A probabilistic multi-scale model for polycrystalline MEMS resonators

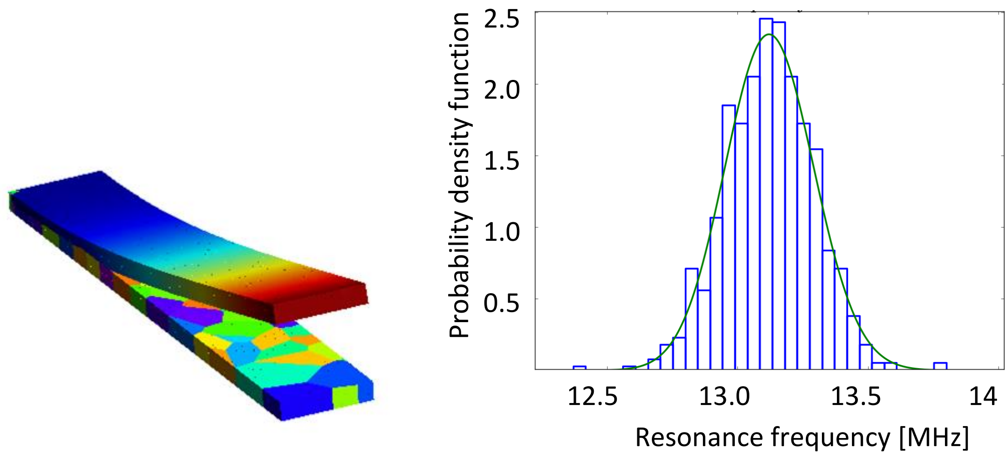
V. Lucas L. Wu[†], S. Paquay[‡], J.-C. Golinval[†], L. Noels[†]

[†]Department of Aerospace and Mechanical Engineering
University of Liege, Chemin des Chevreuils 1, B-4000 Liege, Belgium
{vincent.lucas, l.wu, jc.golinval, l.noels}@ulg.ac.be

[‡]Open Engineering SA,
Rue des Chasseurs-Ardennais 3, B-4031, Liège (Angleur), Belgium.
s.paquay@open-engineering.com

ABSTRACT

The size of micro-electro-mechanical systems (MEMS) is only one or two orders of magnitude higher than the size of their micro-structure, *i.e.* their grain size. As a result, the structural properties exhibit a scatter. As an example we study the beam resonator illustrated in Fig. 1(a), made of poly-silicon material, in which each grain has a random orientation. Solving the problem with a full direct numerical simulation combined to a Monte-Carlo method allows the probability density function to be computed as illustrated in Fig. 1(b). However this methodology is computationally expensive due to the number of degrees of freedom required to study one sample, motivating the development of a non-deterministic 3-scale approach [3].



(a) FEM (b) Probability distribution

Figure 1: Non deterministic prediction of the first resonance frequency of a beam resonator (a) Finite element model (FEM) of the beam resonator. (b) Probability density function obtained using a Monte-Carlo method with a random grain distribution.

In a multiscale approach, at each macro-point of the macro-structure, the resolution of a micro-scale boundary value problem relates the macro-stress tensor to the macro-strain tensor. At the micro-level, the macro-point is viewed as the center of a Representative Volume Element

(RVE). The resolution of the micro-scale boundary problem can be performed using finite-element simulations, as in the computational homogenization framework, *e.g.* [2]. However, to be representative, the micro-volume-element should have a size much bigger than the micro-structure size.

In the context of the MEMS resonator, this representativity is lost and Statistical Volume Elements (SVE) are considered. These SVEs are generated under the form of a Voronoï tessellation with a random orientation for each silicon grain. Hence, a Monte-Carlo procedure combined with a homogenization technique allows a distribution of the material tensor at the meso-scale to be estimated. The correlation between the meso-scale material tensors of two SVEs separated by a given distance can also be evaluated.

A generator at the meso-scale based on the spectral method [4] is implemented. The generator [3] accounts for a lower bound [1] of the meso-scale material tensor in order to ensure the existence of the second-order moment of the Frobenius norm of the generated material tensor inverse [5].

Using the random meso-scale field obtained with the meso-scale generator, which accounts for the spatial correlation, a Monte-Carlo method can be used at the macro-scale to predict the probabilistic behavior of the MEMS resonator.

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