \gamma_s^n (D_s \infty - D_s) \dot{\gamma}_s$

Experimental data from Morelle et al. 2012
Application to epoxy resins

- Creep tests without considering failure damage:
  - Saturation damage law: \[ \dot{D}_s = H_s \bar{\gamma}_s^{n_s} (D_{s\infty} - D_s) \dot{\bar{\gamma}}_s \]

Experimental data from Morelle et al. 2012
Application to epoxy resins

- Local variable for failure stage: based on a pressure-sensitive failure criterion:
  - Failure criterion: \( F_f = \Phi_f \left( \hat{\tau}, \hat{X}_c, \hat{X}_t \right) - r \leq 0 \)
  - Power failure surface:
    \[
    \Phi_f = \frac{m_c + 1}{m_c^{\alpha_f} + 1} \left( \frac{\hat{\tau}_e}{\hat{X}_c} \right)^{\alpha_f} + \frac{1 - m_c^{\alpha_f}}{m_c^{\alpha_f} + m_c} \frac{\text{tr} \hat{\tau}}{\hat{X}_c} - 1
    \]
    \[
    \hat{\tau}_e = \sqrt{\frac{3}{2} \text{dev} \hat{\tau} : \text{dev} \hat{\tau}}
    \]
  - Failure stresses:
    \[
    \hat{X}_{c,t} = \hat{X}_{c,t}(\dot{\gamma})
    \]
    e.g. power law:
    \[
    \hat{X}_c = \hat{X}_c^0 + A_c \dot{\gamma}^{\beta_c}
    \]
    \[
    \hat{X}_t = \hat{X}_t^0 + A_t \dot{\gamma}^{\beta_t}
    \]
  - Failure onset evolution: \( \dot{r} \geq 0, \ F_f \leq 0, \ \dot{r}F_f = 0 \)
  - Local variable for failure stage:
    \[
    \dot{\gamma}_f = \begin{cases} \dot{\gamma} & \text{if } \dot{r} > 0 \\ 0 & \text{if } \dot{r} = 0 \end{cases}
    \]
Application to epoxy resins

- Stress/strain behavior with failure:
  - Saturation damage law: \( \dot{D}_s = H_s \bar{\gamma}_s^n (D_s - D_s) \dot{\bar{\gamma}}_s \)
  - Failure damage law: \( \dot{D}_f = H_f \bar{\gamma}_f^n (1 - D_f) \dot{\bar{\gamma}}_f \)

![Graphs showing stress-strain behavior for traction and compression](image)

Traction

Compression
Application to epoxy resins

- Transition from the continuum damage to a cohesive law (Nguyen V.P et al. 2010)
  - Voided structure made of epoxy resin
  - Uniaxial traction (mode I) using the orthotropic BC

Stress-strain

$$G_c = \int_0^{+\infty} Td[u]$$

$$\approx 42J/m^2$$

Damage pattern

Traction/separation law
Application to epoxy resins

- Uniaxial traction test of the carbon fiber/epoxy composite

\[
f = 30\%
\]
\[
R = 0.05\text{ mm}
\]
\[
L_x = 3.48 \times 10^{-4}\text{ mm}
\]
\[
L_y = 3.01 \times 10^{-4}\text{ mm}
\]
\[
L_z = 1.1 \times 10^{-5}\text{ mm}
\]

\[D = 1 - (1 - D_s)(1 - D_f)\]
Conclusions & future works

• The proposed model can capture:
  • Rate-dependent full-range stress/strain behavior of epoxy
  • Rate-dependent stress/strain behavior of composites
  • Traction/separation law can be extracted on RVEs

• Future works
  • Study the failure behavior of fiber/epoxy composites using the computational homogenization method
Thank you for your attention!