# A Non-Local Damage-Enhanced Incremental-Secant Mean-Field-Homogenization for Composite Laminate Failure Predictions

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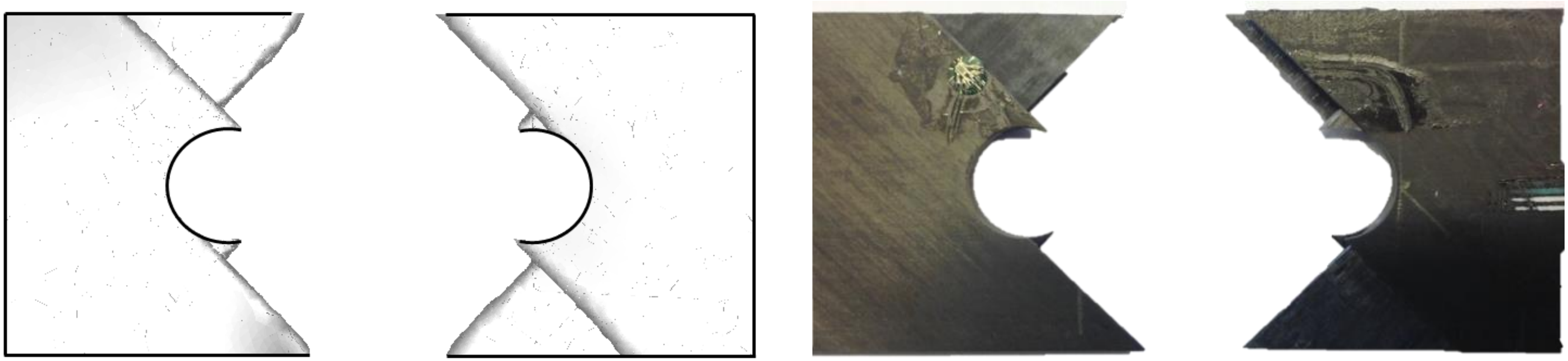
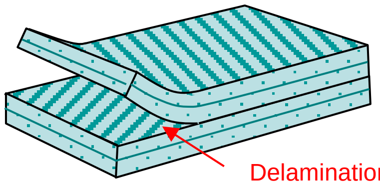
Contributors: F. Sket, J.M. Molina (IMDEA), A. Makradi (Tudor)

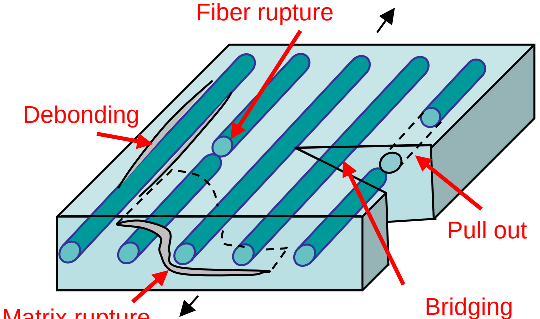


SIMUCOMP The research has been funded by the Walloon Region under the agreement no 1017232 (CT-EUC 2010-10-12) in the context of the ERA-NET +, Matera + framework.

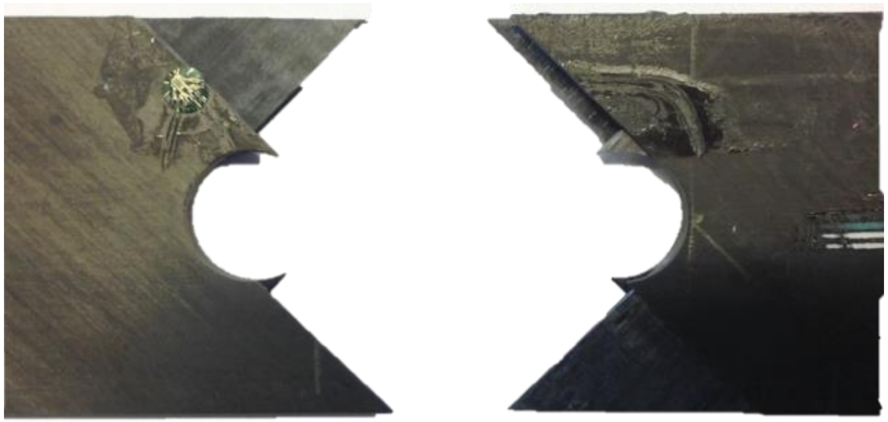
# Failure of composite laminates

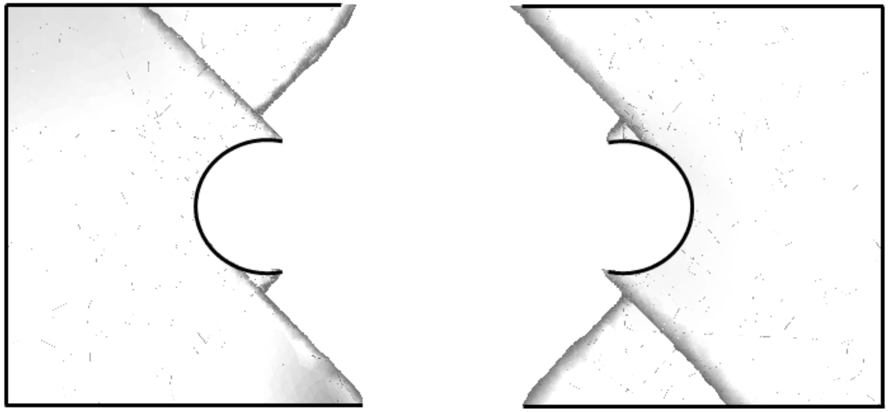
- Difficulties
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure




  - Continuum damage models do not represent accurately the intra-laminar failure
    - Damage propagation direction is not in agreement with experiments

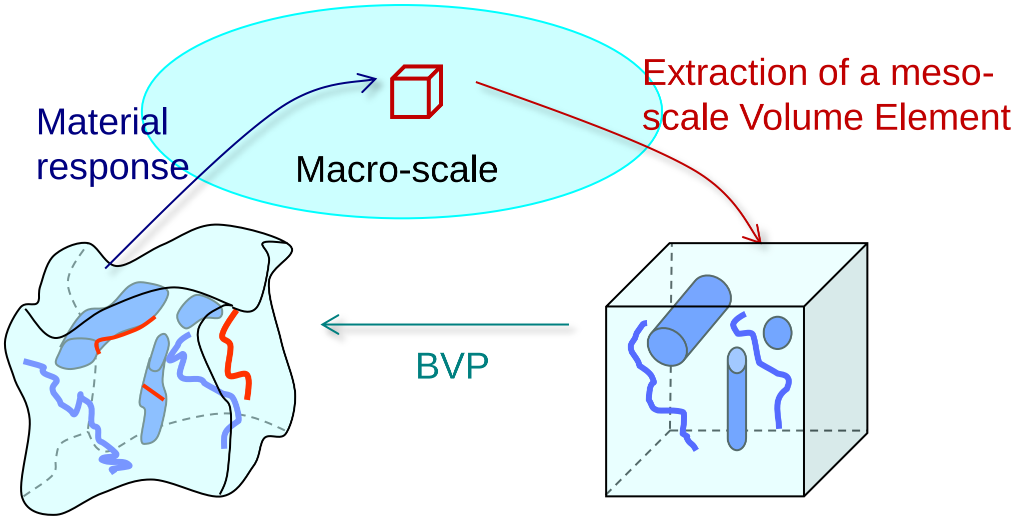




- Solution:
  - Embed damage model in a multi-scale formulation
  - For computational efficiency: use of mean-field-homogenization

# Multi-scale modelling

- Multi-scale modelling
  - One way: homogenization
  - 2 problems are solved (concurrently)
    - The macro-scale problem
    - The meso-scale problem (on a meso-scale Volume Element)




- Length-scales separation

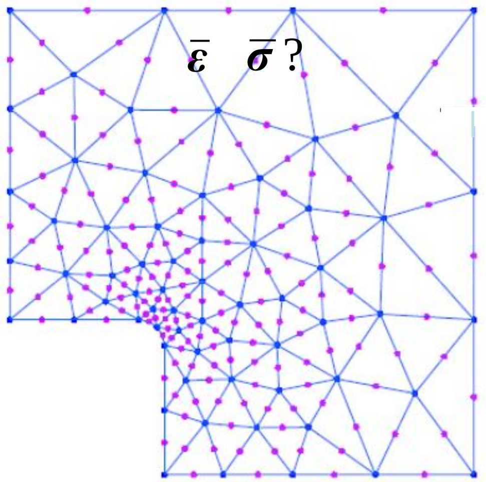
$$L_{macro} >> L_{VE} >> L_{micro}$$

**For accuracy: Size of the meso-scale volume element smaller than the characteristic length of the macro-scale loading**

**To be statistically representative: Size of the meso-scale volume element larger than the characteristic length of the micro-structure**

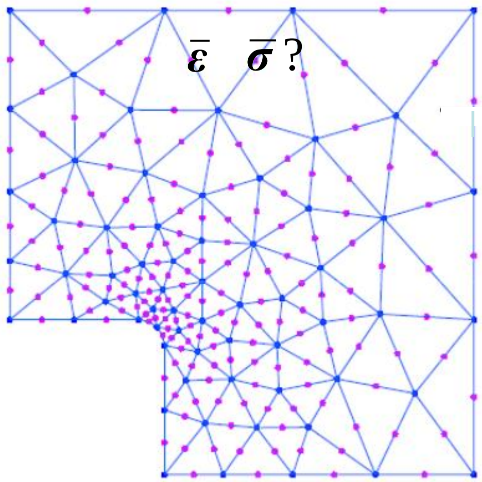
# Multi-scale modelling

- Mean-Field-Homogenization
  - Macro-scale
    - FE model
    - At one integration point $\bar{\boldsymbol{\varepsilon}}$ is know, $\bar{\boldsymbol{\sigma}}$ is sought

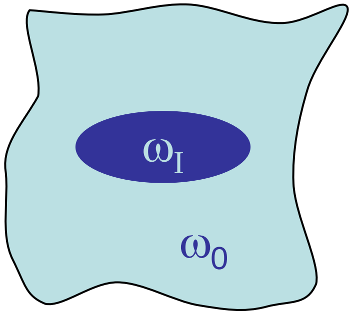


# Multi-scale modelling

- Mean-Field-Homogenization
  - Macro-scale
    - FE model
    - At one integration point $\bar{\boldsymbol{\varepsilon}}$ is know, $\bar{\boldsymbol{\sigma}}$ is sought

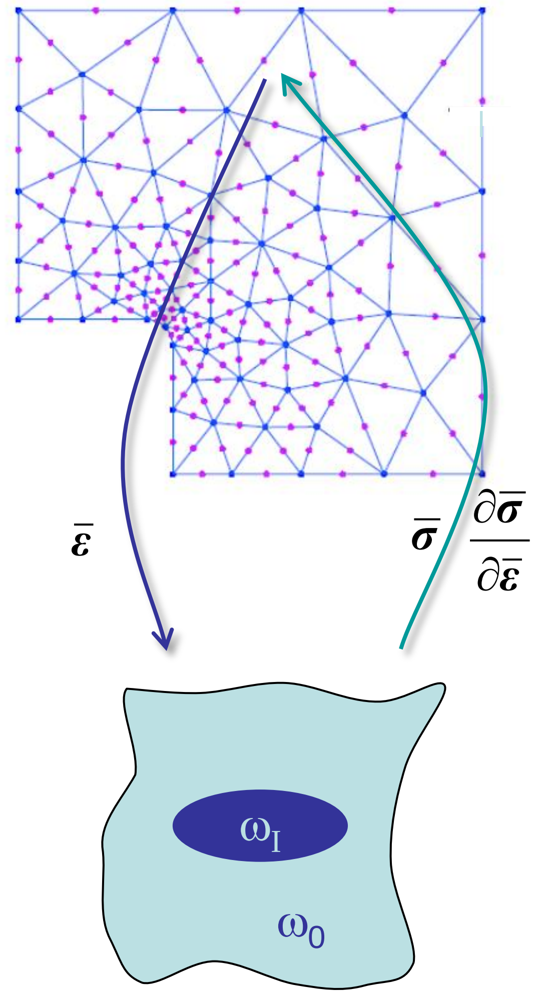



  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models



# Multi-scale modelling

- Mean-Field-Homogenization
  - Macro-scale
    - FE model
    - At one integration point $\bar{\varepsilon}$ is know, $\bar{\sigma}$ is sought
  - Transition
    - Downscaling: $\bar{\varepsilon}$ is used as input of the MFH model
    - Upscaling: $\bar{\sigma}$ is the output of the MFH model
  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models




Mori and Tanaka 73, Hill 65, Ponte Castañeda 91, Suquet 95, Doghri et al 03, Lahellec et al. 11, Brassart et al. 12, …

- Key principles
  - Based on the averaging of the fields

$$\langle a \rangle = \frac{1}{V}\int_V a(\boldsymbol{X})\mathrm{d}V$$

  - Meso-response
    - From the volume ratios ( $v_0 + v_\mathrm{I} = 1$ )

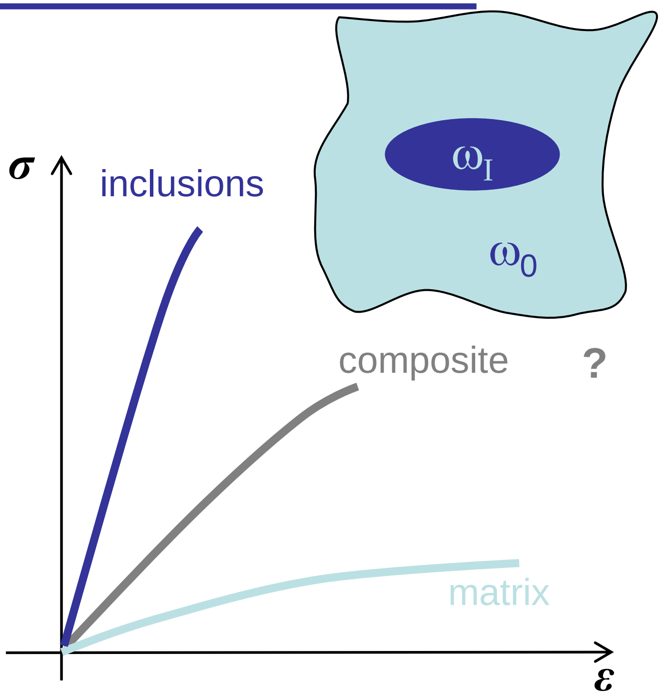
$$\begin{cases} \bar{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma} \rangle = v_0 \langle \boldsymbol{\sigma} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\sigma} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = \langle \boldsymbol{\varepsilon} \rangle = v_0 \langle \boldsymbol{\varepsilon} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\varepsilon} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$

    - One more equation required

$$\boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0$$

  - Difficulty: find the adequate relations

$$\begin{cases} \boldsymbol{\sigma}_\mathrm{I} = f(\boldsymbol{\varepsilon}_\mathrm{I}) \\ \boldsymbol{\sigma}_0 = f(\boldsymbol{\varepsilon}_0) \\ \boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0 \end{cases} \qquad \boldsymbol{B}^{\varepsilon} \ ?$$



- Key principles (2)
  - Linear materials
    - Materials behaviours

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I}} = \overline{\boldsymbol{C}}_{\mathrm{I}} : \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\sigma}_0 = \overline{\boldsymbol{C}}_0 : \boldsymbol{\varepsilon}_0 \end{cases}$$

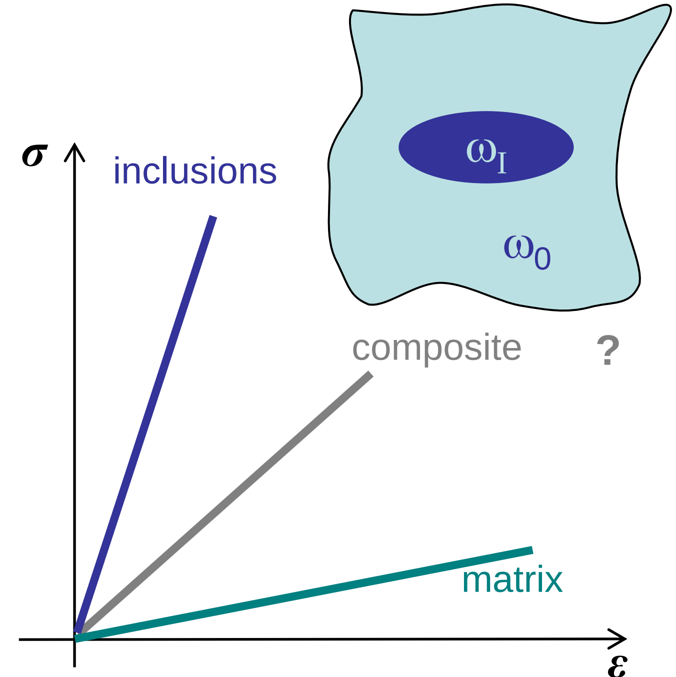
    - Mori-Tanaka assumption $\boldsymbol{\varepsilon}^{\infty} = \boldsymbol{\varepsilon}_0$
    - Use Eshelby tensor

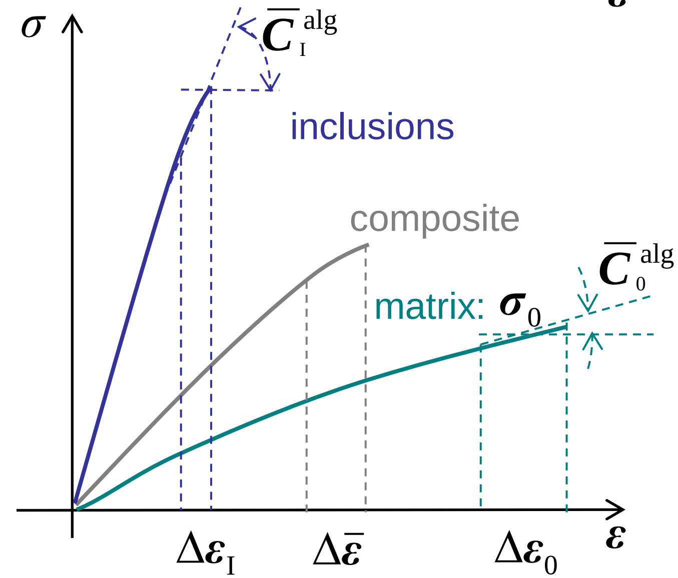
$$\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0, \overline{\boldsymbol{C}}_{\mathrm{I}}\right) : \boldsymbol{\varepsilon}_0$$

with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$

  - Non-linear materials
    - Define a Linear Comparison Composite
    - Common approach: incremental tangent

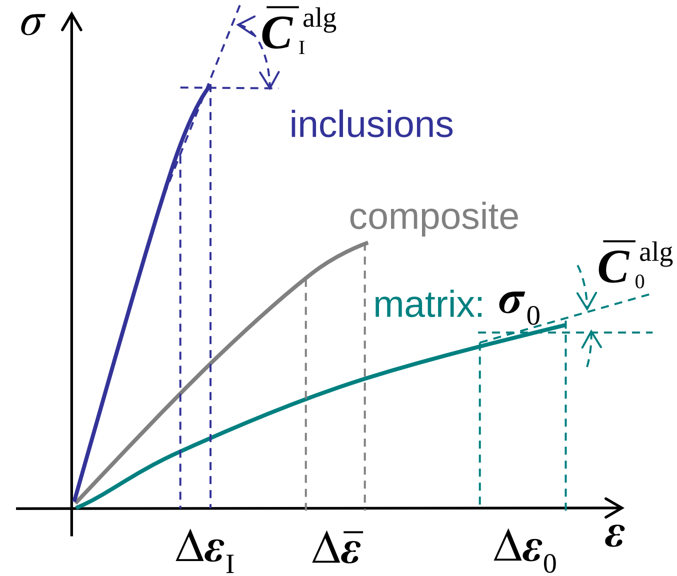
$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_0$$



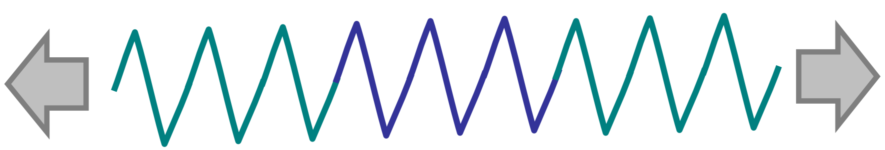



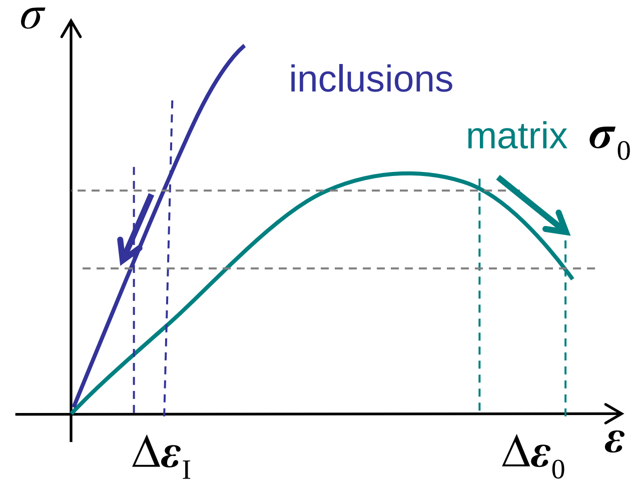
- Problem for materials with strain softening
  - Strain increments in the same direction

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_{0}^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_{0}$$




  - Because of the damaging process, the fiber phase is elastically unloaded during matrix softening






- Solution: new incremental-secant method
  - We need to define the LCC from another stress state

# Content

- Incremental-secant mean-field-homogenization
  - With first statistical moment estimations
  - With second statistical moment estimations

- Damage-enhanced MFH
  - Implicit non-local method
  - Damage-enhanced incremental-secant MFH

- Multi-scale method for the failure analysis of composite laminates
  - Intra-laminar failure: Non-local damage-enhanced mean-field-homogenization
  - Inter-laminar failure: Hybrid DG/cohesive zone model
  - Experimental validation

- Conclusions

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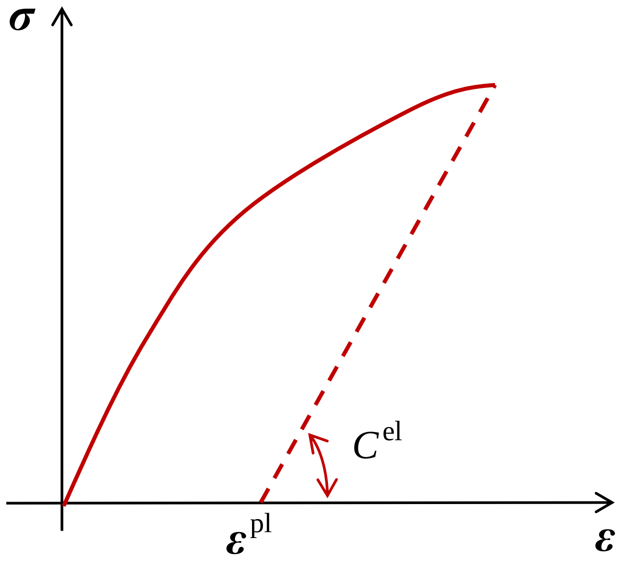
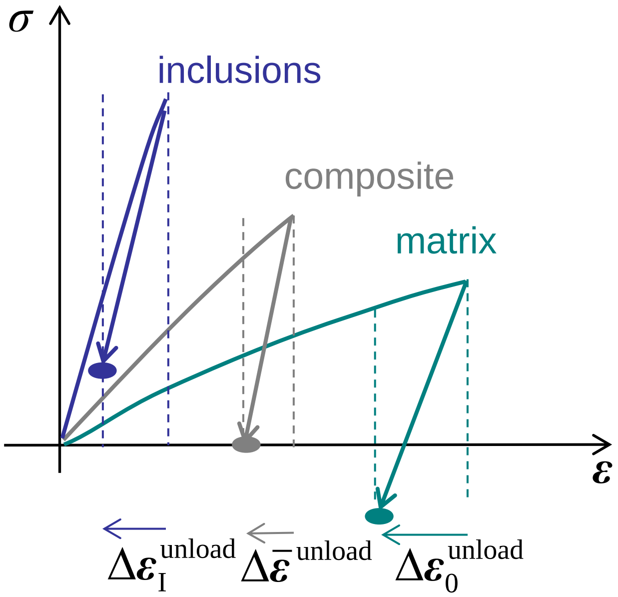
# Incremental-secant mean-field-homogenization

- Material model
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$



# Incremental-secant mean-field-homogenization

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components



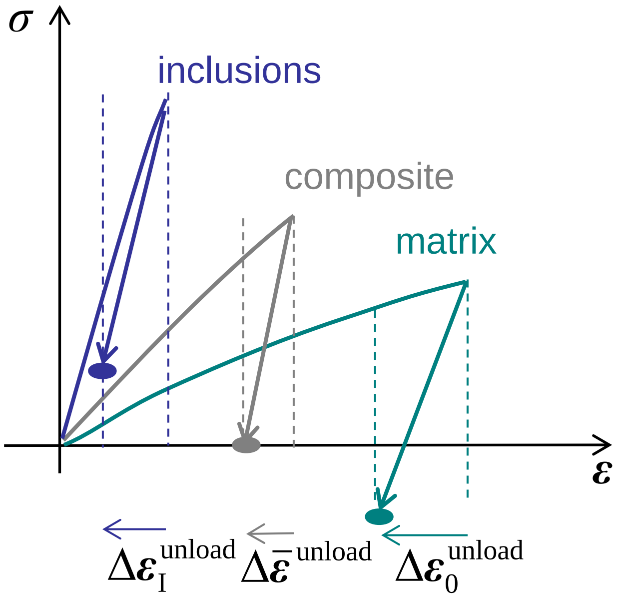
# Incremental-secant mean-field-homogenization

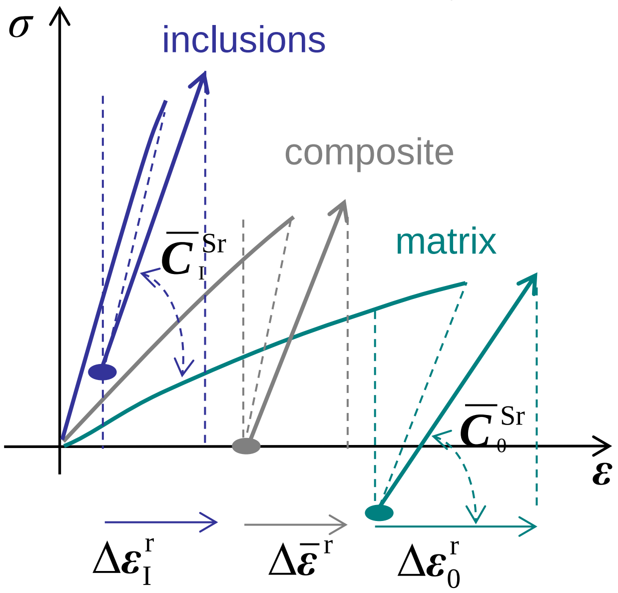
- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components
  - Apply MFH from unloaded state
    - New strain increments (>0)

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}/0} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}/0} + \Delta\boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I}/0}$$

    - Use of secant operators

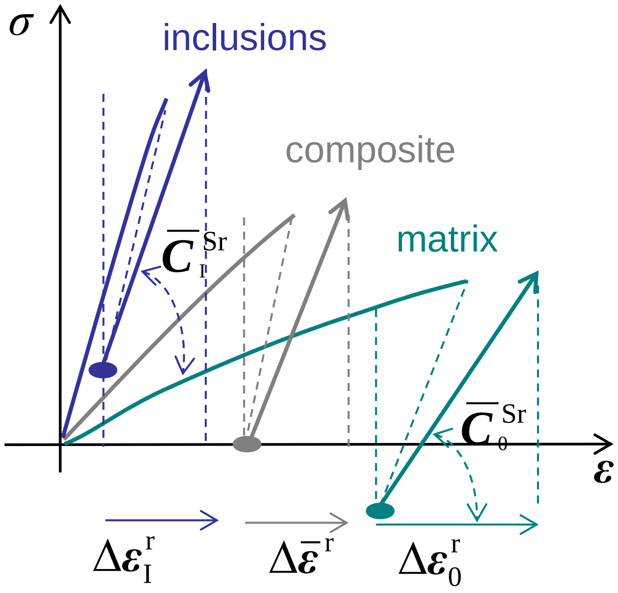
$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{0}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{0}$$



# Incremental-secant mean-field-homogenization

- New incremental-secant approach (2)
  - Equations summary
    - Inputs
      - Internal variables at last increment
      - Residual tensor after virtual unloading
      - $\Delta\bar{\boldsymbol{\varepsilon}}$ from FE resolution




    - Solve iteratively the system

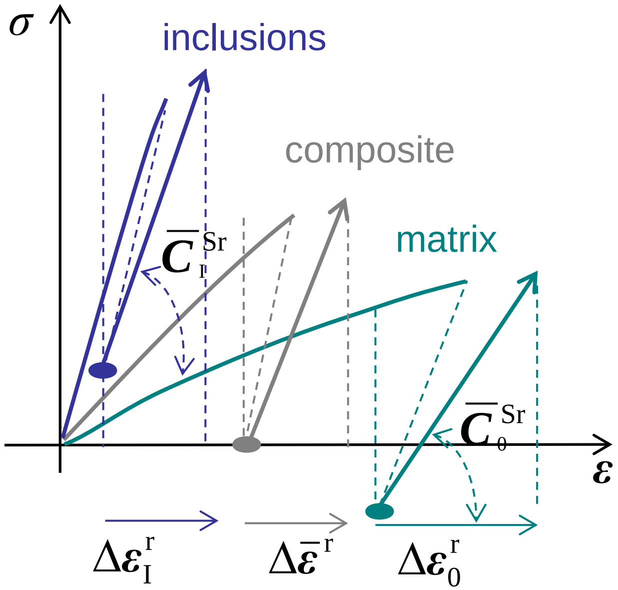
$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_\mathrm{I} \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_\mathrm{I} + \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \bar{\boldsymbol{C}}_0^{\mathrm{Sr}}, \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

    - With the stress tensors

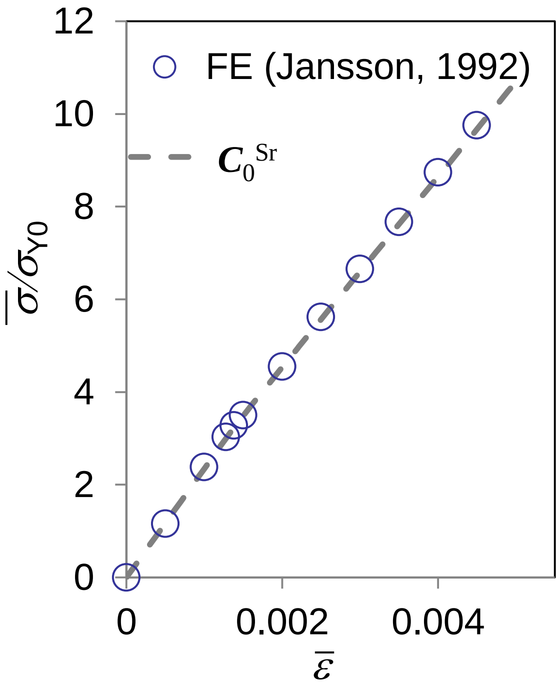
$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \boldsymbol{\sigma}_\mathrm{I} = \boldsymbol{\sigma}_\mathrm{I}^{\mathrm{res}} + \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} \\ \boldsymbol{\sigma}_0 = \boldsymbol{\sigma}_0^{\mathrm{res}} + \bar{\boldsymbol{C}}_0^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

# Incremental-secant mean-field-homogenization

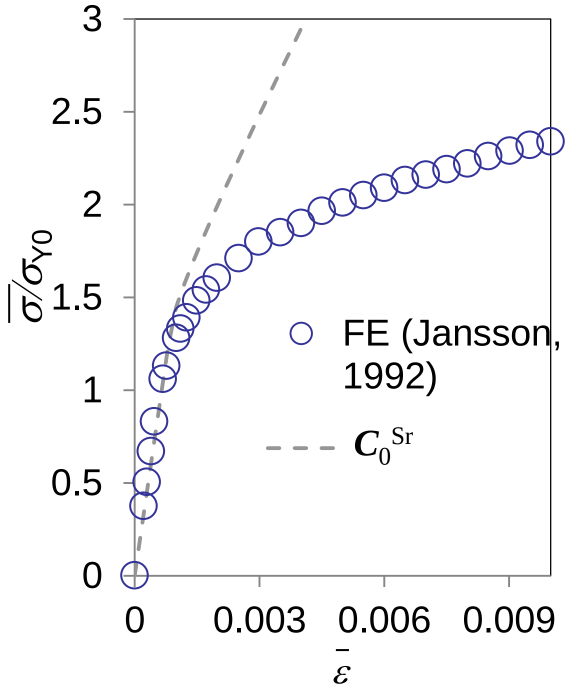
- Zero-incremental-secant method
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - For inclusions with high hardening (elastic)
    - Model is too stiff




## Longitudinal tension



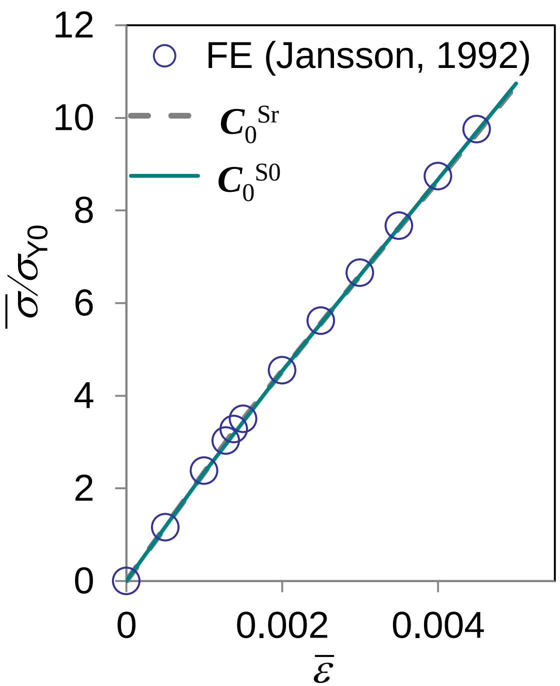

## Transverse loading



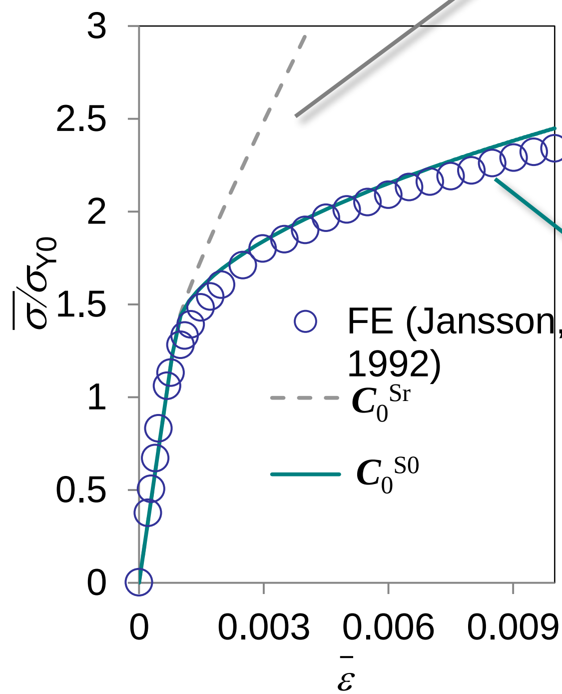
# Incremental-secant mean-field-homogenization

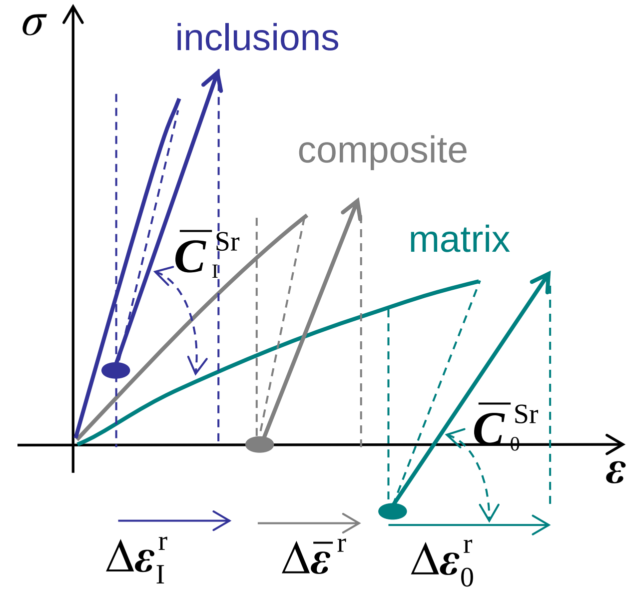
- Zero-incremental-secant method (2)
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - Secant model in the matrix
    - Modified if negative residual stress

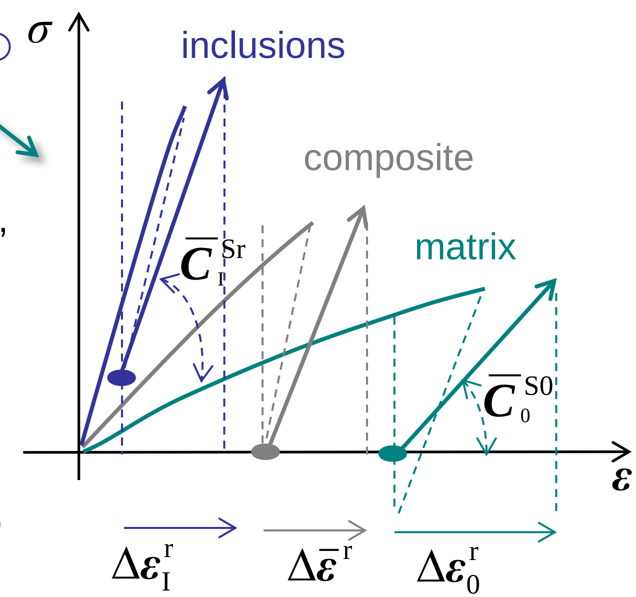
Longitudinal tension




Transverse loading



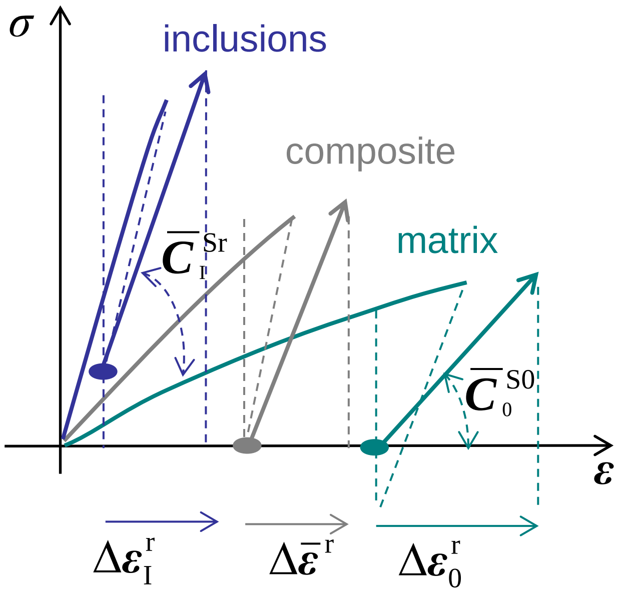
# Incremental-secant mean-field-homogenization

- Zero-incremental-secant method (3)
  - For soft matrix response
    - Remove residual stress in matrix
    - Avoid adding spurious internal energy
    - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_{\mathrm{I}} \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} + \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \bar{\boldsymbol{C}}_0^{\mathrm{S0}}, \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

    - With the stress tensors

$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{\mathrm{I}} = \boldsymbol{\sigma}_{\mathrm{I}}^{\mathrm{res}} + \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} \\ \boldsymbol{\sigma}_0 = \bar{\boldsymbol{C}}_0^{\mathrm{S0}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

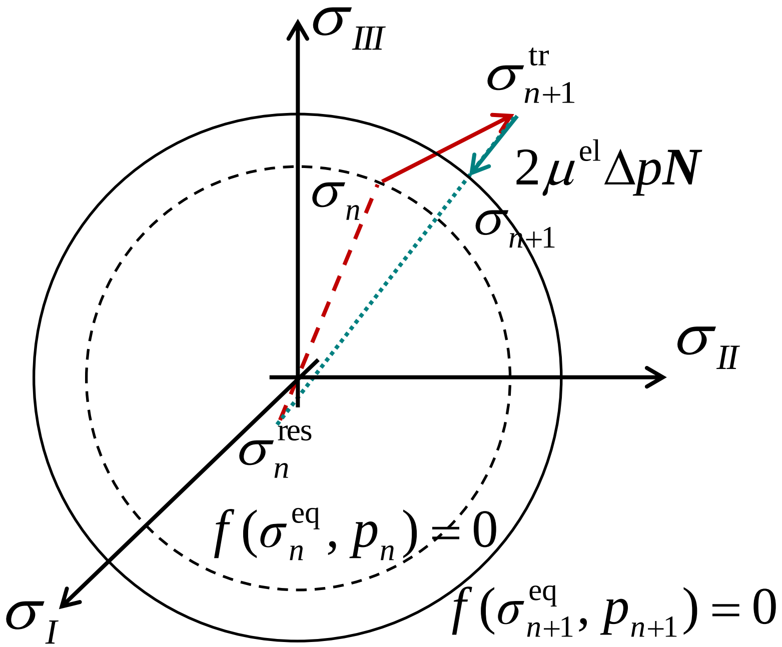


# Incremental-secant mean-field-homogenization

- The secant operators
  - Stress tensor (2 forms)

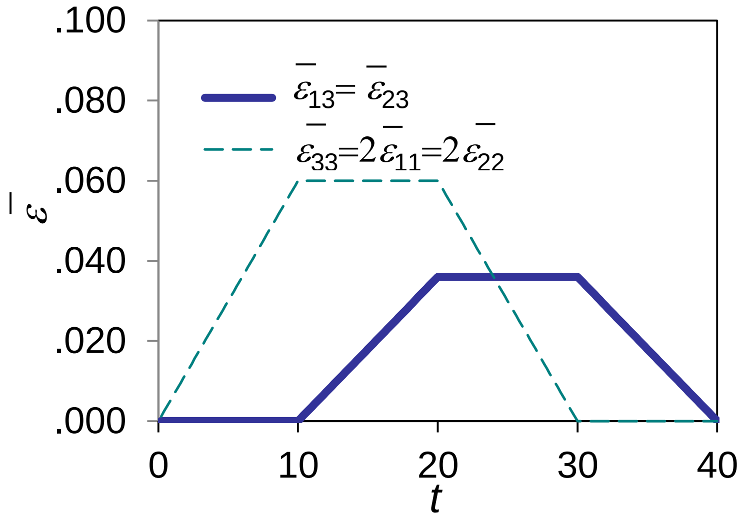
$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I/0}} = \boldsymbol{\sigma}^{\mathrm{res}}_{\mathrm{I/0}} + \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I/0}} : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I/0}} \\ \boldsymbol{\sigma}_{\mathrm{I/0}} = \overline{\boldsymbol{C}}^{\mathrm{S0}}_{\mathrm{I/0}} : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I/0}} \end{cases}$$

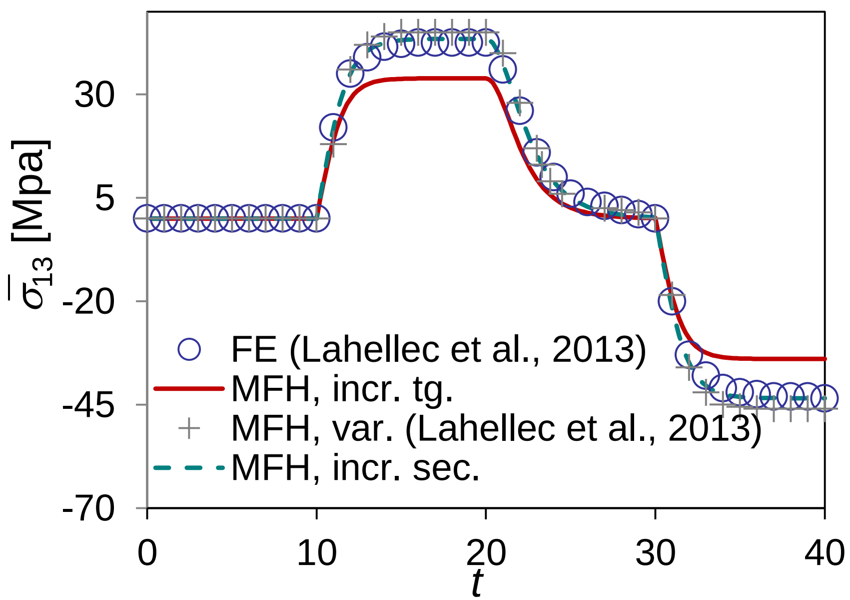
  - Radial return direction toward residual stress
    - First order approximation in the strain increment (and not in the total strain)
    - Exact for the zero-incremental-secant method
  - The secant operators are naturally isotropic

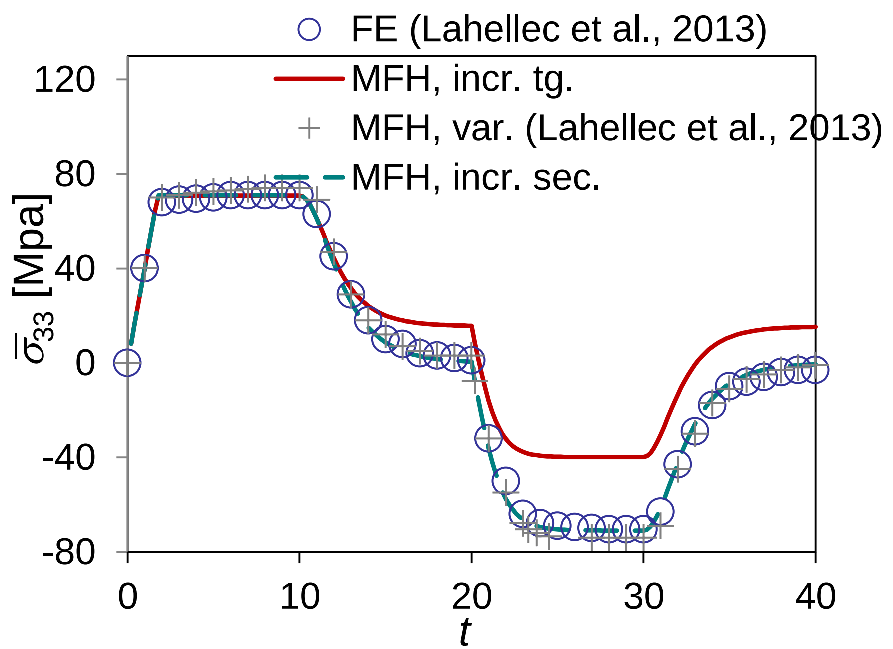



$$\begin{cases} \overline{\boldsymbol{C}}^{\mathrm{Sr}} = 3\kappa^{\mathrm{el}}\boldsymbol{I}^{\mathrm{vol}} + 2\left(\mu^{\mathrm{el}} - 3\dfrac{\mu^{\mathrm{el}^2}\Delta p}{\left(\boldsymbol{\sigma}_{n+1} - \boldsymbol{\sigma}^{\mathrm{res}}_n\right)^{\mathrm{eq}}}\right)\boldsymbol{I}^{\mathrm{dev}} \\ \overline{\boldsymbol{C}}^{\mathrm{S0}} = 3\kappa^{\mathrm{el}}\boldsymbol{I}^{\mathrm{vol}} + 2\left(\mu^{\mathrm{el}} - 3\dfrac{\mu^{\mathrm{el}^2}\Delta p}{\boldsymbol{\sigma}^{\mathrm{eq}}_{n+1}}\right)\boldsymbol{I}^{\mathrm{dev}} \end{cases}$$

- Verification of the method
  - Spherical inclusions
    - 17 % volume fraction
    - Elastic
  - Elastic-perfectly-plastic matrix
  - Non-proportional loading

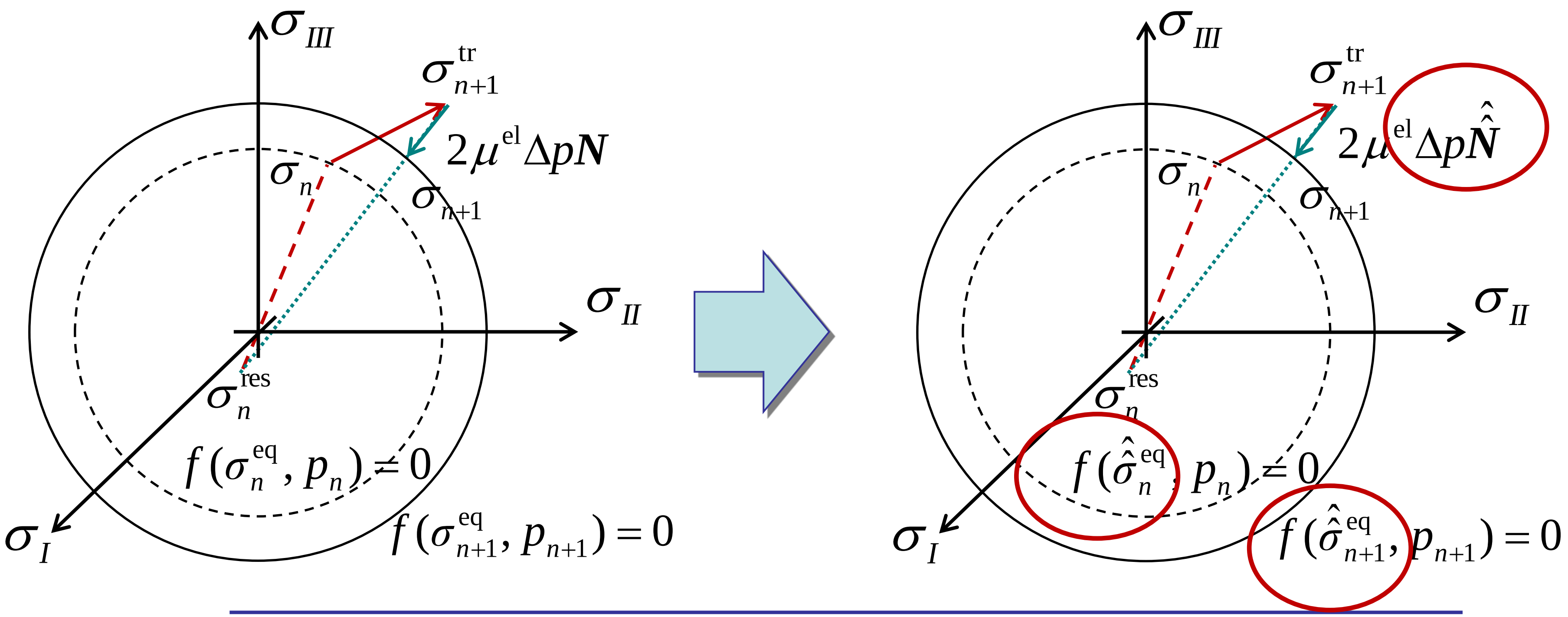


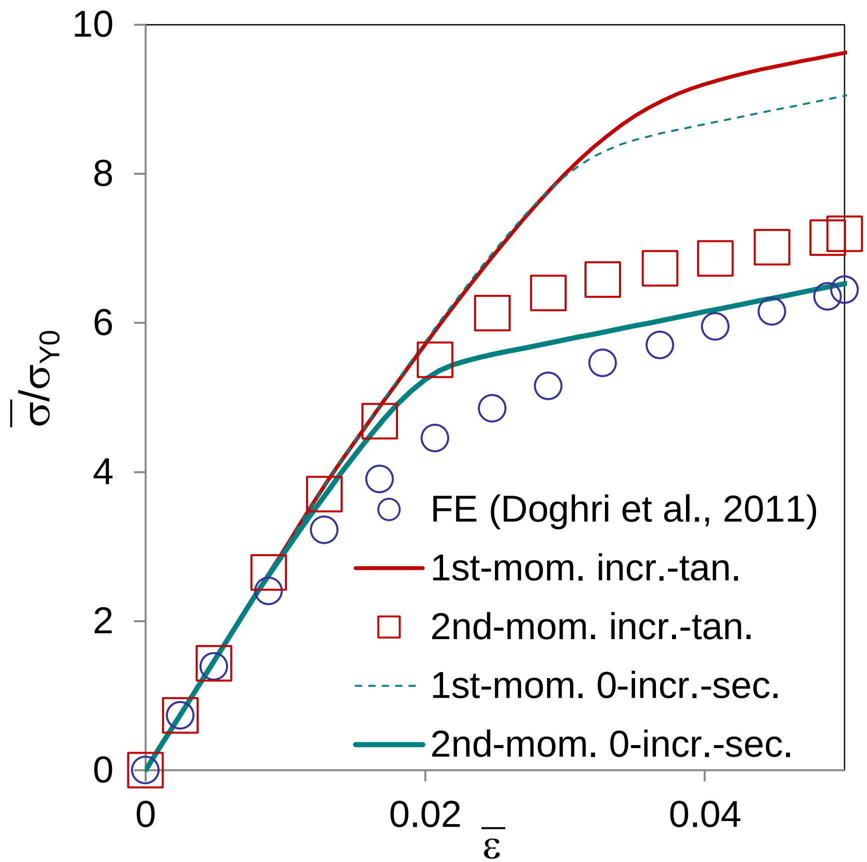



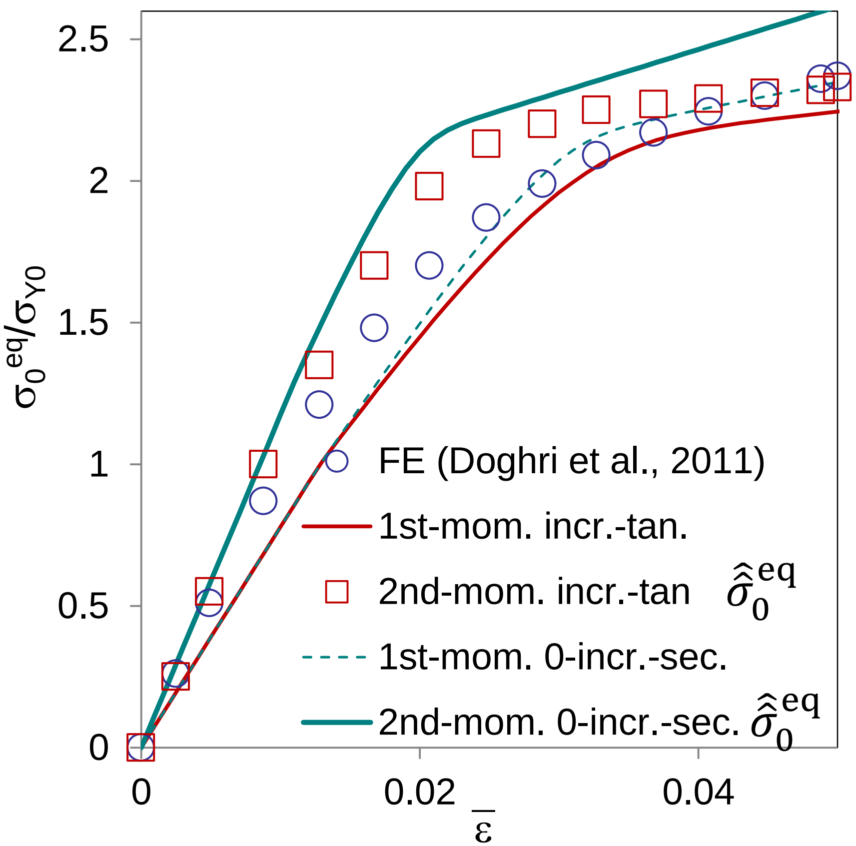
- Second-statistical moment estimation of the von Mises stress
  - J2-plasticity involves quadratic terms
    - First statistical moment (mean value) not fully representative
    - Use second statistical moment estimations to define the yield surface

$$\Delta\hat{\hat{\sigma}}^{\mathrm{r}\ \mathrm{eq}}_{\mathrm{I/0}} = \sqrt{\frac{3}{2}\boldsymbol{I}^{\mathrm{dev}} :: \left\langle \Delta\boldsymbol{\sigma}^{\mathrm{r}}_{\mathrm{I/0}} \otimes \Delta\boldsymbol{\sigma}^{\mathrm{r}}_{\mathrm{I/0}} \right\rangle_{\omega_{\mathrm{I/0}}}}$$



- Short glass fibre reinforced polyamide
  - Short fibres
    - Aspect ratio of 15
    - 17.5 % volume fraction
    - Elastic
  - Elastic-plastic matrix





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  - **Implicit non-local method**
  - **Damage-enhanced incremental-secant MFH**
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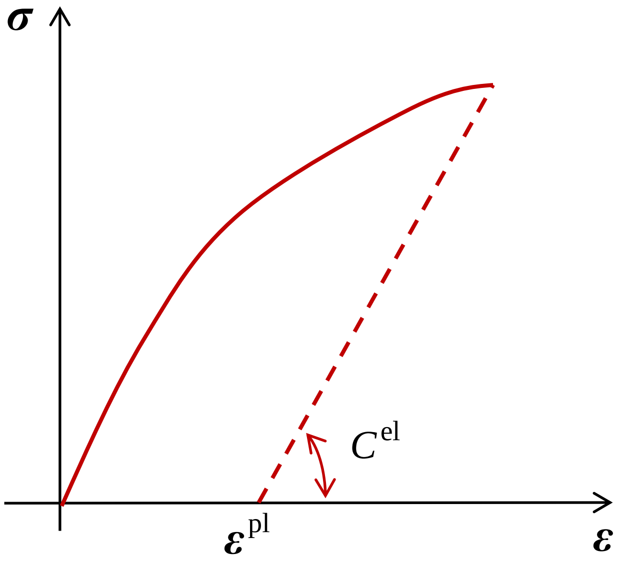
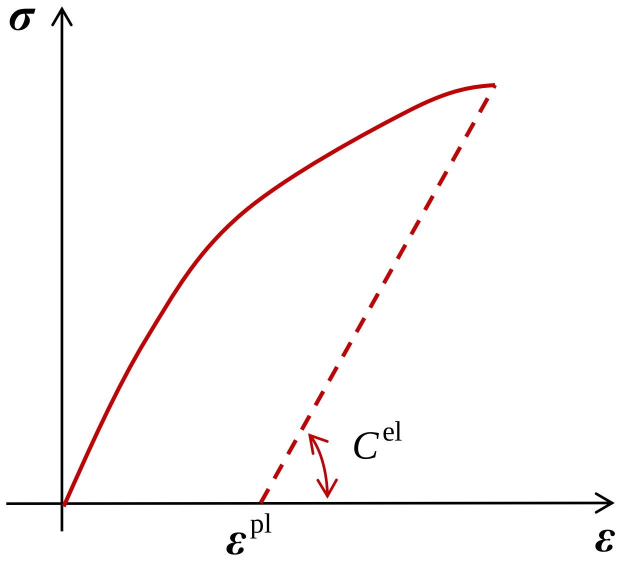
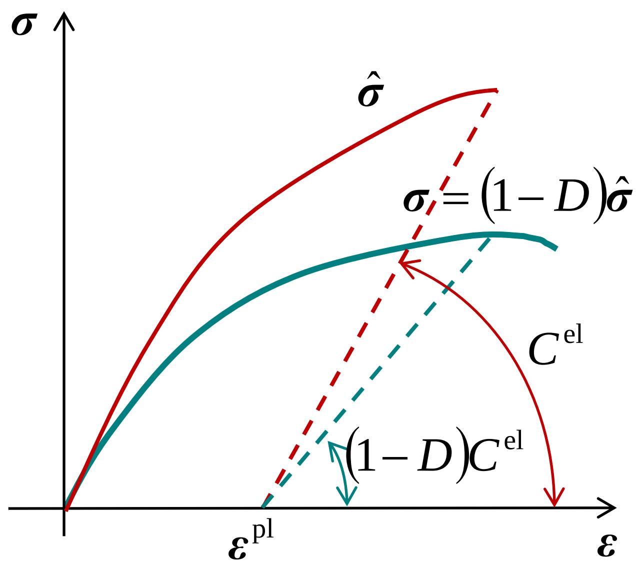
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$



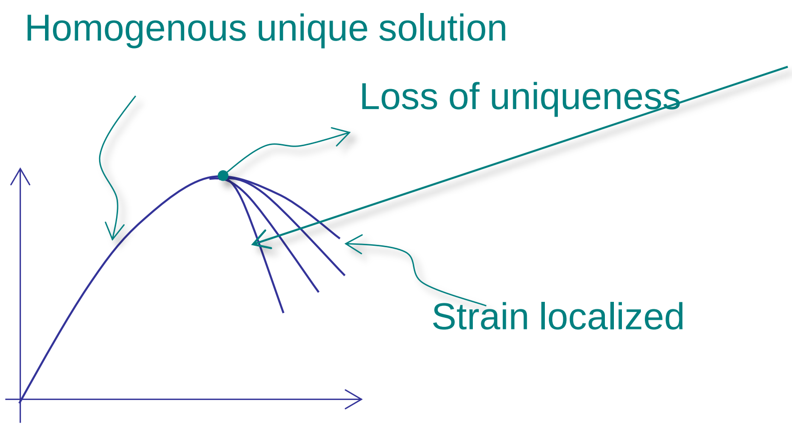
- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$



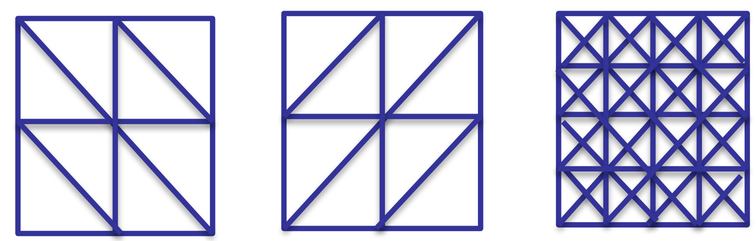



# Non-local damage-enhanced MFH

- Finite element solutions with strain softening suffer from:
  - The loss of the solution uniqueness and strain localization
  - Mesh dependency




The numerical results change with the size of mesh and direction of mesh



The numerical results change without convergence

- Solution: Implicit non-local approach [Peerlings et al 96, Geers et al 97, …]
  - A state variable is replaced by a non-local value reflecting the interaction between neighboring material points

$$\tilde{a}(\boldsymbol{x}) = \frac{1}{V_C} \int_{V_C} a(\boldsymbol{y}) w(\boldsymbol{y}; \boldsymbol{x}) \mathrm{d}V$$

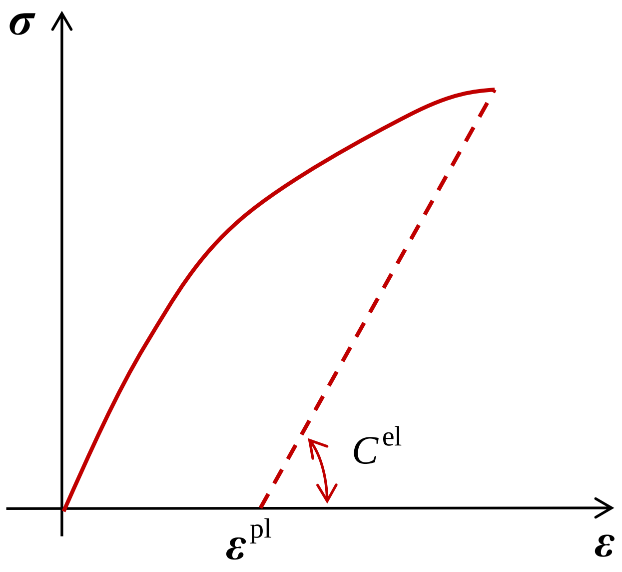
  - Use Green functions as weight *w(y; x)*

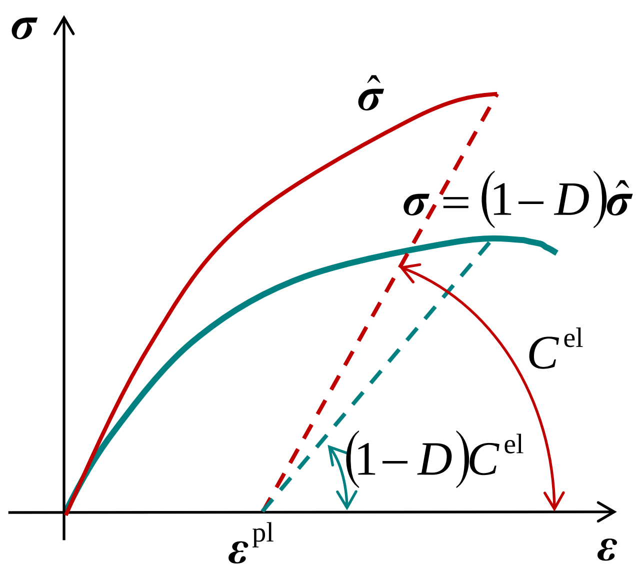
$$\tilde{a} - c\nabla^2 \tilde{a} = a$$ → New degrees of freedom

# Non-local damage-enhanced MFH

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1-D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$
  - Non-Local damage model
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta\tilde{p})$
    - Anisotropic governing equation $\tilde{p} - \nabla \cdot (\boldsymbol{c}_{\text{g}} \cdot \nabla\tilde{p}) = p$
    - Linearization

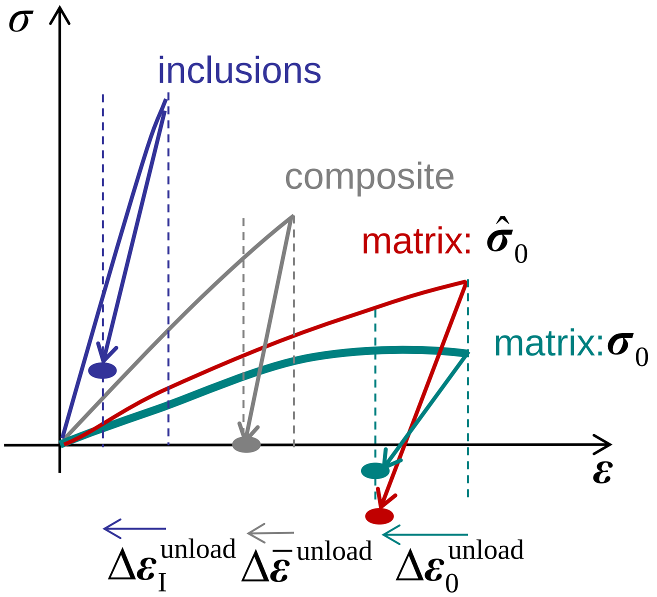
$$\delta\boldsymbol{\sigma} = \left[(1-D)\boldsymbol{C}^{\text{alg}} - \hat{\boldsymbol{\sigma}} \otimes \frac{\partial F_D}{\partial \boldsymbol{\varepsilon}}\right] : \delta\boldsymbol{\varepsilon} - \hat{\boldsymbol{\sigma}} \frac{\partial F_D}{\partial \tilde{p}} \delta\tilde{p}$$



# Non-local damage-enhanced MFH

- Based on the incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components



# Non-local damage-enhanced MFH

- Based on the incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components
    - Apply MFH from unloaded state
      - New strain increments (>0)

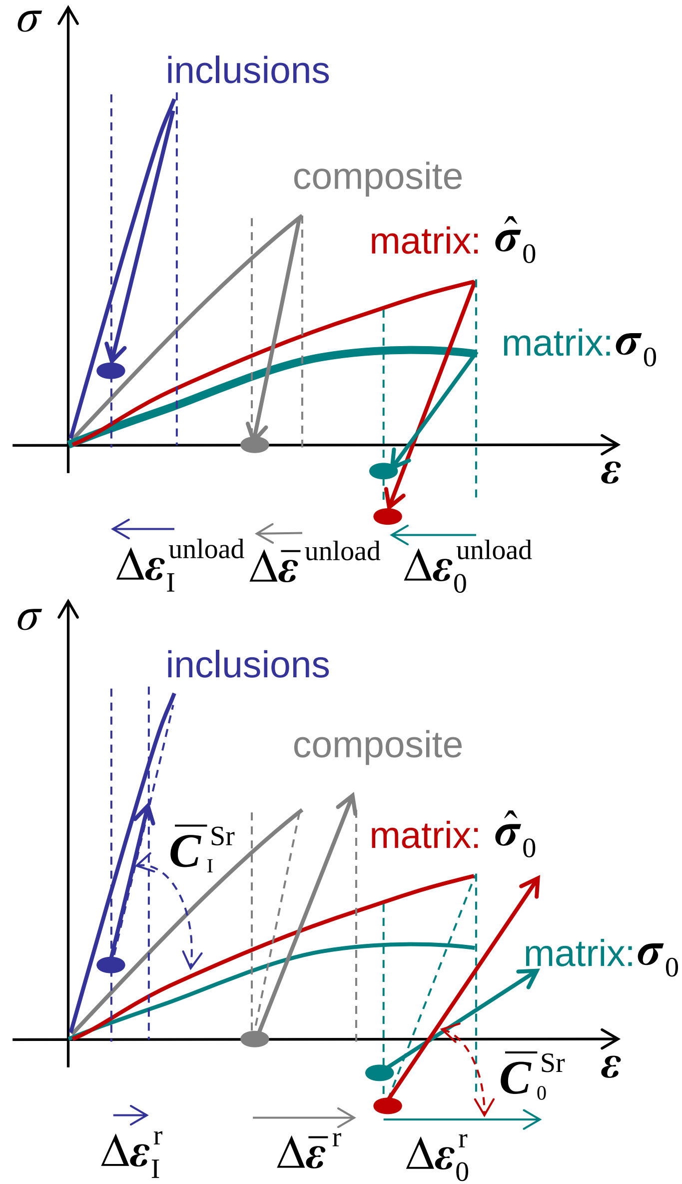
$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}/0} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}/0} + \Delta\boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I}/0}$$

      - Use of secant operators

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\overline{\boldsymbol{C}}^{\mathrm{Sr}}_{0}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{0}$$

      - Possibility of unloading

$$\begin{cases} \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} > 0 \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} < 0 \end{cases}$$



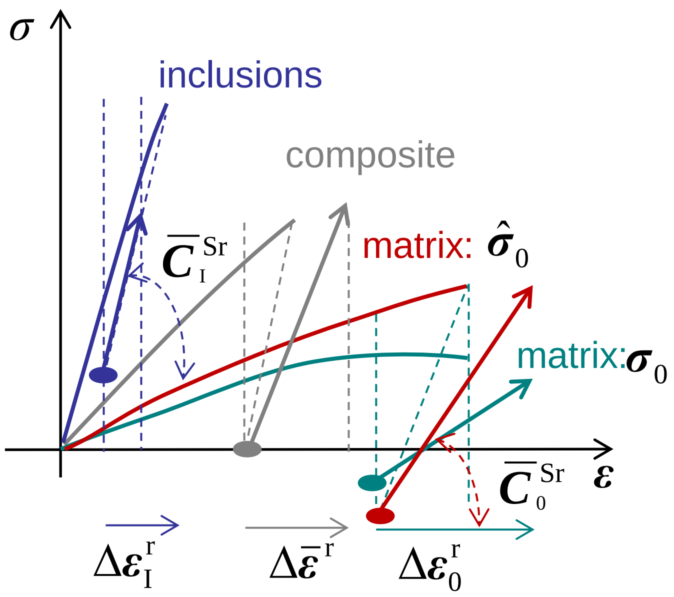
# Non-local damage-enhanced MFH

- Equations summary: residual-approach
  - Inputs
    - Internal variables at last increment
    - Residual tensor after virtual unloading
    - $\Delta\bar{\boldsymbol{\varepsilon}}, \Delta\tilde{p}$ from FE resolution
  - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(r)} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(r)} + v_I \Delta\boldsymbol{\varepsilon}_I^{(r)} \\ \Delta\boldsymbol{\varepsilon}_I^{r} = \Delta\boldsymbol{\varepsilon}_I + \Delta\boldsymbol{\varepsilon}_I^{unload} \\ \Delta\boldsymbol{\varepsilon}_0^{r} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{unload} \\ \Delta\boldsymbol{\varepsilon}_I^{r} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\bar{\boldsymbol{C}}_0^{Sr}, \bar{\boldsymbol{C}}_I^{Sr}\right) : \Delta\boldsymbol{\varepsilon}_0^{r} \end{cases}$$

  - With the stress tensors

$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_I \boldsymbol{\sigma}_I \\ \boldsymbol{\sigma}_I = \boldsymbol{\sigma}_I^{res} + \bar{\boldsymbol{C}}_I^{Sr} : \Delta\boldsymbol{\varepsilon}_I^{r} \\ \boldsymbol{\sigma}_0 = (1-D)\hat{\boldsymbol{\sigma}}_0^{res} + (1-D)\bar{\boldsymbol{C}}_0^{Sr} : \Delta\boldsymbol{\varepsilon}_0^{r} \end{cases}$$



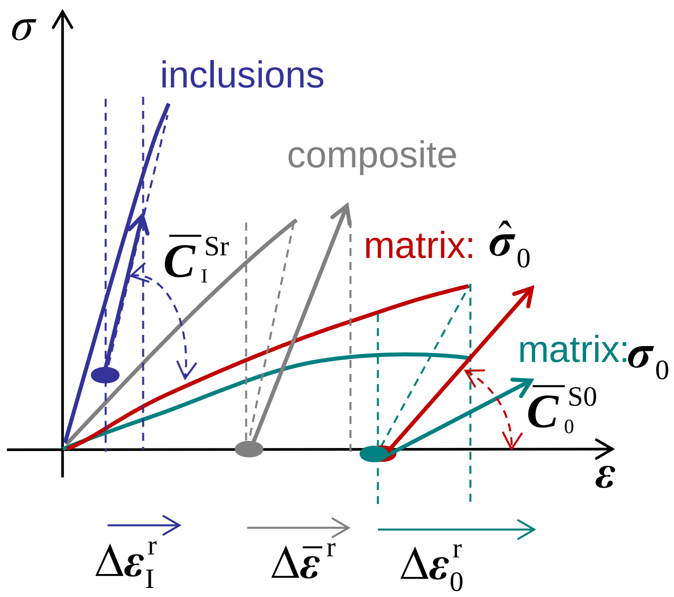
# Non-local damage-enhanced MFH

- Equations summary: zero-approach
  - For soft matrix response
    - Remove residual stress in matrix
    - Avoid adding spurious internal energy
  - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_{\mathrm{I}} \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} + \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{S0}}, \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

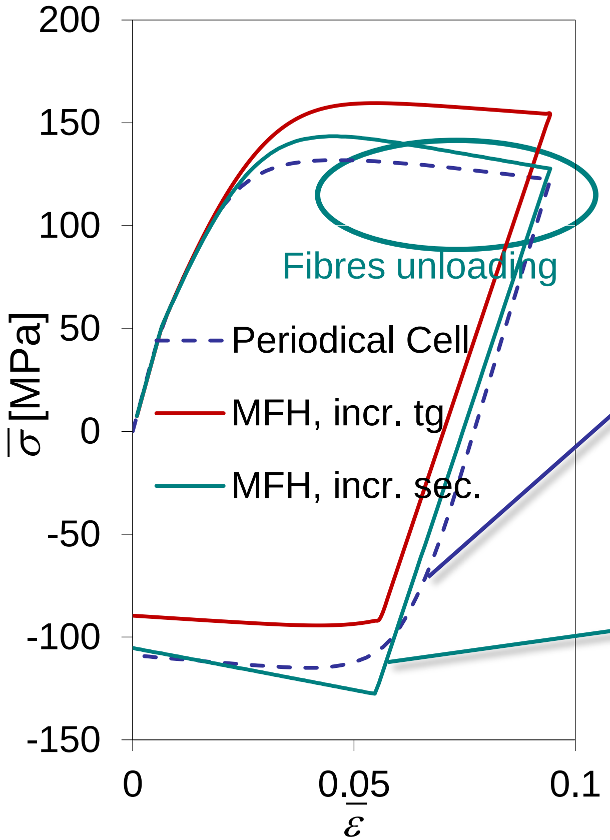
  - With the stress tensors

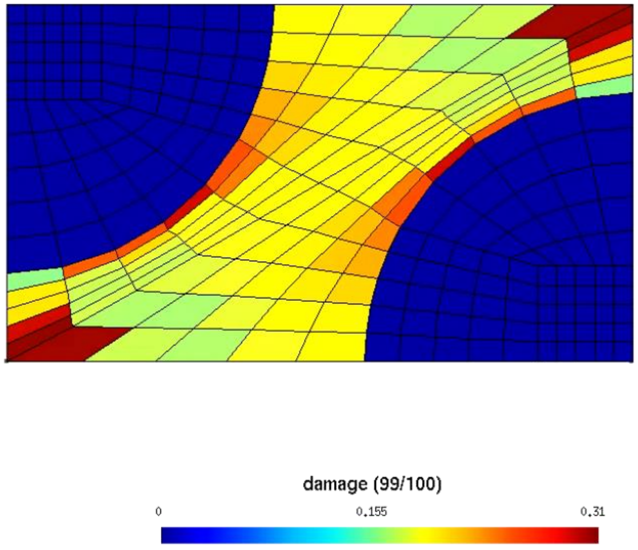
$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{\mathrm{I}} = \boldsymbol{\sigma}_{\mathrm{I}}^{\mathrm{res}} + \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} \\ \boldsymbol{\sigma}_0 = (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{S0}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

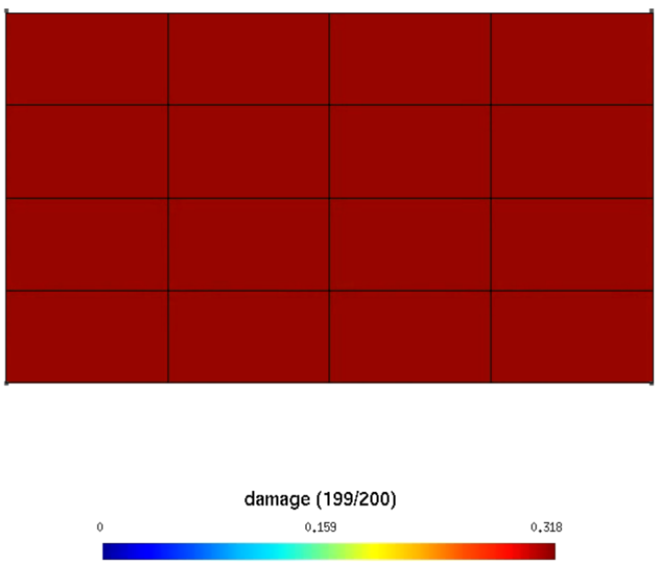


# Non-local damage-enhanced MFH

- New results for damage
  - Fictitious composite
    - 50%-UD fibres
  - Elasto-plastic matrix with damage

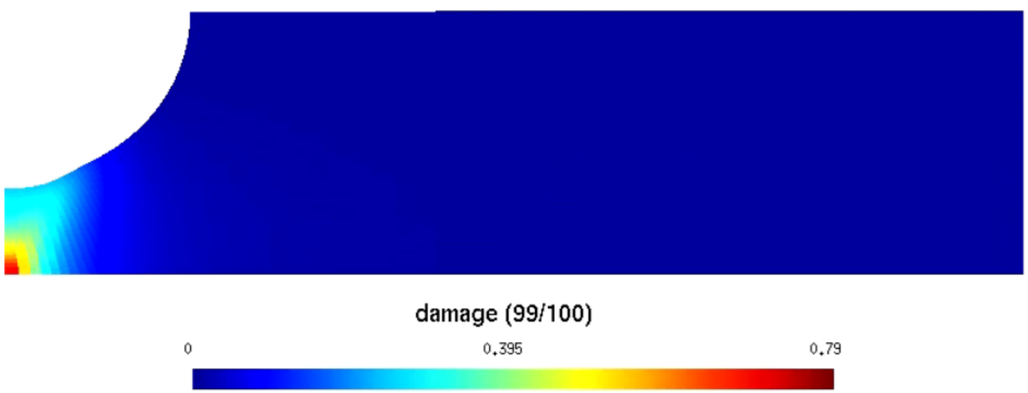


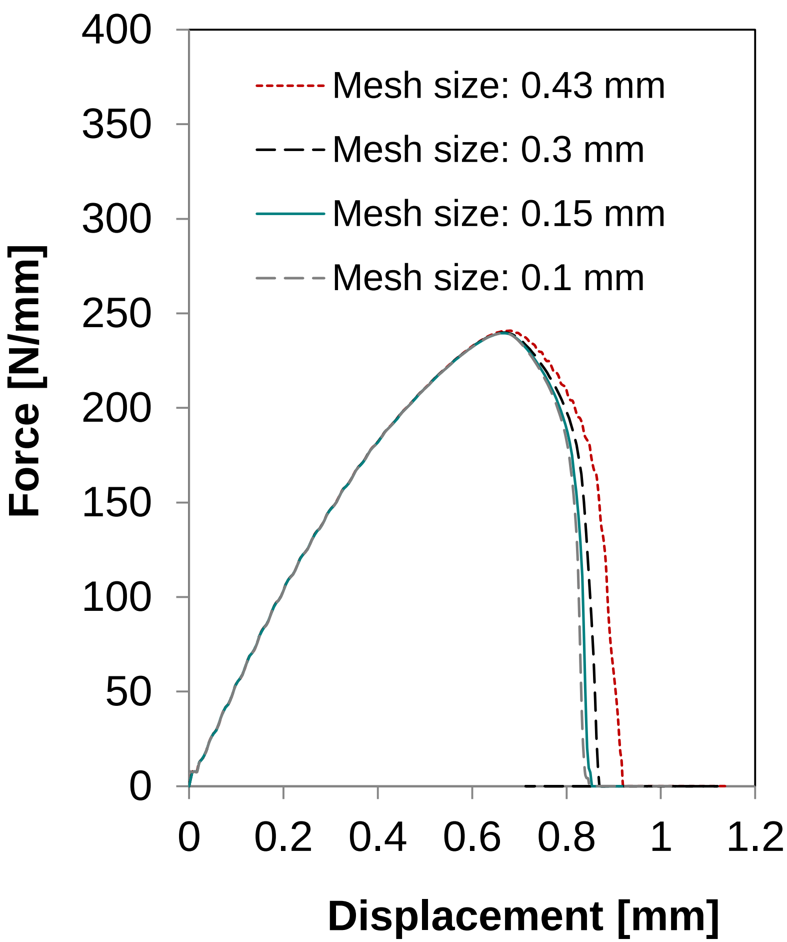



# Non-local damage-enhanced MFH

- Mesh-size effect
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage
  - Notched ply



# Content

- Incremental-secant mean-field-homogenization
  - With first statistical moment estimations
  - With second statistical moment estimations
- Damage-enhanced MFH
  - Implicit non-local method
  - Damage-enhanced incremental-secant MFH
- **Multi-scale method for the failure analysis of composite laminates**
  - **Intra-laminar failure: Non-local damage-enhanced mean-field-homogenization**
  - **Inter-laminar failure: Hybrid DG/cohesive zone model**
  - **Experimental validation**
- Conclusions

- Weak formulation of a composite laminate
  - Strong form

$$\begin{cases} \nabla \cdot \bar{\boldsymbol{\sigma}}^T + \boldsymbol{f} = \boldsymbol{0} & \text{for each homogenized ply } \Omega_i \\ \tilde{p} - \nabla \cdot \left( \boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p} \right) = p & \text{for the matrix phase} \end{cases}$$

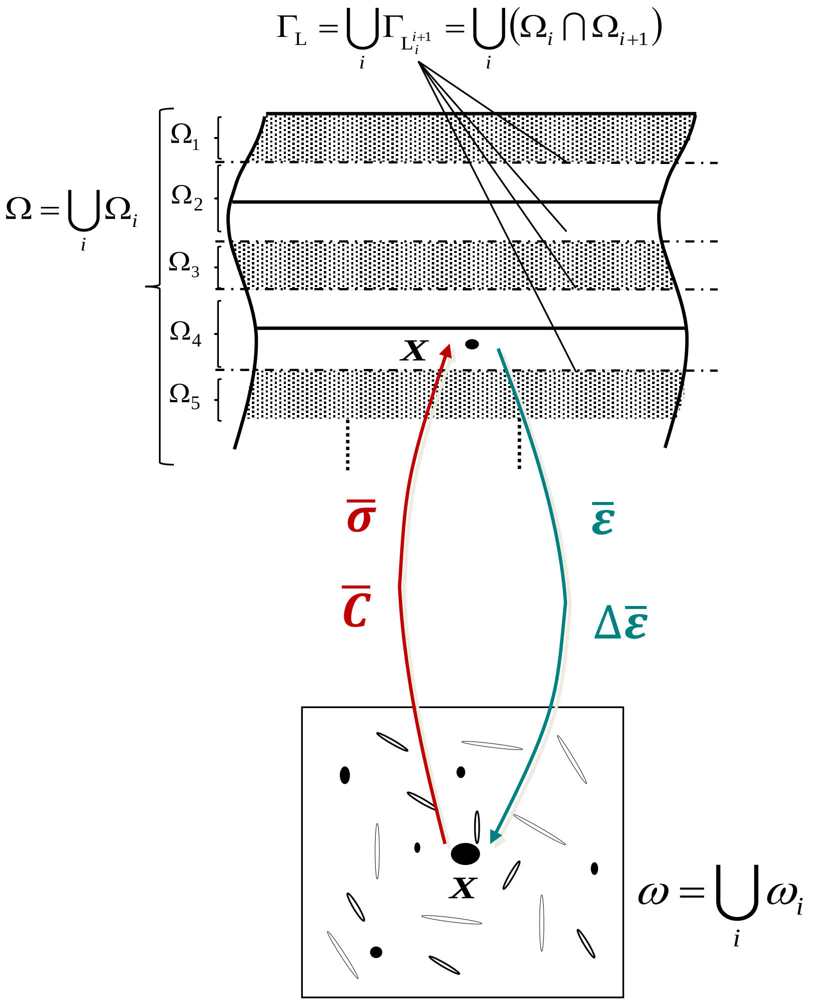
  - Boundary conditions

$$\begin{cases} \bar{\boldsymbol{\sigma}} \cdot \bar{\boldsymbol{n}} = \bar{\boldsymbol{T}} \\ \bar{\boldsymbol{n}} \cdot \left( \boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p} \right) = 0 \end{cases}$$

  - Macro-scale finite-element discretization

$$\begin{cases} \tilde{p} = N^a_{\tilde{p}} \tilde{\mathbf{p}}^a \\ \bar{\boldsymbol{u}} = N^a_u \bar{\boldsymbol{u}}^a \end{cases}$$

$$\Rightarrow \begin{bmatrix} \boldsymbol{K}_{\bar{u}\bar{u}} & \boldsymbol{K}_{\bar{u}\tilde{p}} \\ \boldsymbol{K}_{\tilde{p}\bar{u}} & \boldsymbol{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\bar{\boldsymbol{u}} \\ d\tilde{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{ext}} - \boldsymbol{F}_{\mathrm{int}} \\ \boldsymbol{F}_p - \boldsymbol{F}_{\tilde{p}} \end{bmatrix}$$



- Resolution strategies
  - Fully coupled resolution

$$\begin{bmatrix} \boldsymbol{K}_{\bar{u}\bar{u}} & \boldsymbol{K}_{\bar{u}\tilde{p}} \\ \boldsymbol{K}_{\tilde{p}\bar{u}} & \boldsymbol{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\bar{\mathbf{u}} \\ d\tilde{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{ext}} - \boldsymbol{F}_{\mathrm{int}} \\ \boldsymbol{F}_{p} - \boldsymbol{F}_{\tilde{p}} \end{bmatrix}$$

  - Staggered dynamic resolution
    - Explicit resolution of the displacement dofs

$$\ddot{\bar{\boldsymbol{u}}}^{n+1} = \frac{1}{1-\alpha_M} \boldsymbol{M}\left[\boldsymbol{F}^{n}_{\mathrm{ext}} - \boldsymbol{F}^{n}_{\mathrm{int}}\right] - \frac{\alpha_M}{1-\alpha_M} \ddot{\bar{\boldsymbol{u}}}^{n}$$

$$\dot{\bar{\boldsymbol{u}}}^{n+1} = \dot{\bar{\boldsymbol{u}}}^{n} + \Delta t\left[1-\gamma_M\right]\ddot{\bar{\boldsymbol{u}}}^{n} + \Delta t \gamma_M \ddot{\bar{\boldsymbol{u}}}^{n+1}$$

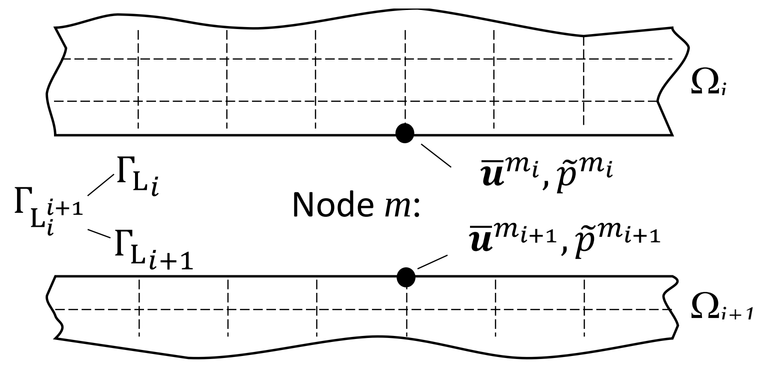
$$\bar{\boldsymbol{u}}^{n+1} = \bar{\boldsymbol{u}}^{n} + \Delta t \dot{\bar{\boldsymbol{u}}}^{n} + \Delta t^2 \left[\frac{1}{2} - \beta_M\right] \ddot{\bar{\boldsymbol{u}}}^{n+1} + \Delta t^2 \beta_M \ddot{\bar{\boldsymbol{u}}}^{n+1}$$

    - Resolution of the non-local equation once every $N$ steps

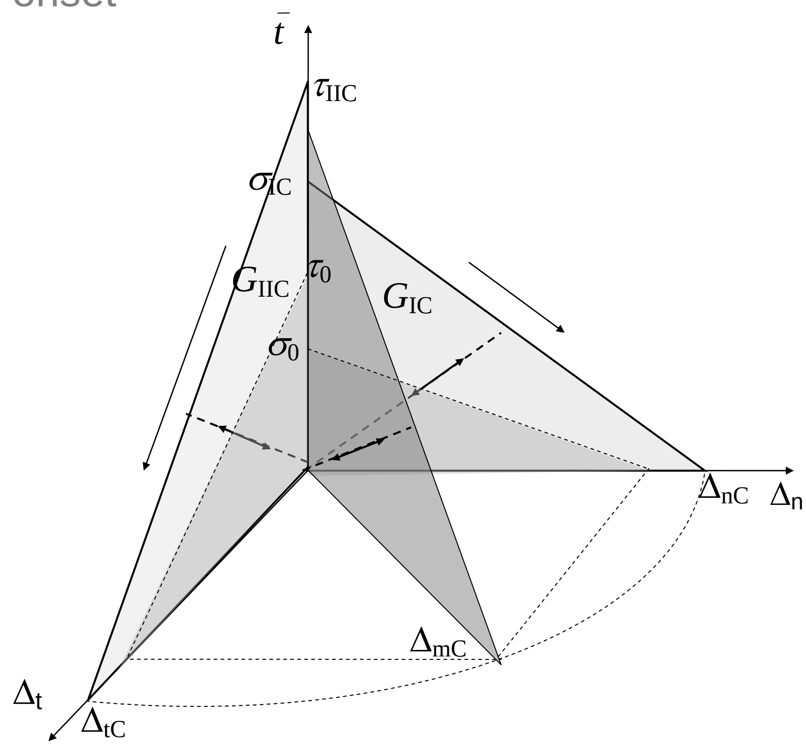
$$\boldsymbol{K}_{\tilde{p}\tilde{p}} d\tilde{\mathbf{p}} = \boldsymbol{F}_{p} - \boldsymbol{F}_{\tilde{p}}$$

# Inter-laminar failure: Hybrid DG/cohesive zone model

- Hybrid DG/Cohesive zone model
  - Interface elements inserted at interface $\Gamma_{\mathrm{L}_i i+1}$ between two plies $i$ & $i+1$
  - Consistent discontinuous Galerkin framework before fracture onset
    - No mesh dependency
    - Easily parallelizable
  - Traction Separation Law (TSL) at fracture onset

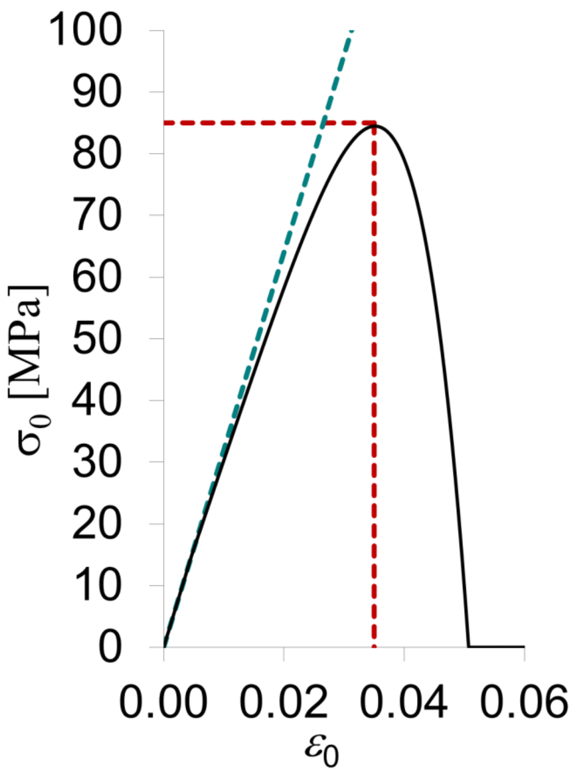



- Definition of an extrinsic TSL
  - Accounting for mixed loading
  - Characterized by
    - Strengths $\sigma_{\mathrm{IC}}$ & $\sigma_{\mathrm{IIC}}$
    - Critical energy release rates $G_{\mathrm{IC}}$ & $G_{\mathrm{IIC}}$

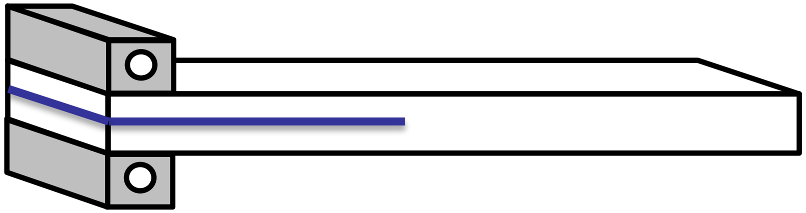


# Experimental validation

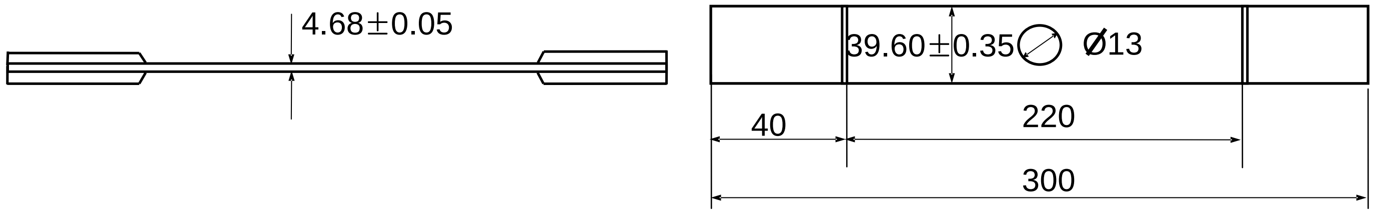
- 60%-UD Carbon-fibres reinforced epoxy
  - Carbon fibres
    - Use of transverse isotropic elastic material
  - Elasto-plastic matrix with damage
    - Use manufacturer Young's modulus
    - Use manufacturer strength



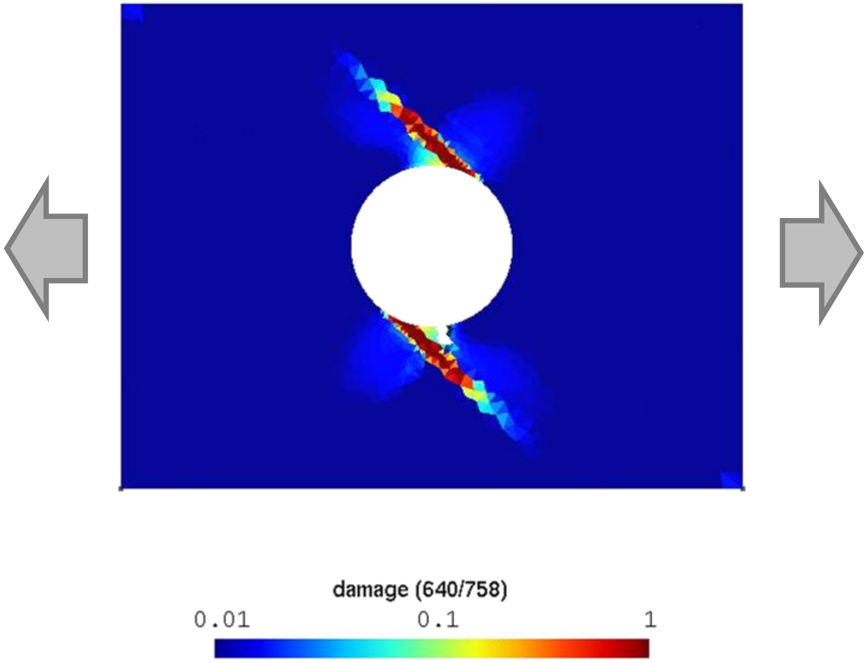

- Delamination: Double Cantilever Beam
  - Critical energy realise rates
    - $G_{IC}$ = 600 J/m$^2$
    - $G_{IIC}$ = 1200 J/m$^2$ (assumption)



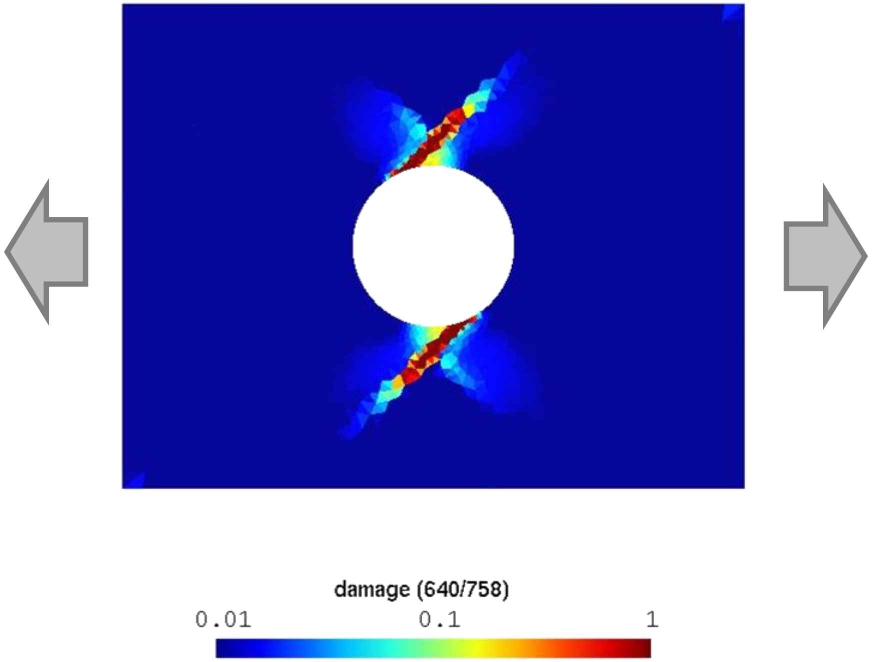
- $[45^o_4 / -45^o_4]_S$- open hole laminate
  - Tensile test on several coupons




  - Propagation of the damaged zones in agreement with the fibre direction



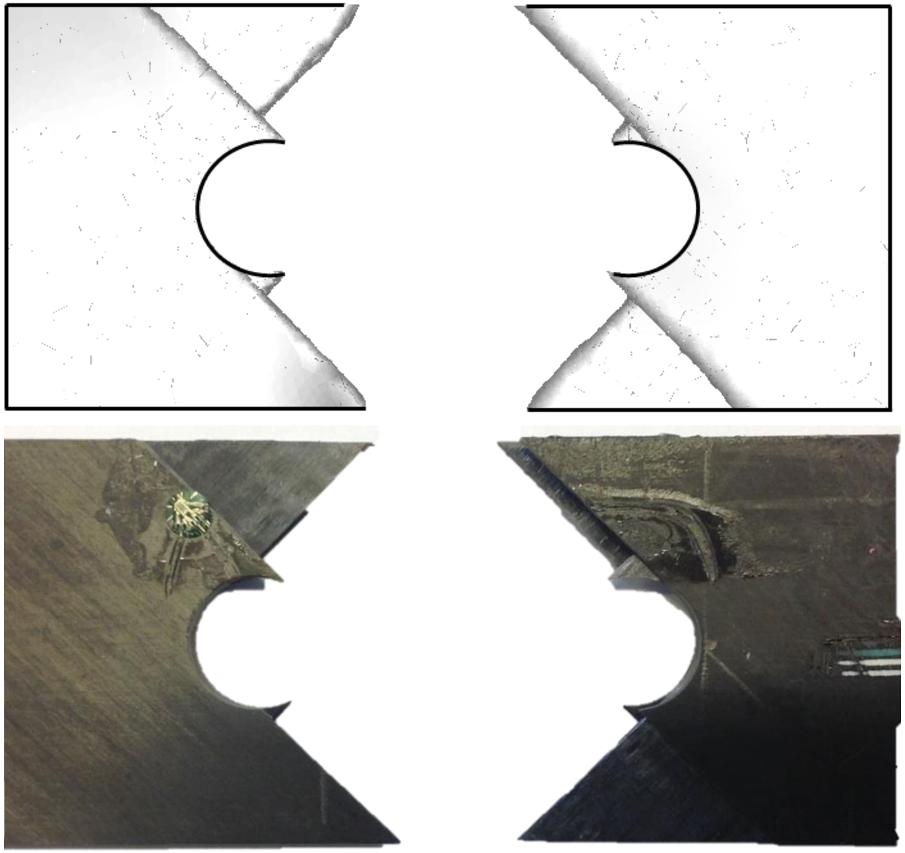

-45º-ply

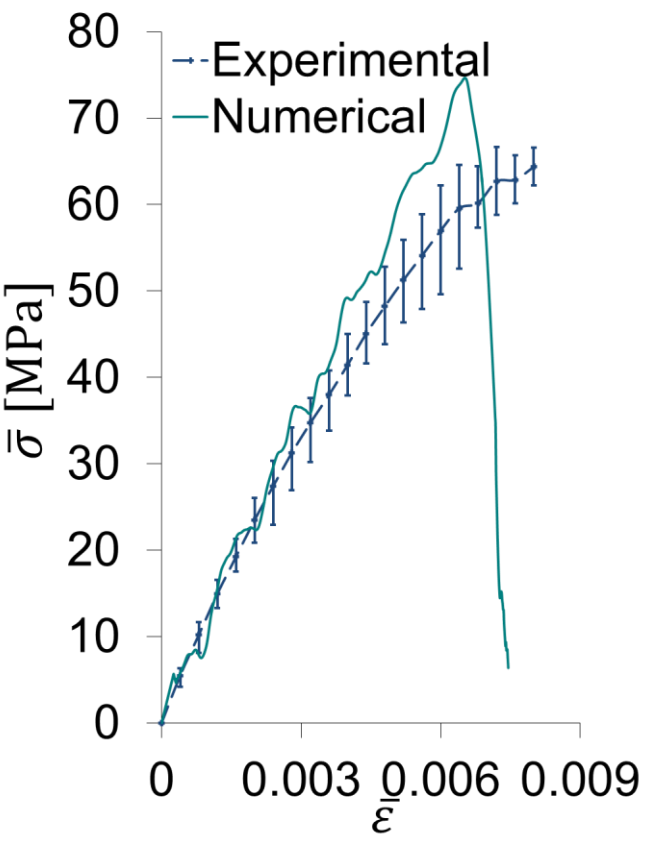



45º-ply

# Experimental validation

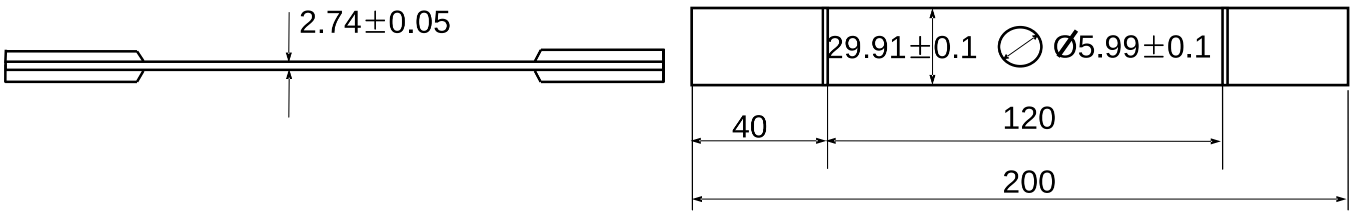
- $[45^o{}_4 / -45^o{}_4]_S$- open hole laminate (2)
  - Predicted delamination zones in agreement with experiments
  - Tensile stress within 15 %



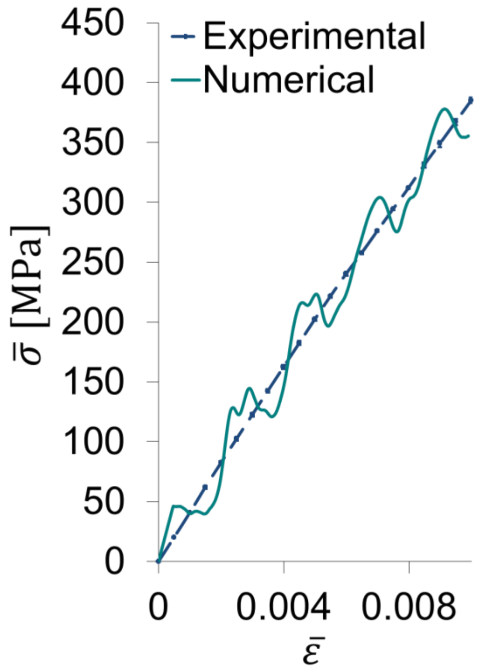


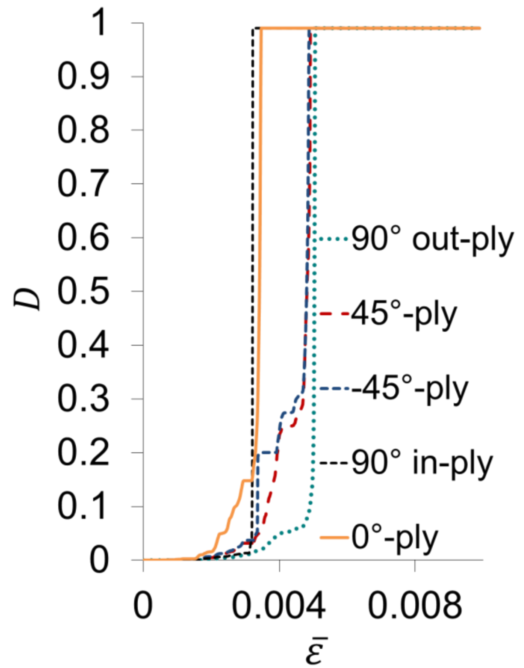
# Experimental validation

- [90º / 45º / -45º / 90º / 0º]$_S$- open hole laminate
  - Tensile test on several coupons




  - Predicted response (stress & maximum damage in each ply)





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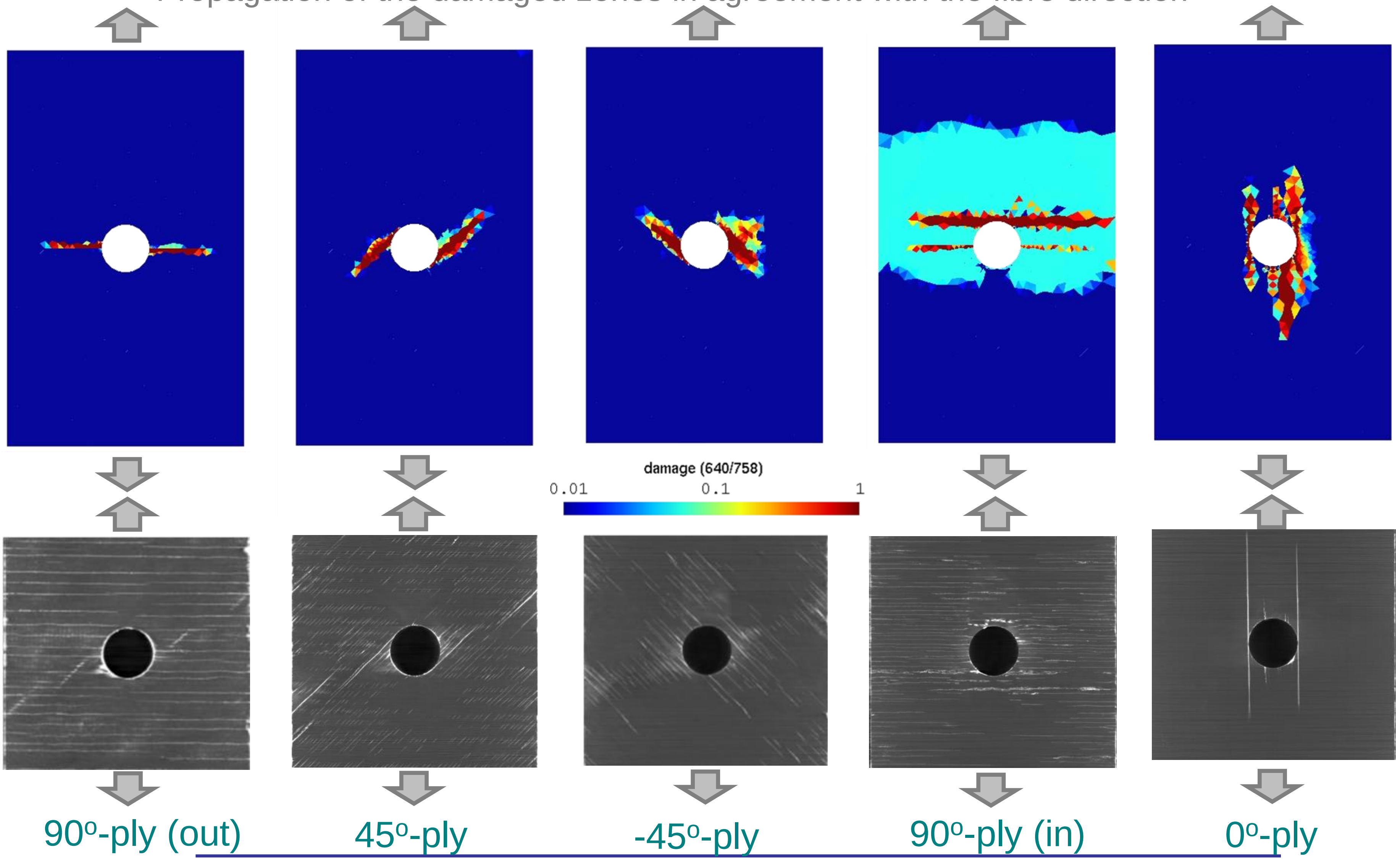
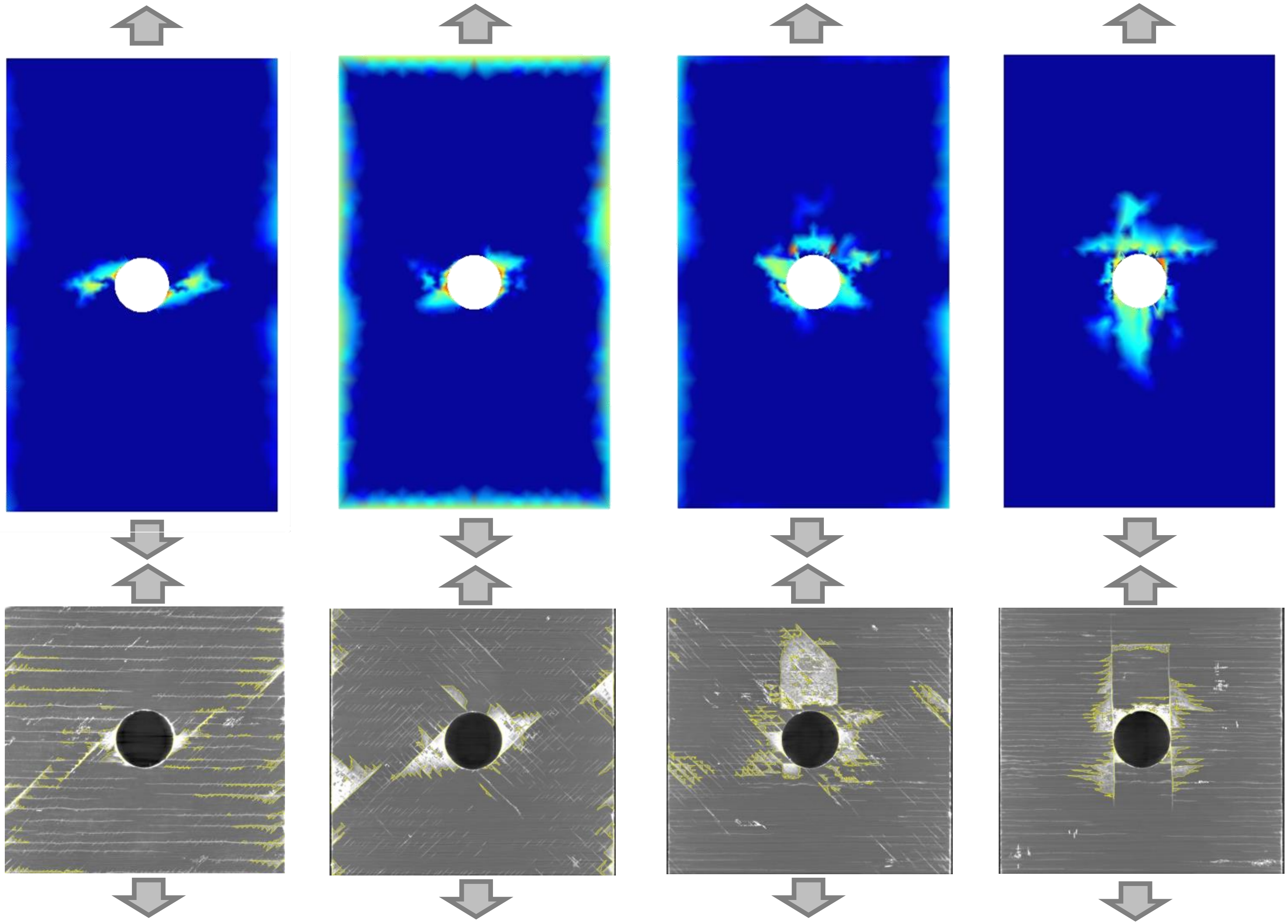
# Experimental validation

- $[90^o / 45^o / -45^o / 90^o / 0^o]_S$- open hole laminate (2)
  - Propagation of the damaged zones in agreement with the fibre direction



# Experimental validation

- $[90^o / 45^o / -45^o / 90^o / 0^o]_S$- open hole laminate (3)
  - Predicted delamination zones in agreement with experiments

$90^o$ (out) / $45^o$ | $45^o$ / $-45^o$ | $-45^o$ / $90^o$ (in) | $90^o$ (in) / $0^o$

# Conclusions

- Multi-scale method for the failure analysis of composite laminates
  - Damage-enhanced MFH
  - Non-local implicit formulation
  - Hybrid DG/CZM for delamination

- Experimental validation
  - Open-hole laminates
  - Different stacking sequences

- Papers
  - Can be downloaded on
    - www.ltas-cm3.ulg.ac.be